Recent Advances in Nonlinear Response Structural Optimization Using the Equivalent Static Loads Method

Oct. 28, 2014

VR&D 2014 Users Conference

Monterey, CA

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Optimization

$$
\begin{aligned}
&\text{Find} && \mathbf{b} \in R^n \\
&\text{to minimize} && f(\mathbf{b}) \\
&\text{subject to} && h_i(\mathbf{b}) = 0 \quad i = 1,\ldots,l \\
& && g_j(\mathbf{b}) \le 0 \quad j = 1,\ldots,m \\
& && \mathbf{b}_{\mathrm{L}} \le \mathbf{b} \le \mathbf{b}_{\mathrm{U}}
\end{aligned}
$$

- Problem formulation is important.
- Do we have to understand the details of the optimization theory?
- Various software systems with various algorithms are available.

Linear Static Response Structural Optimization

$$
\begin{aligned}
&\text{Find} && \mathbf{b} \in R^n, \mathbf{z} \in R^l \\
&\text{to minimize} && f(\mathbf{b}, \mathbf{z}) \\
&\text{subject to} && \mathbf{h}: \ \mathbf{K}(\mathbf{b})\mathbf{z} = \mathbf{f} \\
& && g_j(\mathbf{b}, \mathbf{z}) \le 0 \quad j = 1, \ldots, m \\
& && \mathbf{b}_{\mathrm{L}} \le \mathbf{b} \le \mathbf{b}_{\mathrm{U}}
\end{aligned}
$$

- **Popular**
- **Easy**
- **Well developed software systems are available.**

Linear Static Response Structural Optimization

- Size Optimization: The FEM data are fixed.
- Shape Optimization: Node and element data of FEM analysis are changed during optimization.
- Topology Optimization: Material distribution is optimized.

Not – Linear Static Response Structural Optimization

(1) Dynamic Response Optimization

$$\mathbf{h}: \ \mathbf{M}(\mathbf{b})\ddot{\mathbf{z}} + \mathbf{K}(\mathbf{b})\mathbf{z} = \mathbf{f}$$

(2) Structural Optimization for Multibody Dynamic Systems

$\mathbf{h}$: Governing equation of multibody dynamic system

(3) Structural Optimization for Flexible Multibody Dynamic Systems

$\mathbf{h}$: Governing equation of flexible multibody dynamic systems

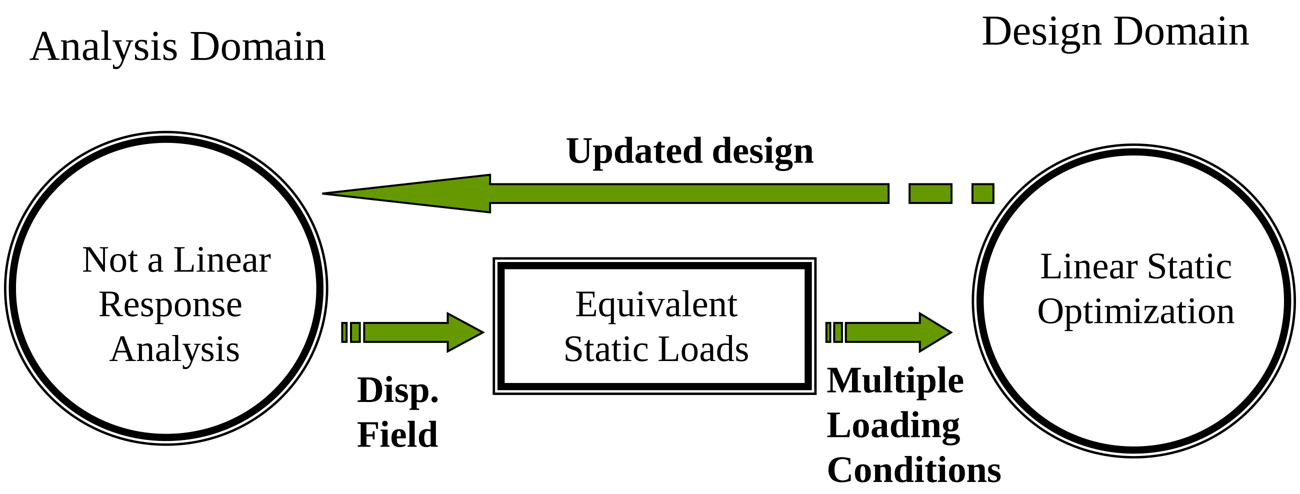
(4) Nonlinear Static Response Structural Optimization

$$\mathbf{h}: \mathbf{K}(\mathbf{b}, \mathbf{z})\mathbf{z} = \mathbf{f}$$

(5) Nonlinear Transient Response Structural Optimization

$$\mathbf{h}: \ \mathbf{M}(\mathbf{b}, \mathbf{z})\ddot{\mathbf{z}} + \mathbf{K}(\mathbf{b}, \mathbf{z})\mathbf{z} = \mathbf{f}$$

The Equivalent Static Loads Method



- This method has been developed for not-linear static response structural optimization.
- Analysis is performed in the analysis domain.
- Equivalent loads are calculated.
- Linear response optimization is performed using the equivalent static loads in the design domain.
- The process proceeds in a cyclic manner.

Contents

(1) Linear Dynamic Response Optimization

(2) Structural Optimization for Multibody Dynamic Systems

(3) Structural Optimization for Flexible Multibody Dynamic Systems

(4) Nonlinear Static Response Structural Optimization

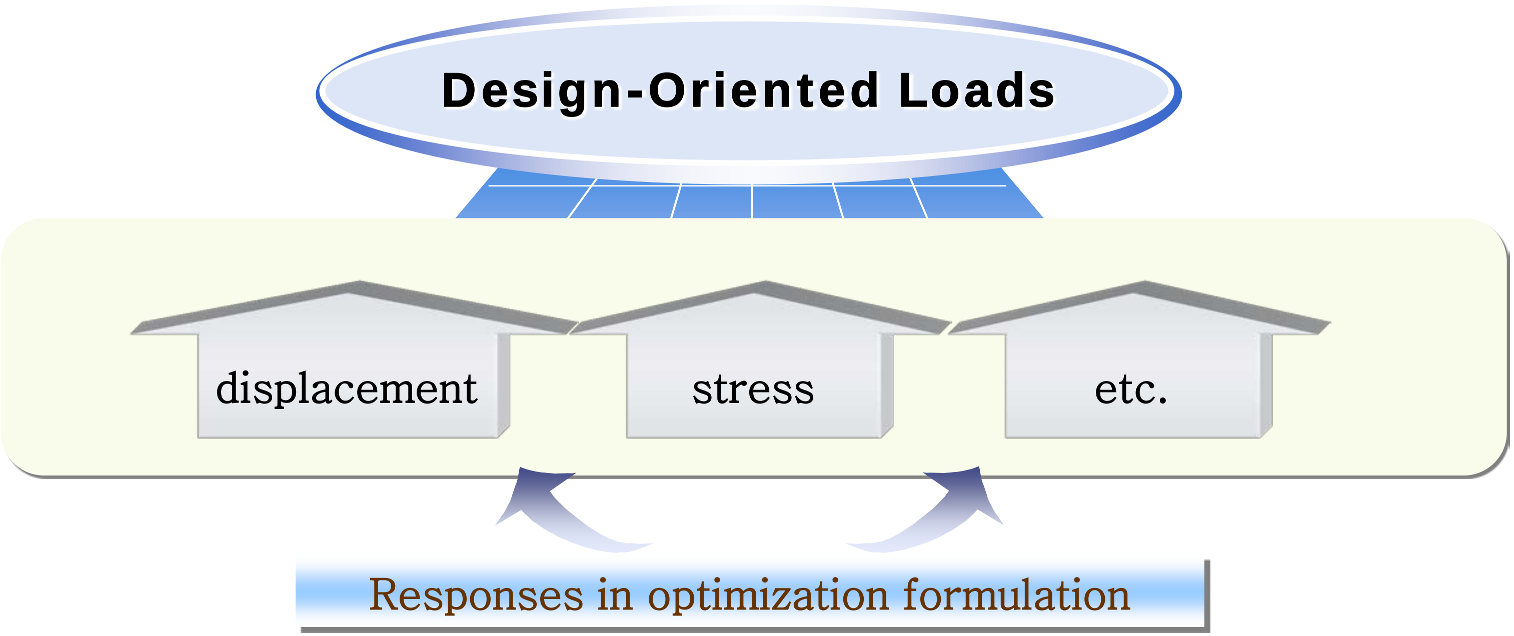
(5) Nonlinear Transient Response Structural Optimization

Nonlinear Static Response Optimization Using Equivalent Loads (NROEL) – Installed in NASTRAN

General Formulation

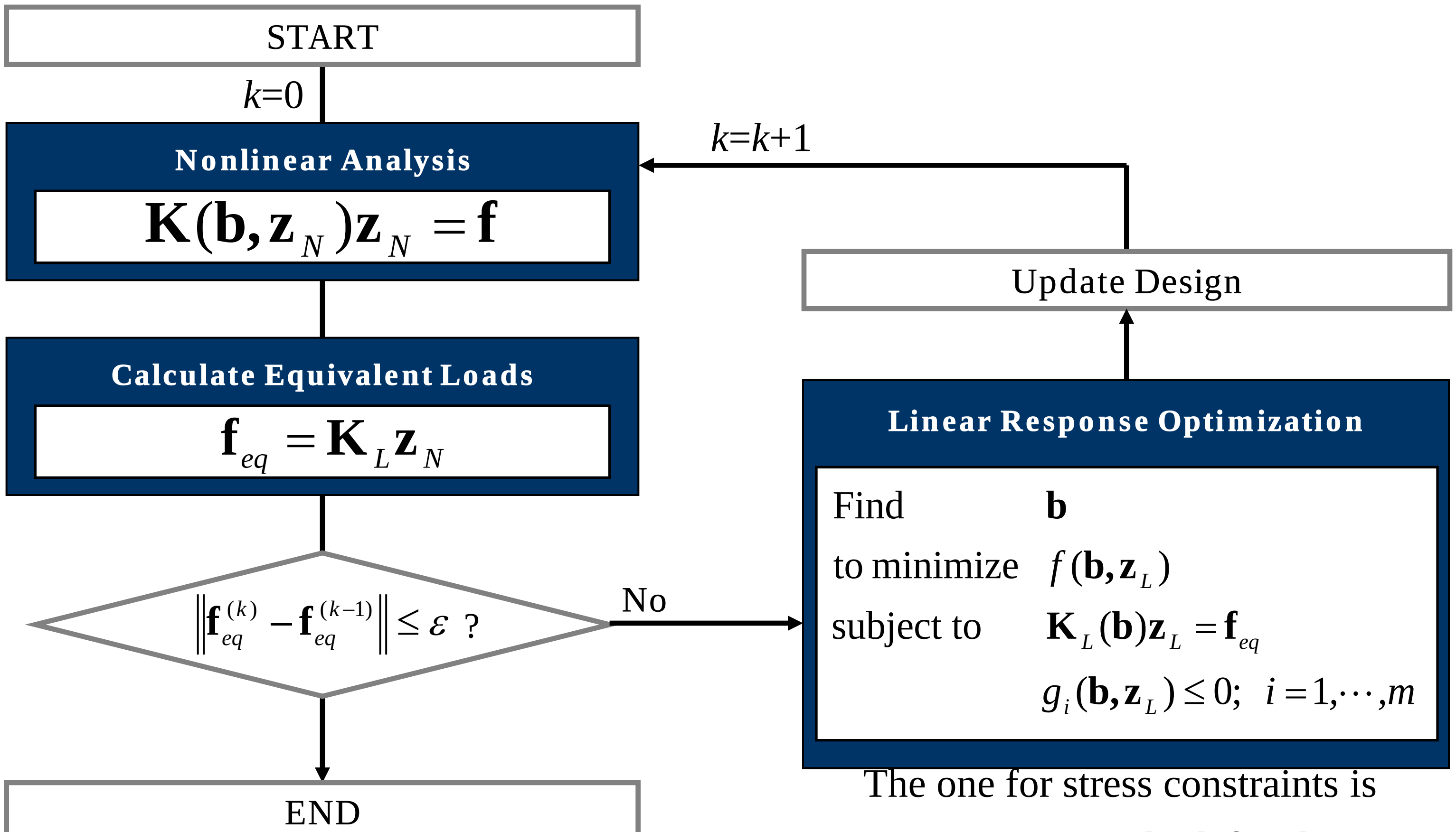
Find	$\mathbf{b}$
to minimize	$f(\mathbf{b},\mathbf{z})$
subject to	$\mathbf{K}(\mathbf{b},\mathbf{z})\mathbf{z}=\mathbf{f}$
	$g_i(\mathbf{b},\mathbf{z})\le 0;\quad i=1,\cdots,m$

Definition: **An Equivalent Load is a load in a linear static system that makes an identical response to that in a nonlinear system.**



Flow of NROEL

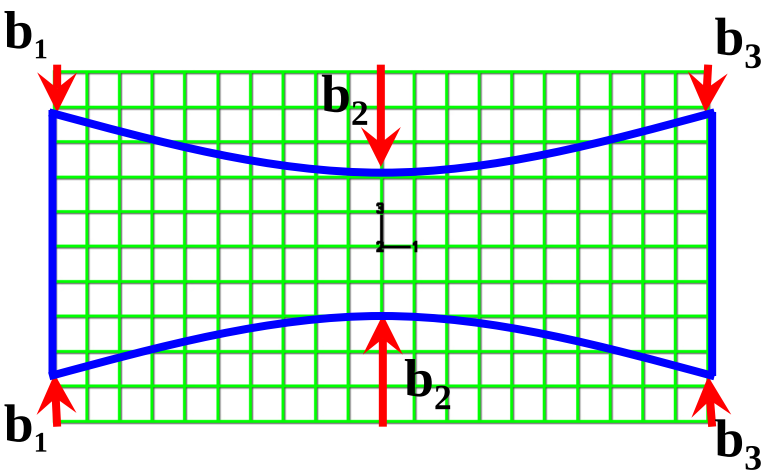
Optimization process using equivalent loads



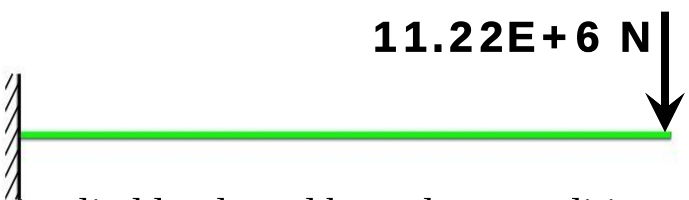
The one for stress constraints is separately defined.

2. A plate

- **Shape change**
  – using domain element



<D.V. and perturbation of the shape >



<Applied loads and boundary conditions>

- **Geometric and Material Nonlinearity**

-Linear hardening

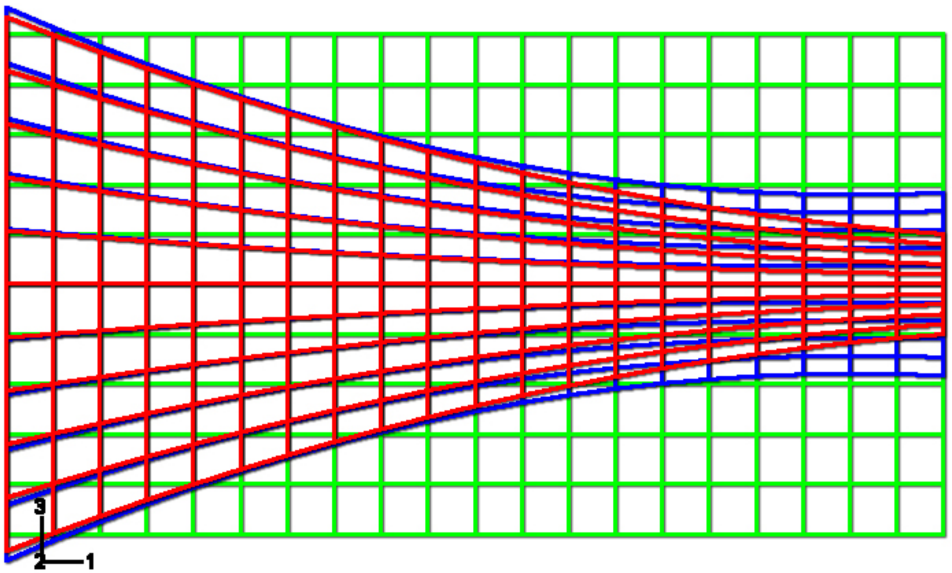
- ✓ E = 200.0 GPa
- ✓ σ_y = 300.0 Mpa
- ✓ E_h = 50.0 GPa

Nonlinear Response Optimization

Find	b_1, b_2, b_3 (shape change)
to minimize	Mass
subject to	$\mathbf{K}(\mathbf{b},\mathbf{z}_N)\mathbf{z}_N = \mathbf{f}$
	$\sigma_j / 350. - 1.0 \le 0;\ j = 1,\cdots,200$

Shape Optimization 1

Results of Optimization

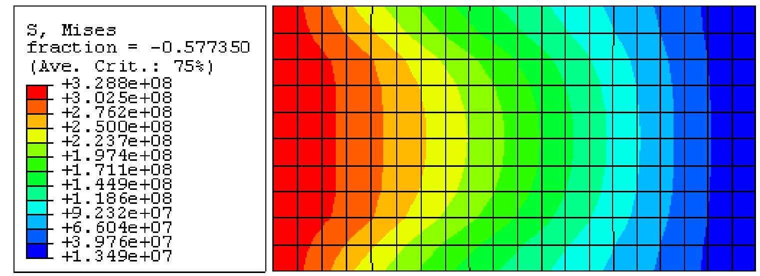


Initial design

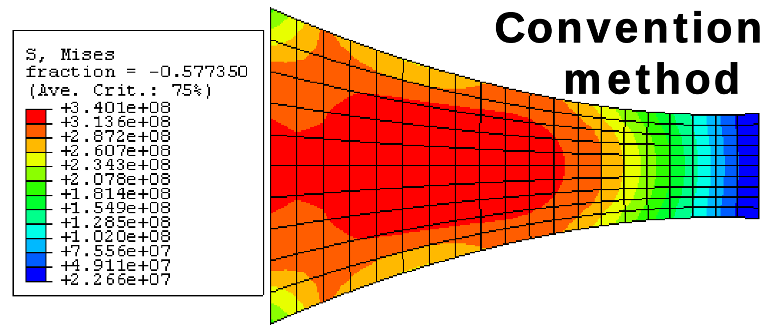
Con. Meth. (215)

NROEL (7)

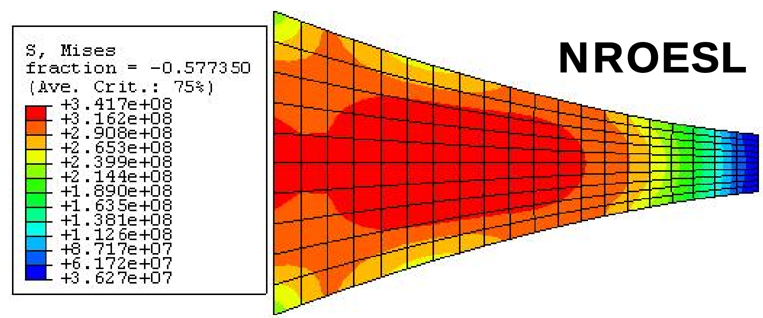
Initial design



Conventional method

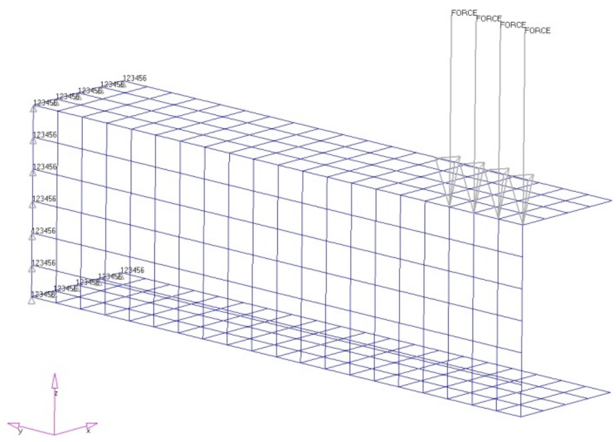


NROESL



4. 280 shell structure

- **Loading and Boundary Conditions**



- **Only Geometric Nonlinearity**

✓ E = 68.9 GPa

Nonlinear Response Optimization

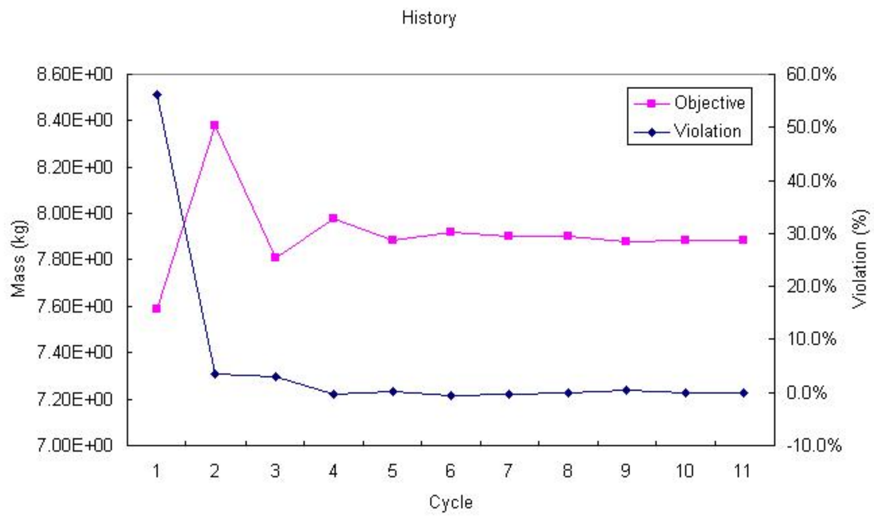
Find	t_i	$(i=1,\cdots,280)$
to minimize	Mass	
subject to	$\mathbf{K}(\mathbf{b},\mathbf{z}_N)\mathbf{z}_N=\mathbf{f}$	
	$\sigma_i/60.-1.0\le 0$	$(i=1,\cdots,280)$
	$\lvert\delta_j\rvert\le 0.005$	$(j=1,\cdots,315)$

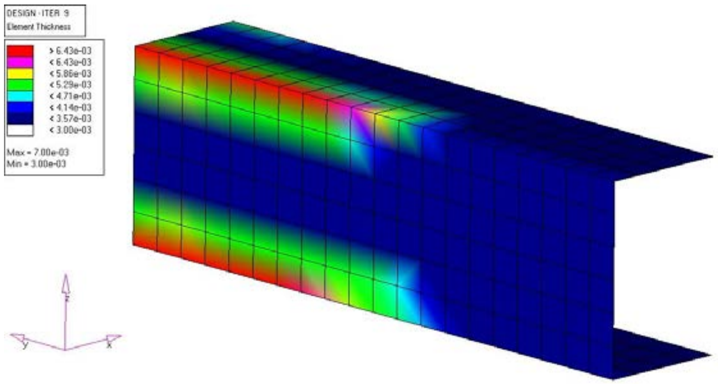
- **NRO using EL**

Abaqus 6.4 – Optistruct 7.0

Large Scale Size Optimization 1

Results of Optimization





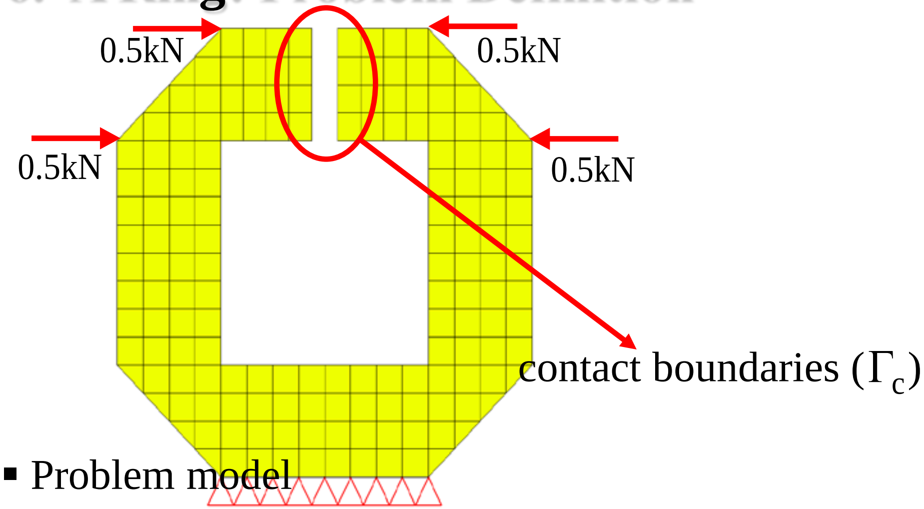
< Design history graph >

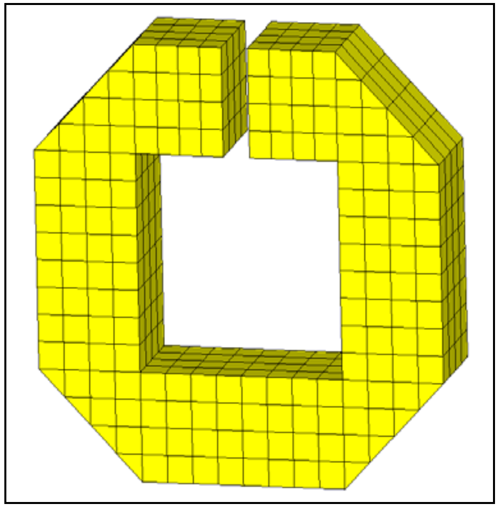
< Optimum thickness contour >

Optimization using the conventional method is fairly expensive.

Shape Optimization with Linear Contact

6. A Ring: Problem Definition

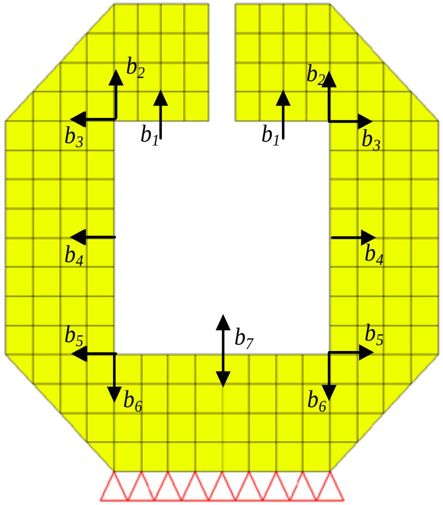




- Problem model
  - Loading condition: The forces are applied at the elements of the upper parts.
  - The element property: PSOLID
  - The total number of elements: 672 (64 CPENTA + 608 CHEXA)
  - Only the boundary nonlinearity is considered.
  - NASTRAN is used for contact analysis and linear response optimization.
  - NASTRAN DMAP is utilized for calculating the equivalent loads.
- Solver: SOL 101
  - Linear contact: Linear analysis + Nonlinear contact parameters

Shape Optimization with Linear Contact

Design condition



Formulation

Find	$b_1, b_2, \cdots, b_7$ (shape of the ring)
to min.	mass
subject to	$\lvert\sigma_i\rvert - 2.0\,\text{KPa} \le 0 \qquad (i = 1, \cdots, 672)$

- Design perturbation vectors

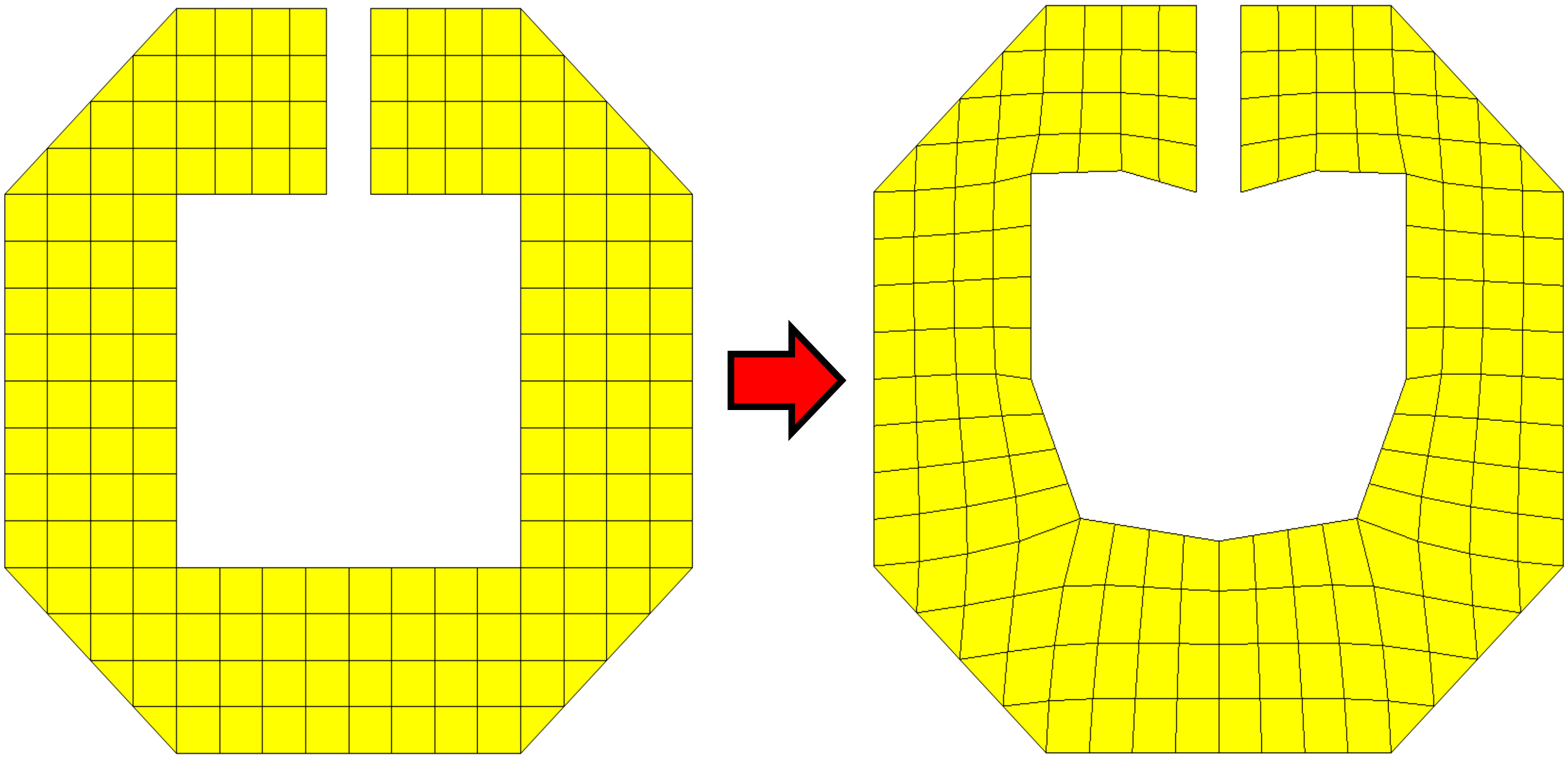
  - Perturbation vectors are utilized for shape change in shape optimization.

  - Each arrow is a perturbation vector.

  - Seven design variables are selected based on the perturbation vectors.

Shape Optimization with Linear Contact

Optimization result – shape change



Initial shape

Optimum shape

Nonlinear Dynamic Response Optimization Using Equivalent Static Loads – Installed in GENESIS and OptiStruct

Equivalent Static Loads for Displacement

$$\mathbf{M}\ddot{\mathbf{z}}_{\mathrm{N}}(\mathrm{t}) + \mathbf{K}(\mathbf{b}, \mathbf{z}_{\mathrm{N}}(\mathrm{t}))\mathbf{z}_{\mathrm{N}}(\mathrm{t}) = \mathbf{f}(\mathrm{t})$$

Nonlinear transient analysis

$\mathbf{z}_{\mathrm{N}}(\mathrm{t})$

$$\mathbf{f}_{eq}^{\mathrm{z}}(\mathrm{t}) = \mathbf{K}_{\mathrm{L}}\mathbf{z}_{\mathrm{N}}(\mathrm{t})$$

Transformation to equivalent loads

$\mathrm{t} \rightarrow \mathrm{s}$

$$\mathbf{K}_{\mathrm{L}}(\mathbf{b})\mathbf{z}_{\mathrm{L}}(\mathrm{s}) = \mathbf{f}_{eq}^{\mathrm{z}}(\mathrm{s})$$

Linear analysis with ESLs

Find $\mathbf{b}$

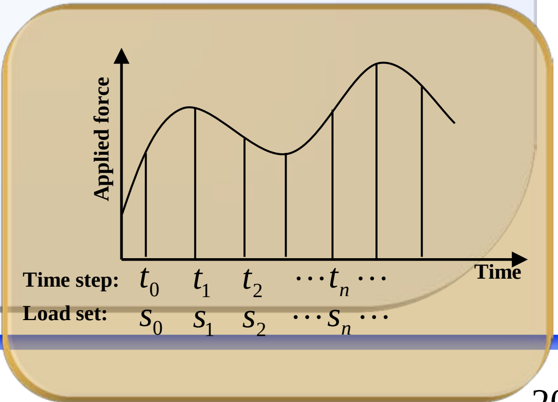
to minimize $F(\mathbf{b}, \mathbf{z}_{\mathrm{L}})$

subject to $\mathbf{K}_{\mathrm{L}}(\mathbf{b})\mathbf{z}_{\mathrm{L}}(\mathrm{s}) = \mathbf{f}_{eq}(\mathrm{s})$

$\mathbf{z}_{\mathrm{L}}(\mathrm{s}) - \mathbf{z}_{\mathrm{allowable}} \le \mathbf{0}$

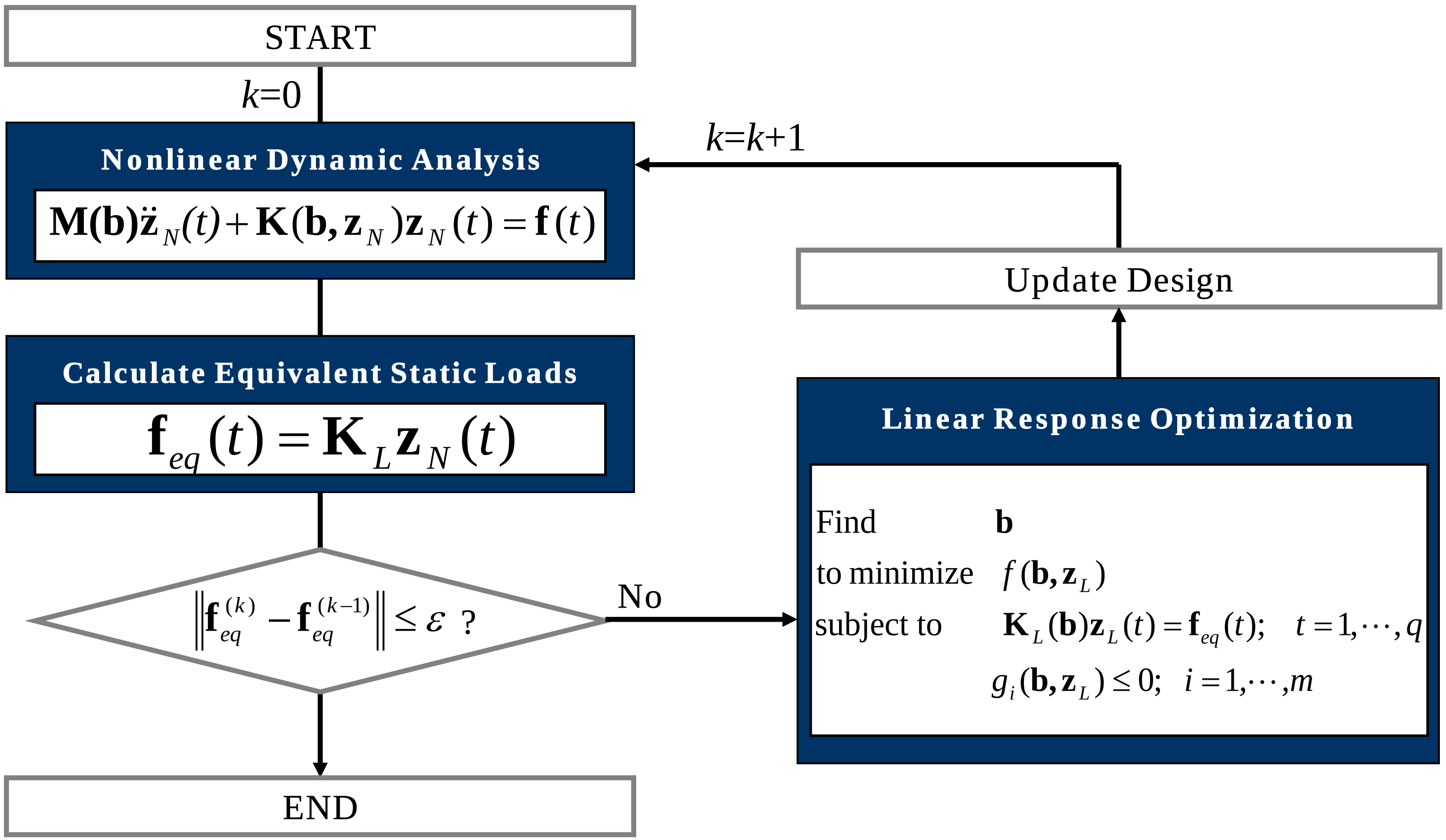
Linear response optimization

$\mathbf{b}_{new}$



Flow of NDROESL

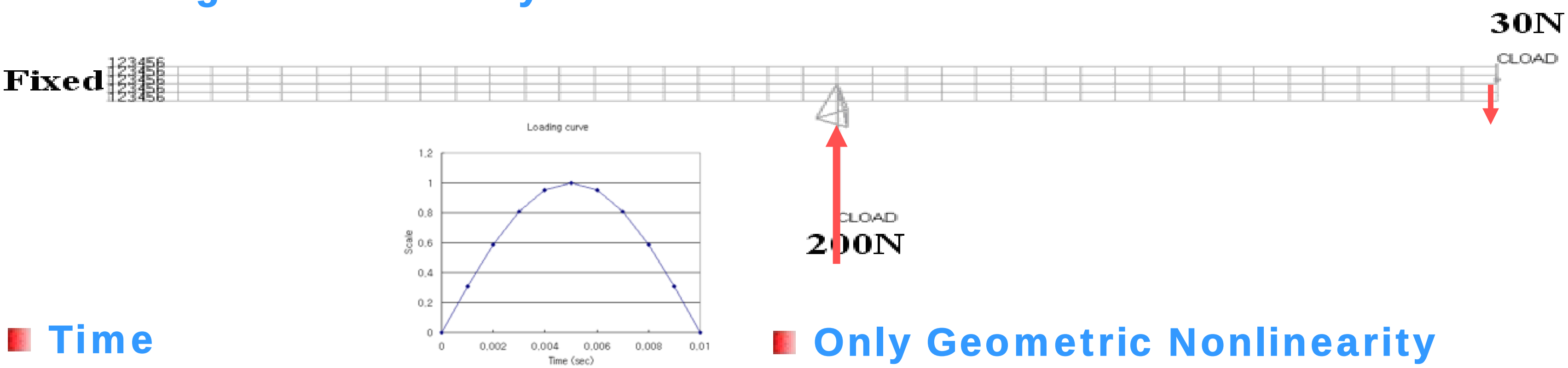
Optimization process using equivalent static loads



Nonlinear Transient Size Optimization

1. 160 shell structure

Loading and Boundary Conditions



Time

Loading time: 0.01 sec

Total analysis time: 2.0 sec

NRO using ESL

Abaqus 6.4 – Optistruct 7.0

Only Geometric Nonlinearity

Nonlinear Response Optimization

Find $t_i \quad (i = 1, \cdots, 160)$

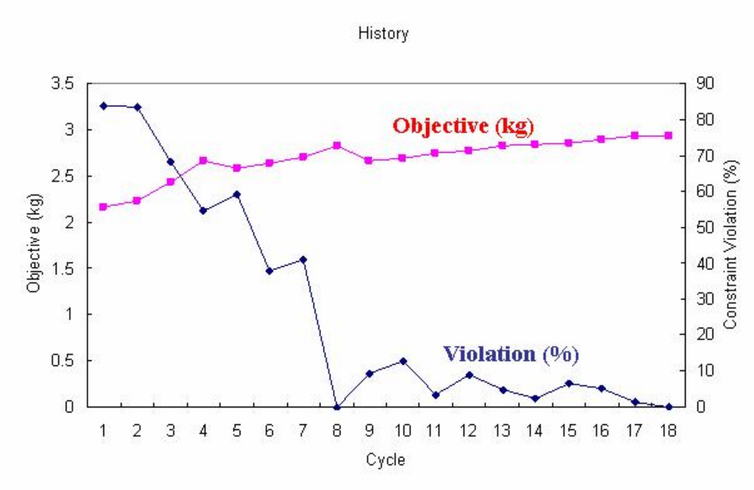
to minimize Mass

subject to $\mathbf{M}(\mathbf{b})\ddot{\mathbf{z}}_N + \mathbf{K}(\mathbf{b}, \mathbf{z}_N)\mathbf{z}_N = \mathbf{f}$

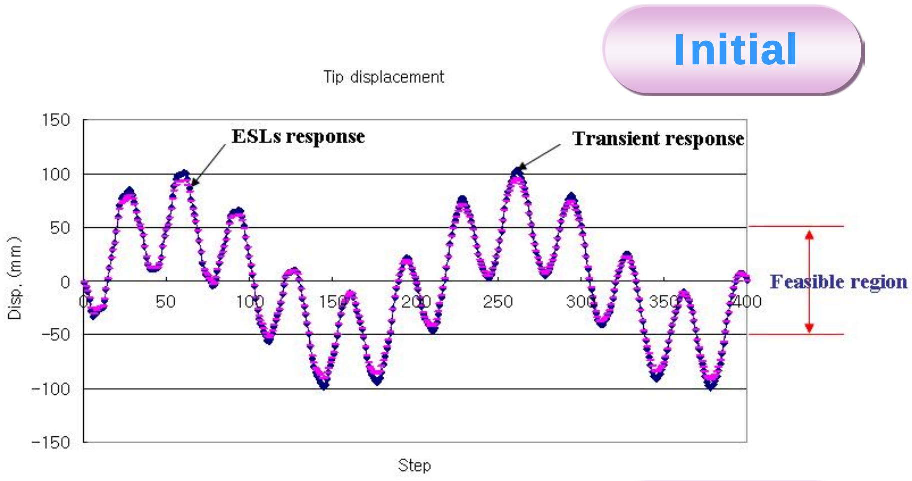
$|\delta_j| \le 0.005 \quad (j = 1, \cdots, 205)$

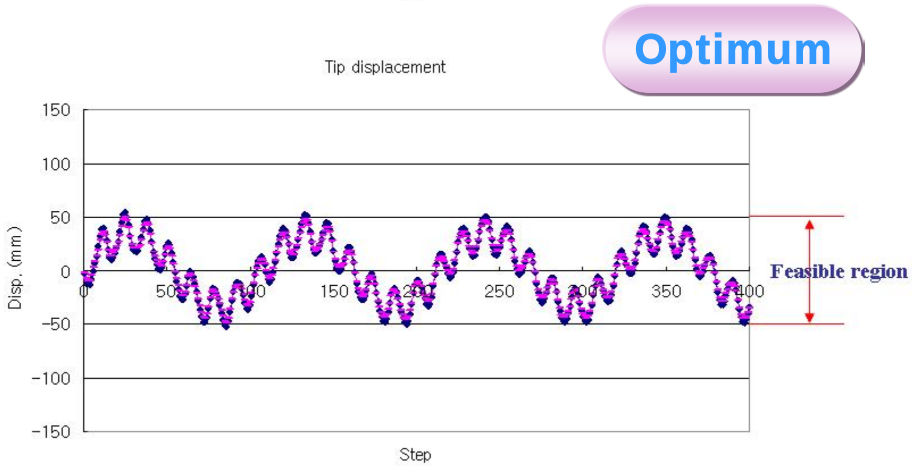
Nonlinear Transient Size Optimization

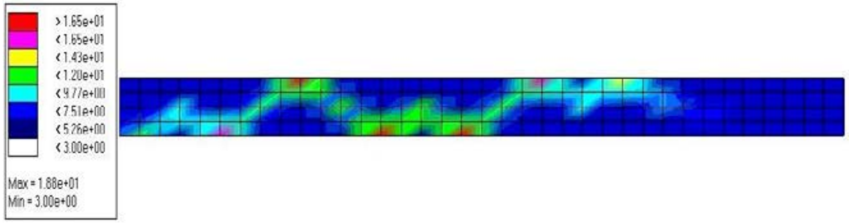
Results of Optimization



< Design history graph >



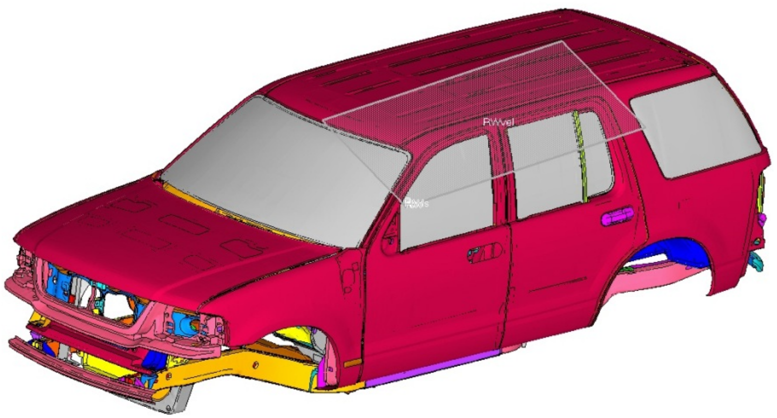




< Optimum thickness contour >

< Tip displacement >

3. Roof crush problem - Ford Explorer Model: developed by the GWU



Finite Element Model

- Number of Parts : 394
- Number of Elements : 432,596
- Number of Nodes : 431,629
- Number of total DOFs : 2,589,774

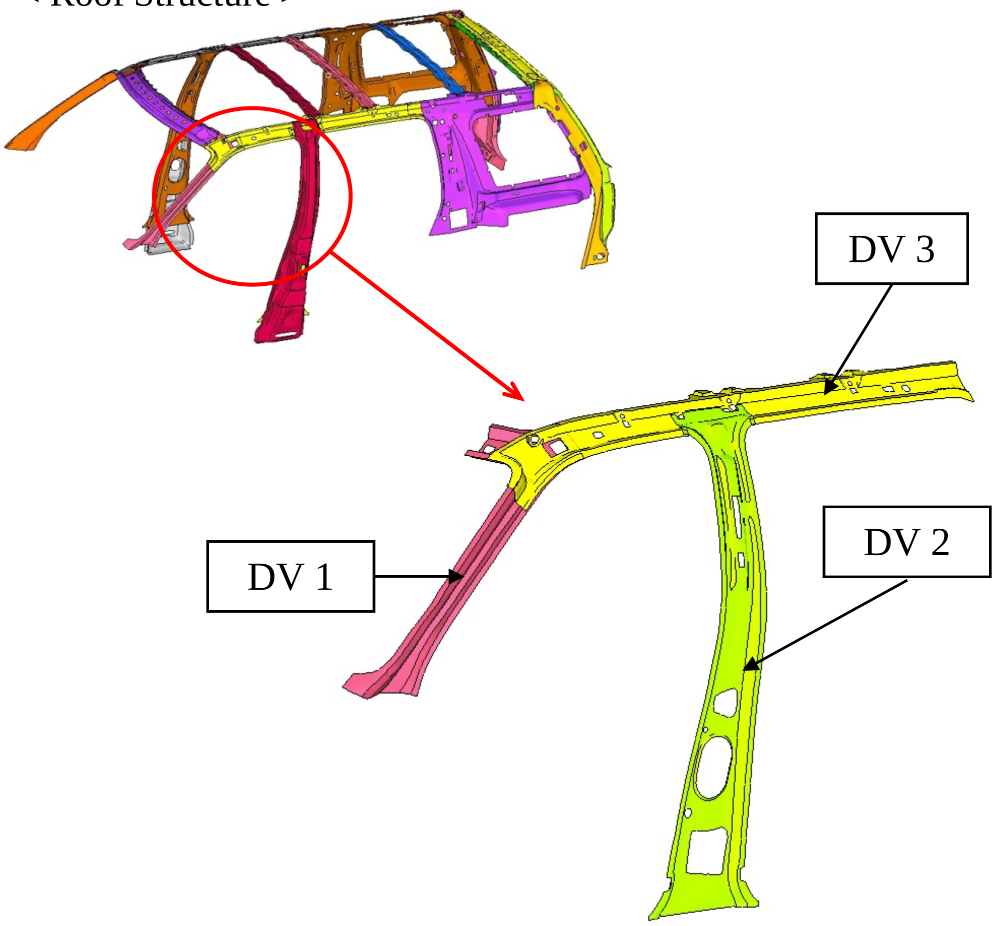
FMVSS 216 Standard (Roof crush resistance)

The current FMVSS 216 standard requires that a passenger car roof withstand a load of 1.5 times the vehicle's unloaded weight in kilograms multiplied by 9.8 or 22,240 Newton's, whichever is less, to either side of the forward edge of the vehicle's roof with no more than 127 mm of crush.

Roof Crush Optimization

Definition of design variables

< Roof Structure >



DV 1: Thickness of A-Pillar (t_1)

DV 2: Thickness of B-Pillar (t_2)

DV 3: Thickness of Roof-Rail (t_3)

Roof Crush Optimization

Modified formulation for the ESL method

Find $t_i \quad (i = 1,2,3)$

to minimize mass

subject to

$$\mathbf{M(b)}\ddot{\mathbf{z}}_N + \mathbf{K(b, z}_N)\mathbf{z}_N = \mathbf{f}$$

$$1.65 \times \text{weight} - \text{rigid wall force} \geq 0.0 \ \ (\text{roof crush} = 127\text{mm})$$

$$0.6 \leq \text{dv}1 \leq 2.0$$

$$0.6 \leq \text{dv}2 \leq 2.0$$

$$0.6 \leq \text{dv}3 \leq 2.0$$

Modify

Find $t_i \quad (i = 1,2,3)$

to minimize mass

subject to

$$\mathbf{M(b)}\ddot{\mathbf{z}}_N + \mathbf{K(b, z}_N)\mathbf{z}_N = \mathbf{f}$$

$$127\text{mm} - \text{distance of the roof crush} \leq 0.0 \ \ (\text{t}_{\text{step}} = 67.5\text{ms})$$

$$0.6 \leq \text{dv}1 \leq 2.0$$

$$0.6 \leq \text{dv}2 \leq 2.0$$

$$0.6 \leq \text{dv}3 \leq 2.0$$

Roof Crush Optimization Using RSM and ESL

Response Surface Method

- Software : LS-DYNA 971, LS-OPT
- Linear + Interaction terms are used for RSM.
- D-Optimal method as the sampling method is used.
- The number of experimental points is eight.
- Nonlinear analysis time : about 30 hours (1CPU) for a full car model
- The CPU time per 1 iteration : about 240 hours

Equivalent Static Loads Method

- LS-DYNA 971 is used for the roof crush analysis.
- DMAP of NASTRAN 2006 is used for the calculation of equivalent static loads.
- NASTRAN 2006 is used for linear static optimization using equivalent static loads.
- Nonlinear analysis time : about 30 hours for a full car model
- Linear optimization time : about 6 hours
- The CPU time per 1 cycle : about 36 hours

* Equipment of solver : HP-UX Itanium II (4CPU)

Roof Crush Optimization Using RSM and ESL

Results

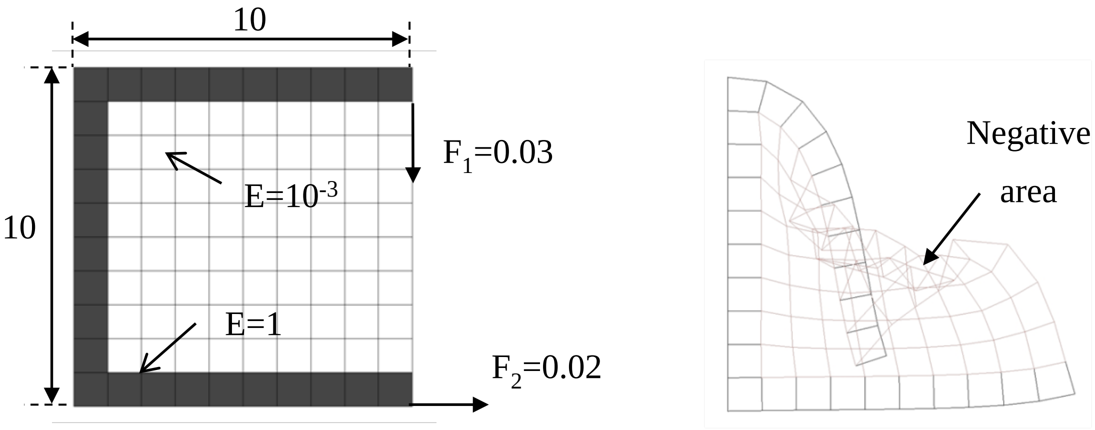
	Initial model	RSM result	ESL result
DV 1	1.2 mm	1.16 mm	0.86 mm
DV 2	1.1 mm	0.6 mm	0.96 mm
DV 3	1.0 mm	0.6 mm	0.6 mm
Mass	4.481 kg	3.346 kg	3.329 kg
Constraint violation		-11.2%	+0.7%
Number of nonlinear analyses		33	5
Number of iterations		4	
Number of cycles			5
Total CPU time (1CPU)		990 hours	180 hours

Nonlinear Dynamic Response Topology Optimization Using the Equivalent Static Loads

Issues of Nonlinear Dynamic Response Topology Optimization

*** Mesh distortion problem**

- Low-density elements appear during and even after the optimization process.
- Low-density elements cause excessive mesh distortion.
- This phenomenon leads to many Newton-Raphson iterations or divergence in the numerical analysis.



(a) Finite element model (b) Deformed shape

Example of unstable elements under large deformation

Issues of Nonlinear Dynamic Response Topology Optimization

*** Definition of the objective function**

- The purpose of topology optimization

  Maximization of the stiffness of the structure = Minimization of the compliance

- The general objective function for linear topology optimization → $\mathrm{f}^{\mathrm{T}}\mathrm{z}$
- When topology optimization in the time domain is performed, the objective functions are as follows:

  1) The weighted summation compliance

$$\sum_{u=1}^{l} \omega_u \left(\mathbf{f}_u^{\mathrm{T}} \mathbf{z}_u \right); \qquad u = 1, \ldots, l$$

  2) The weighted summation compliance near the peaks

$$\sum_{u=1}^{p} \omega_u \left(\mathbf{f}_u^{\mathrm{T}} \mathbf{z}_u \right); \qquad u = 1, \ldots, p$$

l : the number of time steps in the time domain

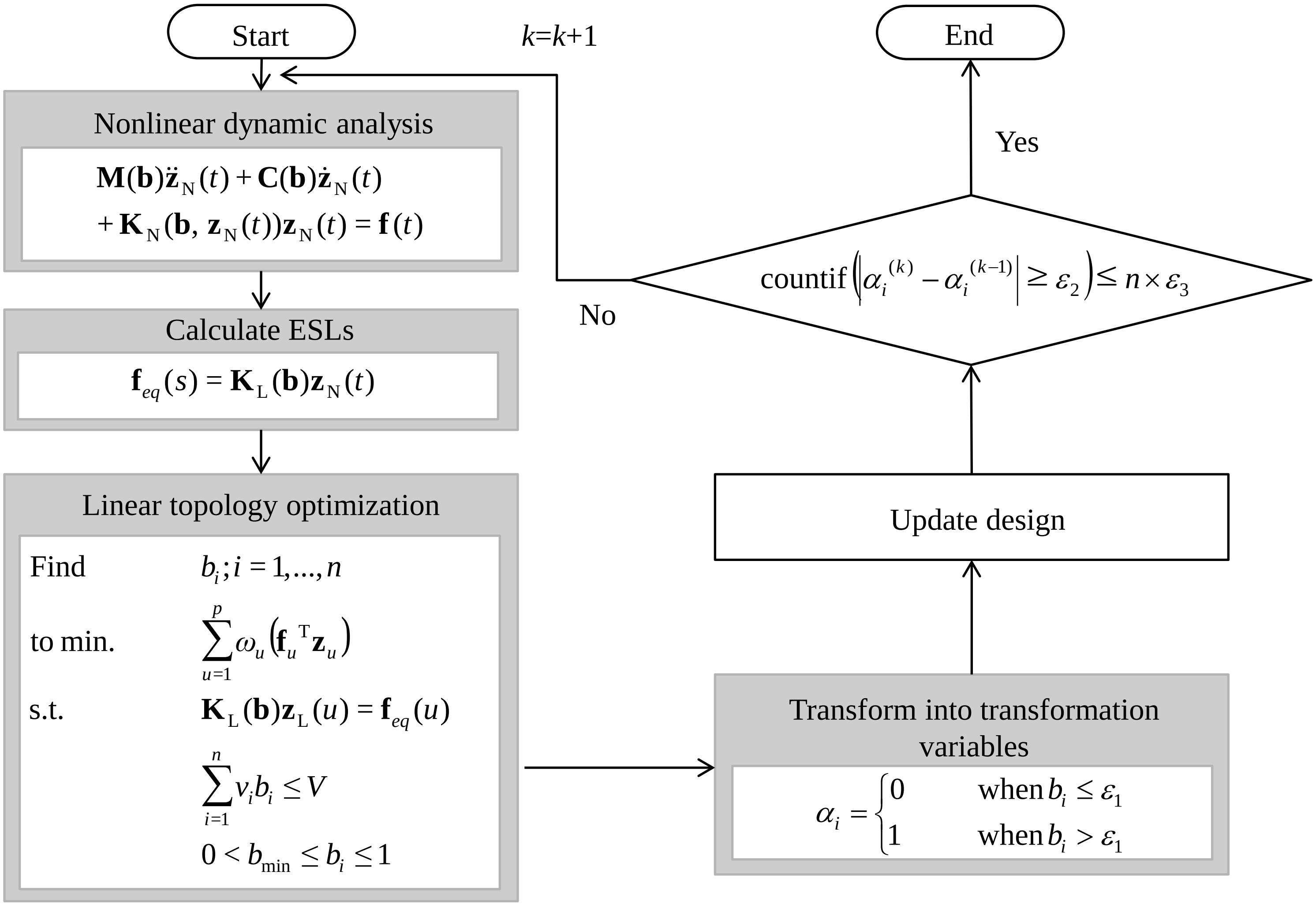
ω_u : the weighting factor

$\mathbf{f}_u$: the magnitude of the dynamic load vector at the *u*th time step

$\mathbf{z}_u$: the displacement vector of the *u*th time step

p : the number of time steps near the peaks

* ESLSO for nonlinear dynamic response topology optimization



Example (I): A Plate Fixed along Both Ends

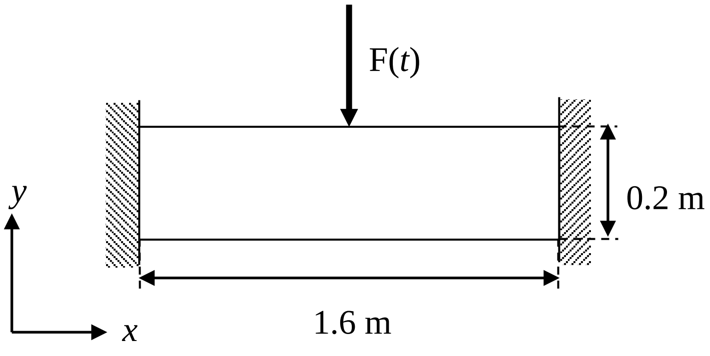
*** Problem definition**

- Information of the problem

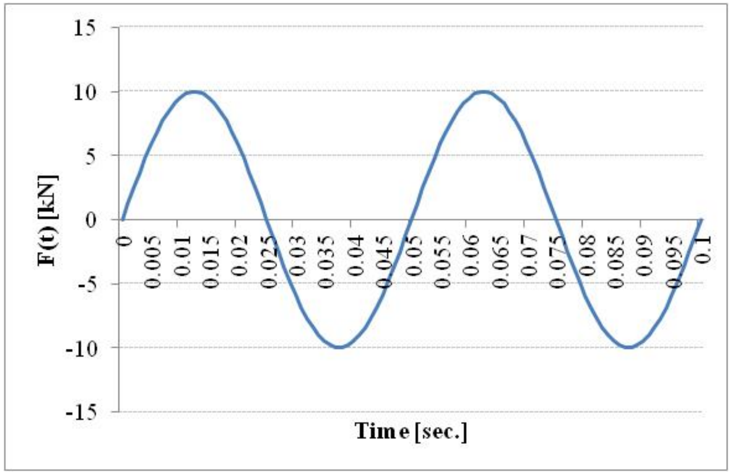
  - The load duration time: 0.1 sec.

  - The maximum magnitude of a dynamic load: 10 kN

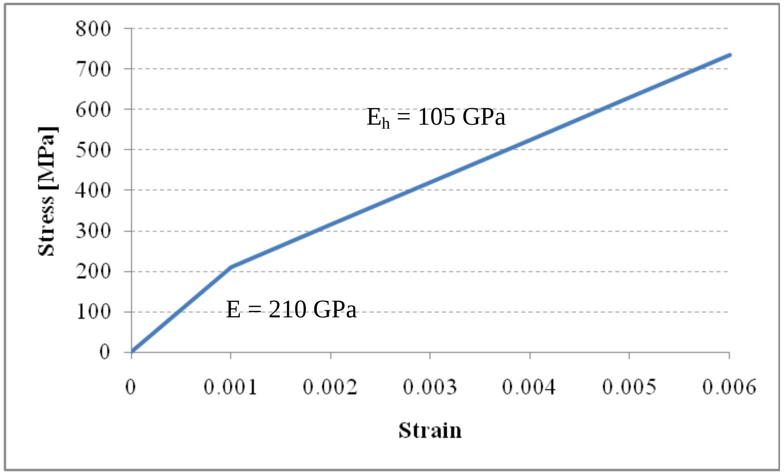
  - Commercial software: ABAQUS (analysis), GENESIS (optimization), NASTRAN (ESLs)



Problem description



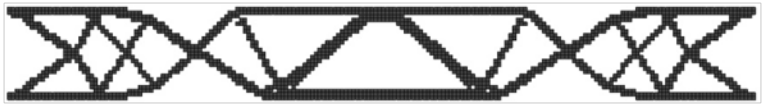
Dynamic load profile



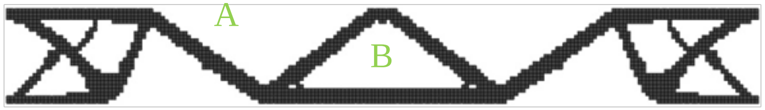
Bilinear elastoplastic stress-strain curve

Example (I): A Plate Fixed along Both Ends

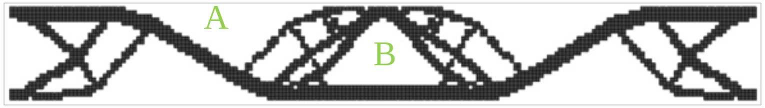
* Optimization results



a) Linear static topology optimization



b) Geometric nonlinearity



c) Material nonlinearity

d) Material and geometric nonlinearity

Optimization results for a plate fixed at both ends

	Linear optimization	Nonlinear dynamic optimization		
		considering geom. nonlinearity	considering mat. nonlinearity	considering geom. & mat. nonlinearities
No. of iterations	19	-	-	-
No. of cycles	-	3	4	3
No. of nonlinear dynamic analyses	-	3	4	3

Example (II): A Crash Box for Crashworthiness

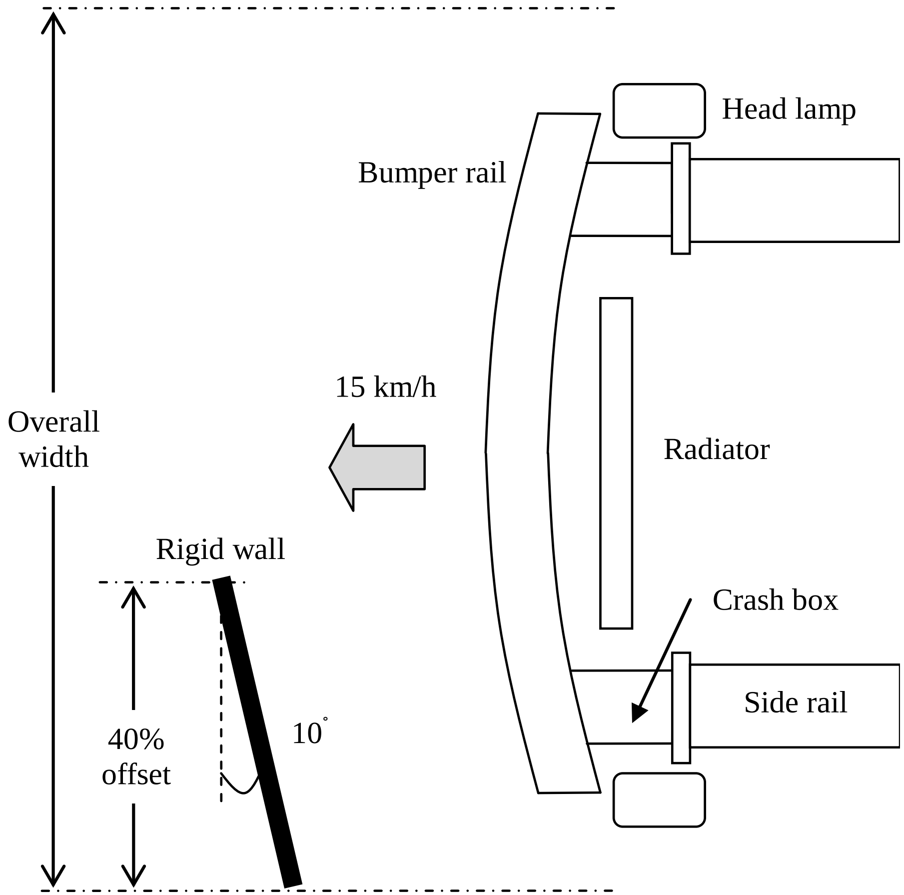
*** Background**

- Crash box

  - Location: Between the bumper rail and the side rail

  - Role: Preventing the transmitted impact energy to the vehicle body by absorbing the energy in the event of a crash.

- RCAR (Research Council for Automobile Repairs): International organization that works toward reducing insurance costs by improving automotive damageability, repairability, safety and security.



RCAR test conditions of the frontal structure

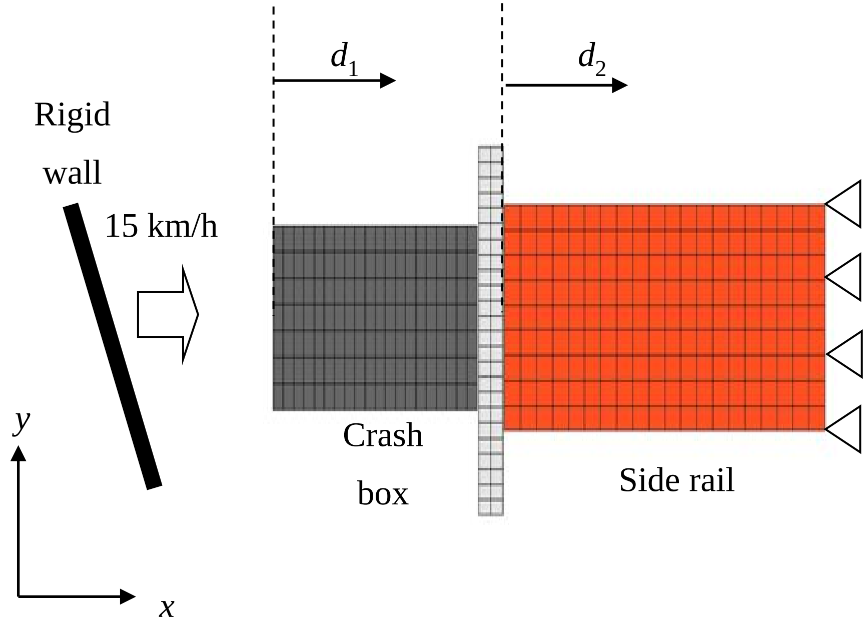
Example (II): A Crash Box for Crashworthiness

*** Problem definition**

- Geometric, material and contact nonlinearities are considered.
- Design domain: Only the crash box
- The objective function: Maximizing the strain energy of the crash box at some time steps near the end time of the impact.

Formulation

Find	$b_i ; i = 1, \ldots, 6804$
to max.	$\sum_{s=1}^{q} \text{strain energy}_s$
s.t.	$\mathbf{K}_{\mathrm{L}}(\mathbf{b})\mathbf{z}_{\mathrm{L}}(s) = \mathbf{f}_{eq}(s); s = 1, \ldots, q$
	$\sum_{i=1}^{6804} v_i b_i \le V_{\mathrm{ref}} \times 50\%$
	$d_1 \le d_{1,\mathrm{allowable}}$
	$d_2 \le d_{2,\mathrm{allowable}}$
	$0 < b_{\min} \le b_i \le 1$



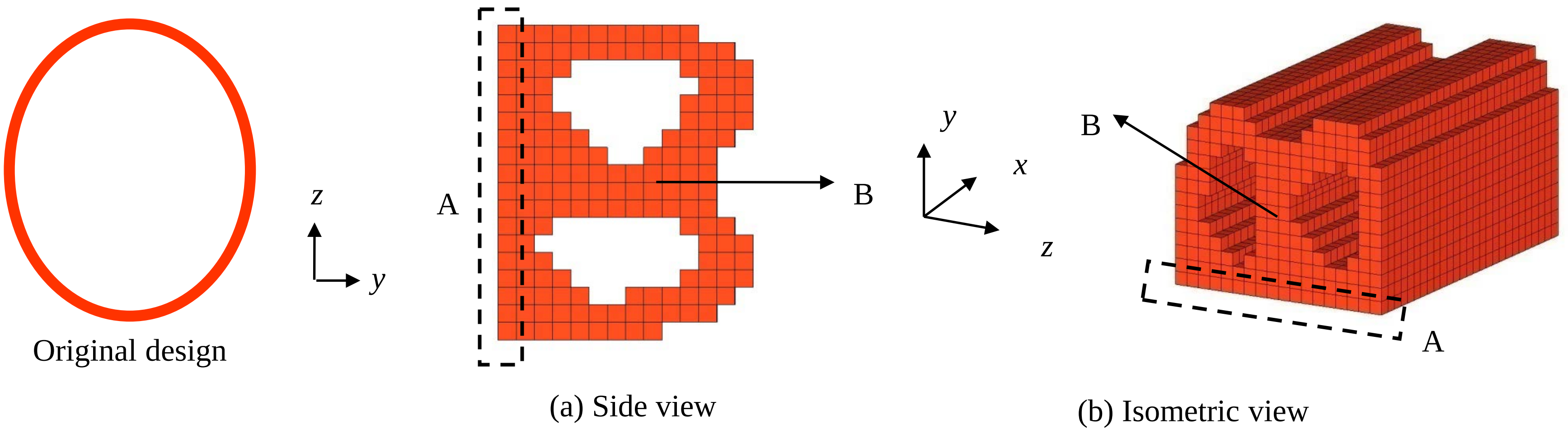
Finite element model of the crash box

Commercial software: LS-DYNA (analysis), GENESIS (optimization), NASTRAN (ESLs)

Example (II): A Crash Box for Crashworthiness

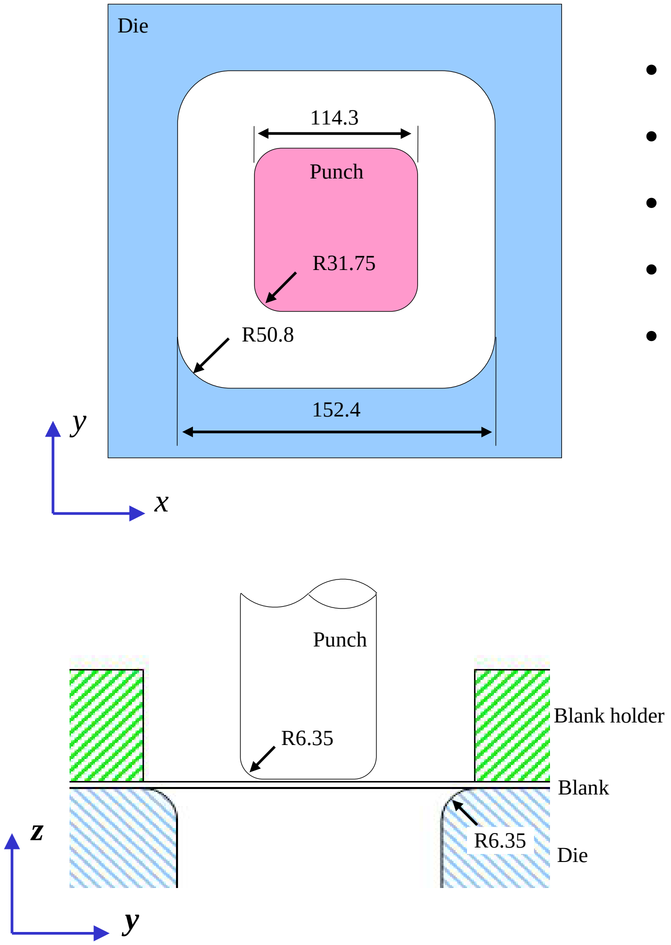
*** Optimization results**

- Optimization process converges in 7 cycles.
- Part A: Primary contacted part when the crash box is impacted on the rigid wall
- Part B: Increasing the absorbed energy of the crash box



Optimization results of the crash box

Sheet Metal Forming with a Tapered Square Cup



Geometric description of the tooling
for the oblique square cup

- The tooling: the die, the punch and the blank holder
- Blank holding force: 100 kN
- Stroke of the punch: 40 mm (-z direction)
- Wrinkling occurrence part: the side-wall
- Reason of wrinkling occurrence at the side-wall

  \- The geometry of the wall is not constrained by the die and the punch.

* Formulation

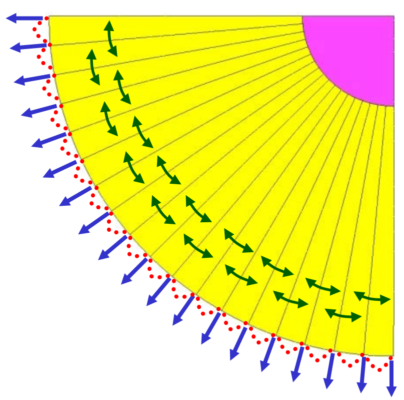
Find $b_i \quad (i = 1, 2, \ldots, 21)$

to minimize $s_j \quad (j = 1, 2, \ldots, 700)$

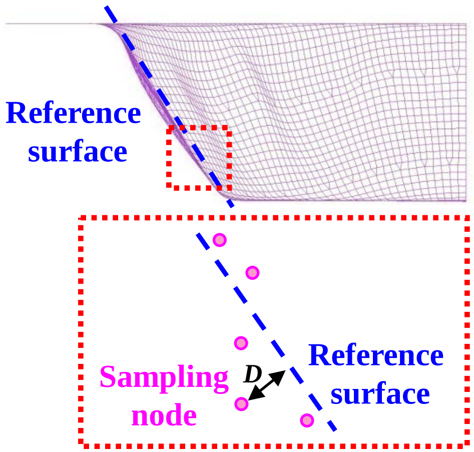
subject to $D_j \leq 10.0$

$$\text{where, } s_j = \left[\frac{1}{699} \left(\sum_{j=1}^{700} \left(D_j - \bar{D} \right)^2 \right) \right]^{1/2}$$

- b_i: the scale factors for the perturbation vectors
- s_j: standard deviation
- i: the perturbation vector number
- j: the sampling node number at the side-wall
- D_j: the distance between sampling nodes and the reference surface
- The reference surface is made on the assumption that the wrinkling disappeared.



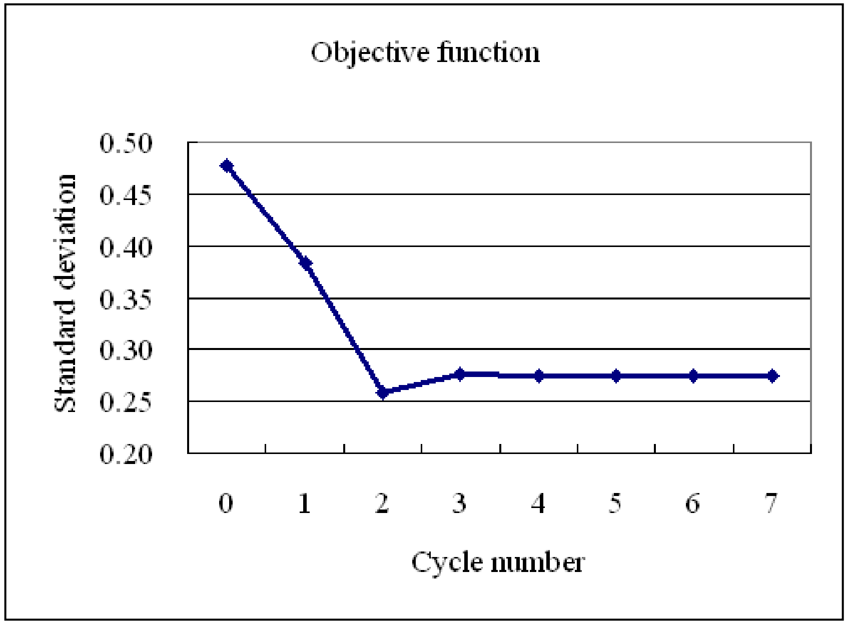
The initial blank shape and perturbation vectors



The distance between the sampling nodes and reference surface

Sheet Metal Forming with a Tapered Square Cup

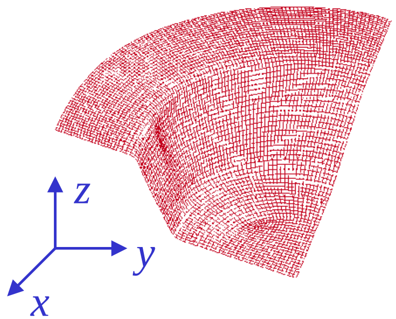
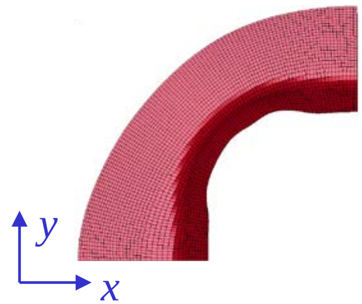
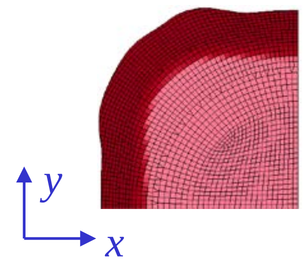
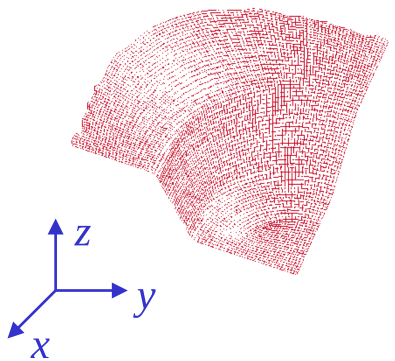
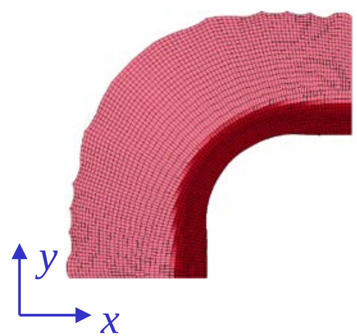
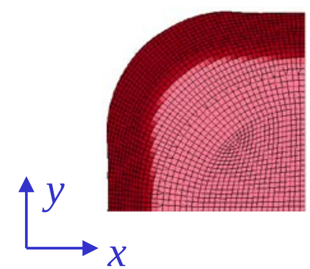
*** Results**



- The ranges of each design variable are changed after the second cycle.
- The move limit strategy is used.
- Objective function: 0.4781 → 0.2741 (convergence criteria: 2.0%)

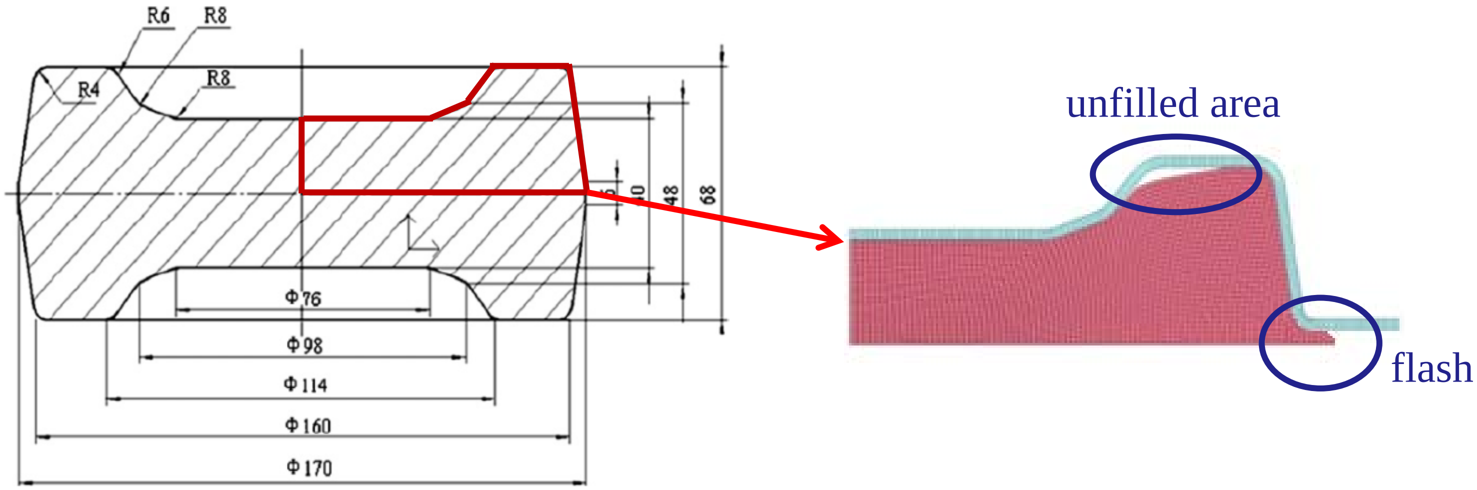
Sheet Metal Forming with a Tapered Square Cup

*** Results**

	Shape of blank after the sheet metal forming	Split plane – height 20 mm		Standard deviation
		Clip (+)	Clip (-)	
Initial model				0.4781
Optimum model				0.2741

Optimization of the Prefrom Shape for H Shape Forging

*** Problem information**

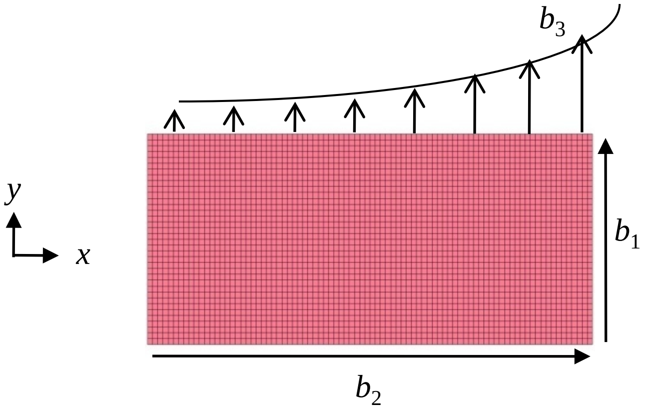


- Model: ¼ H shape forging
- The type of element: the axe-symmetric 2D solid element
- The unfilled area: low quality of the product
- The flash: cost is high because of material loss
- For reduction of the unfilled area and flash, the optimization of the preform shape is needed

Optimization of the Prefrom Shape for H Shape Forging

*** Formulation**

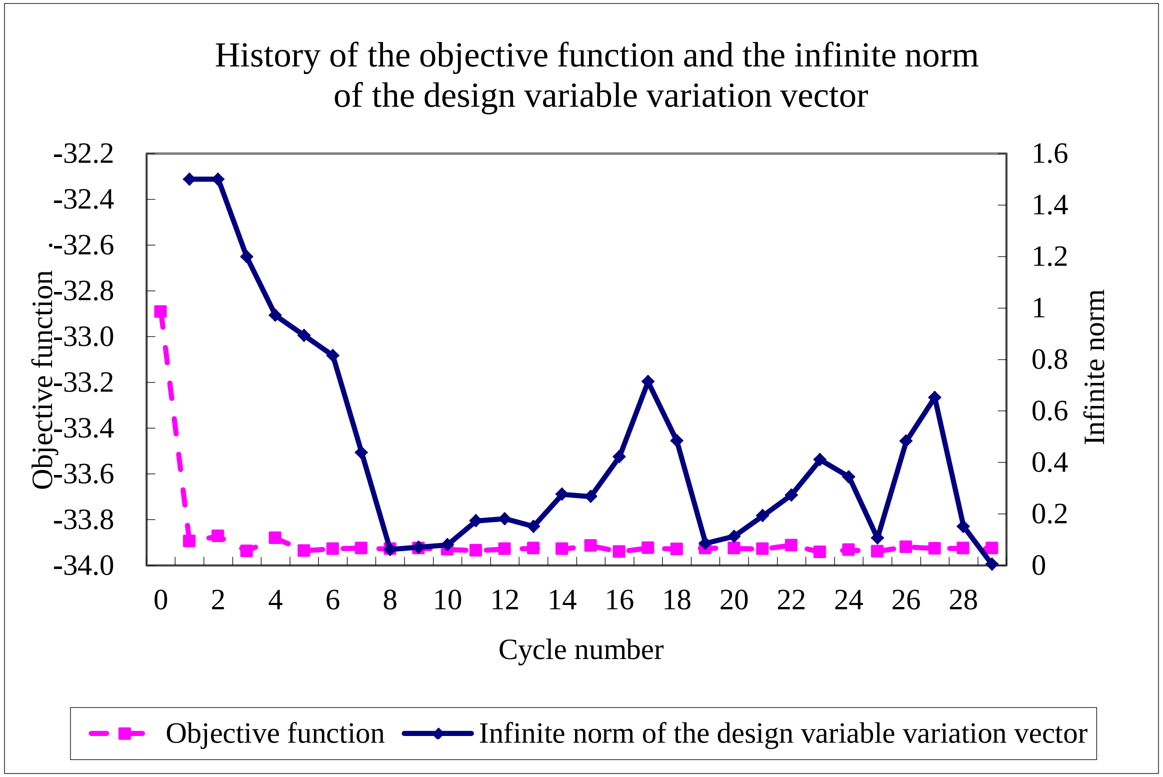
$$
\begin{aligned}
&\text{Find} && b_i && (i = 1, 2, 3) \\
&\text{to minimize} && -Y_j && (j = 1, 2, 3, \ldots, 28) \\
&\text{subject to} && 0.0 \le m_h \le 0.2 && \\
& && S_h \le 0.2 && (h = 1, 2, 3, \ldots, 21)
\end{aligned}
$$



- Design variable: Shape change of the preform
  - b_i : The scale factors for the perturbation vectors
- Objective function: Reduction of the unfilled area
  - Y_j : The mean value of the sample nodes in the top corner (y-direction)
- Constraint: Removal the flash
  - m_h : The mean distance between the sample nodes and the target line (x-direction)
  - S_h : The standard deviation of the sample nodes (x-direction)

Optimization of the Prefrom Shape for H Shape Forging

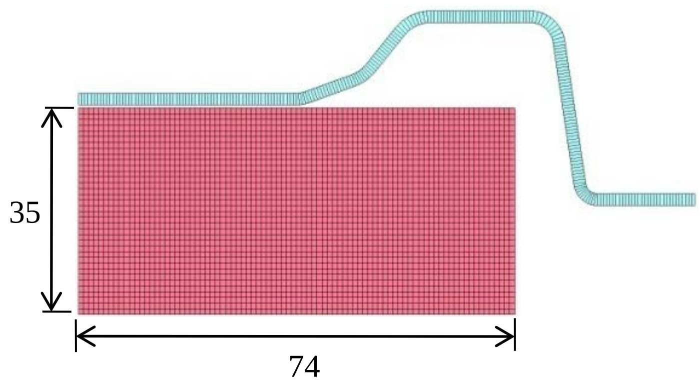
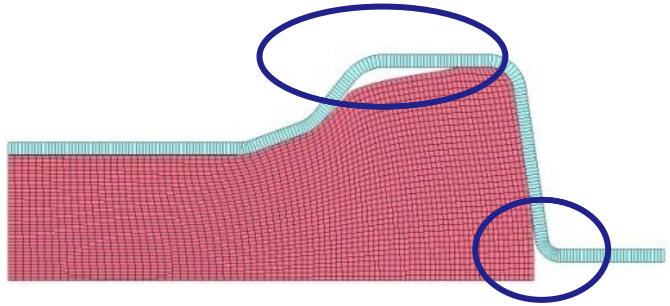
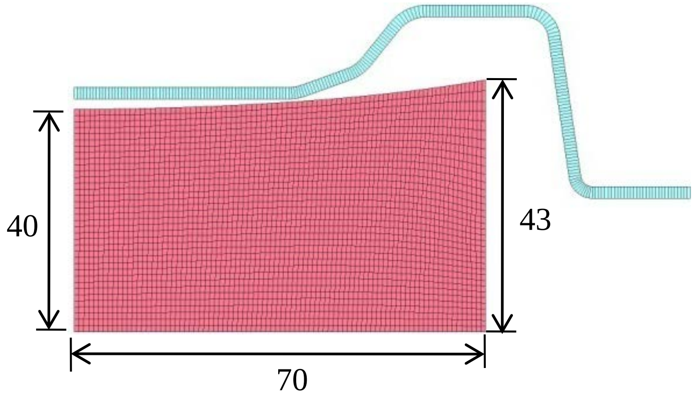
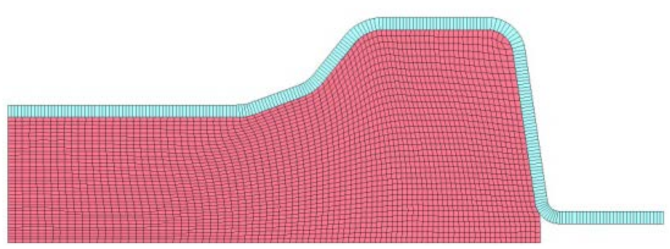
*** Result**



- Objective function: -32.892 → -33.925
- Constraint violation: 175.4% → 0%
- Number of cycles: 30

Optimization of the Prefrom Shape for H Shape Forging

* Result

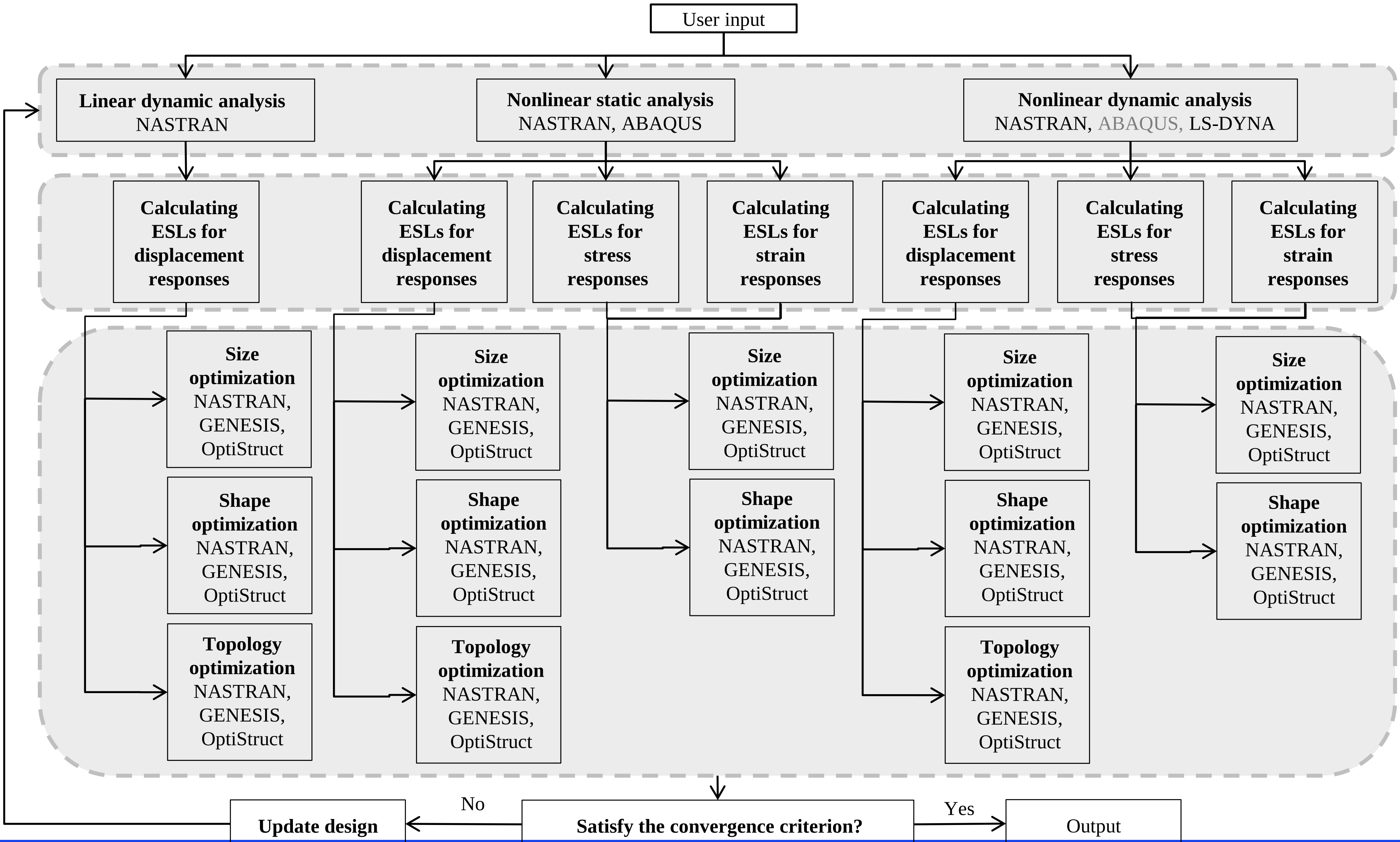
		Analysis Result
Initial Model		
Optimum Model		**No unfilled area and flash** 

ESLSO Software

Software Development

- Software is developed using C and C++ on the Windows system.
- ESLSO software system has been developed based on the theory of ESLSO.
- Linear dynamic, nonlinear static and nonlinear dynamic response optimization using ESLs are supported in the software system.
- Ls-DYNA and Nastran can be utilized for finite element analysis while linear static response optimization using Nastran, Genesis and OptiStruct.
- ESLs for displacement, stress and/or strain constraints are included in the current software system.

Flow of the Software System

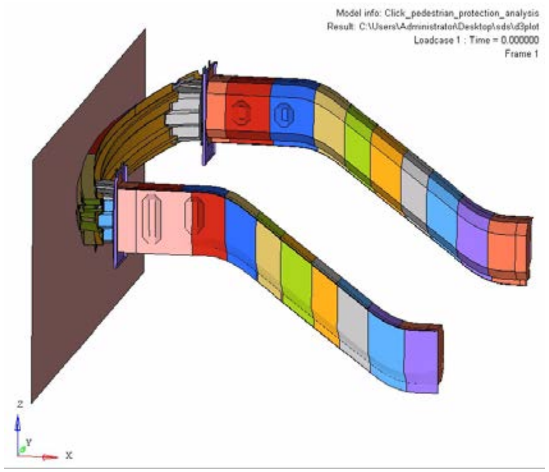


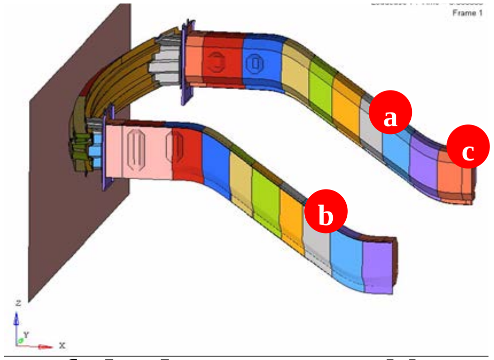
Current development of ESLSO

Automobile Crash Optimization

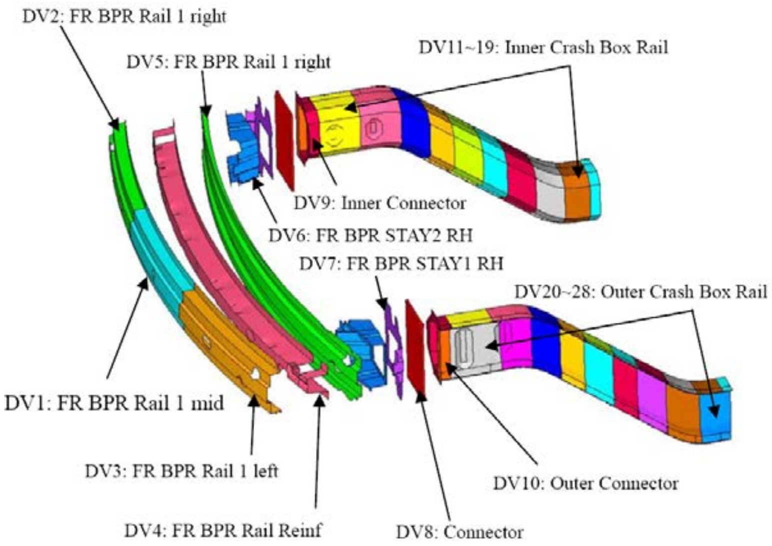
- **Optimization of a Frontal Structure**
  - **Formulation**

$$
\begin{aligned}
&\text{Find} && b_i \quad (i = 1, \ldots, 28) \\
&\text{to minimize} && \text{mass} \\
&\text{subject to} && \left|\delta_{x,\mathrm{a}} - \delta_{x,\mathrm{c}}\right| \geq 142.5(mm) \\
& && \left|\delta_{x,\mathrm{b}} - \delta_{x,\mathrm{c}}\right| \geq 142.5(mm) \\
& && 0.7\ \mathrm{mm} \leq b_i \leq 2.5\,\mathrm{mm}
\end{aligned}
$$





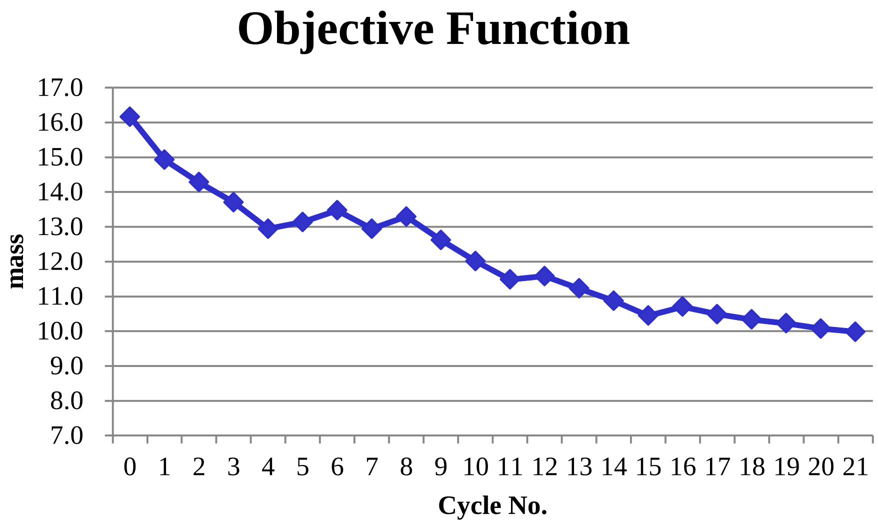
- Design variables



- **The size of the bumper:** width: 1127 mm, length: 762 mm, height: 412 mm
- **Initial velocity of the frontal structure:** 8 km/h
- **In optimization, inertia relief is used.**

Automobile Crash Optimization

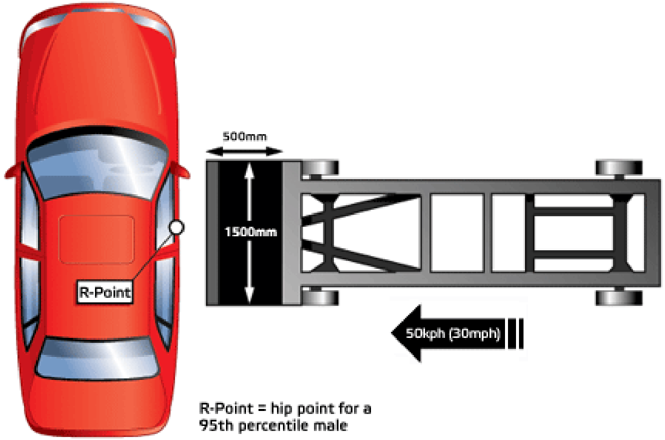
- **Frontal Structure Optimization**
  - **Results**

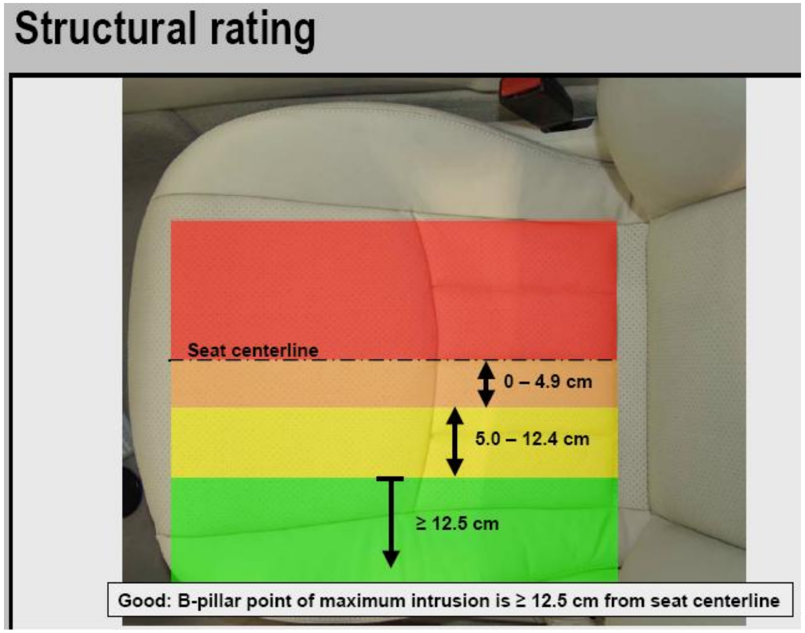


- **Objective function: 16.16 kg → 9.98 kg**
- **The total number of nonlinear analyses: 21**
- **The total number of cycles: 21**
- **The total CPU time per one cycle: 9 minutes**

Automobile Crash Optimization

- **Side Impact Optimization**
  1. Initial velocity of the barrier : 50 km/h
  2. Rating (Good): The distance between B-pillar point of maximum intrusion and the center line of the seat > 125mm





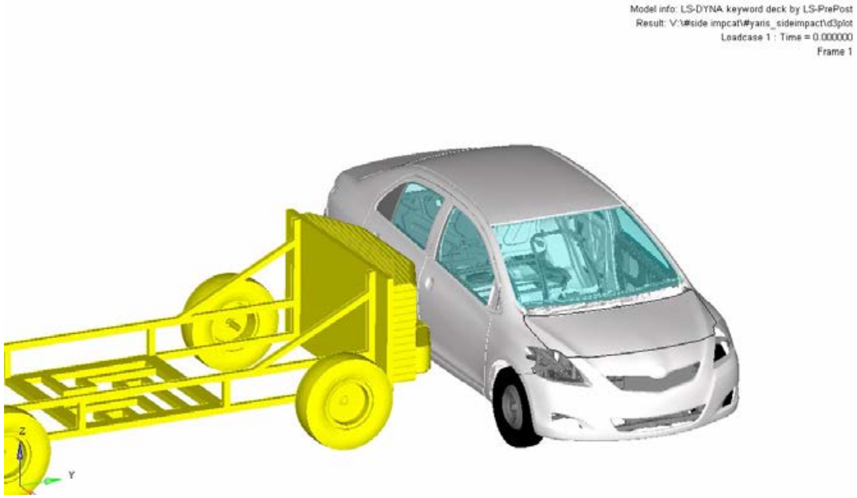
Automobile Crash Optimization

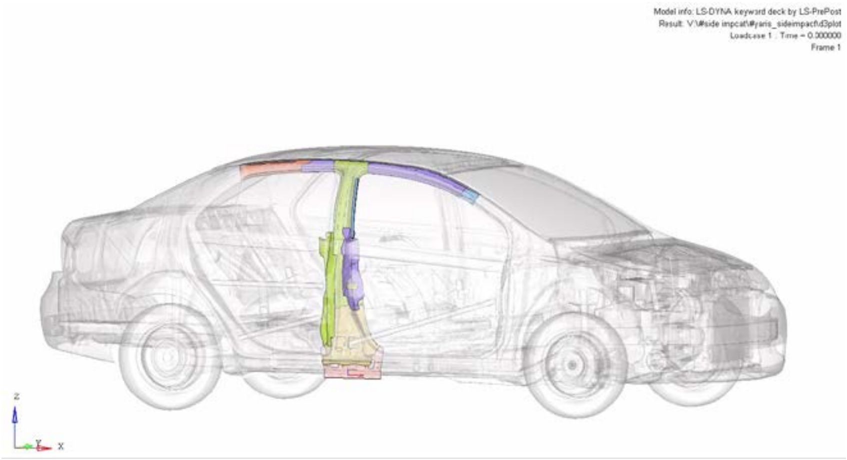
- **Side Impact Optimization**
  - **Model information**

  - **TOYOTA YARIS** [National Crash Analysis Center, NCAC]

  - **No. of elements**: 977,810

  - **CPU time** : 8 hours (LS-DYNA R5)





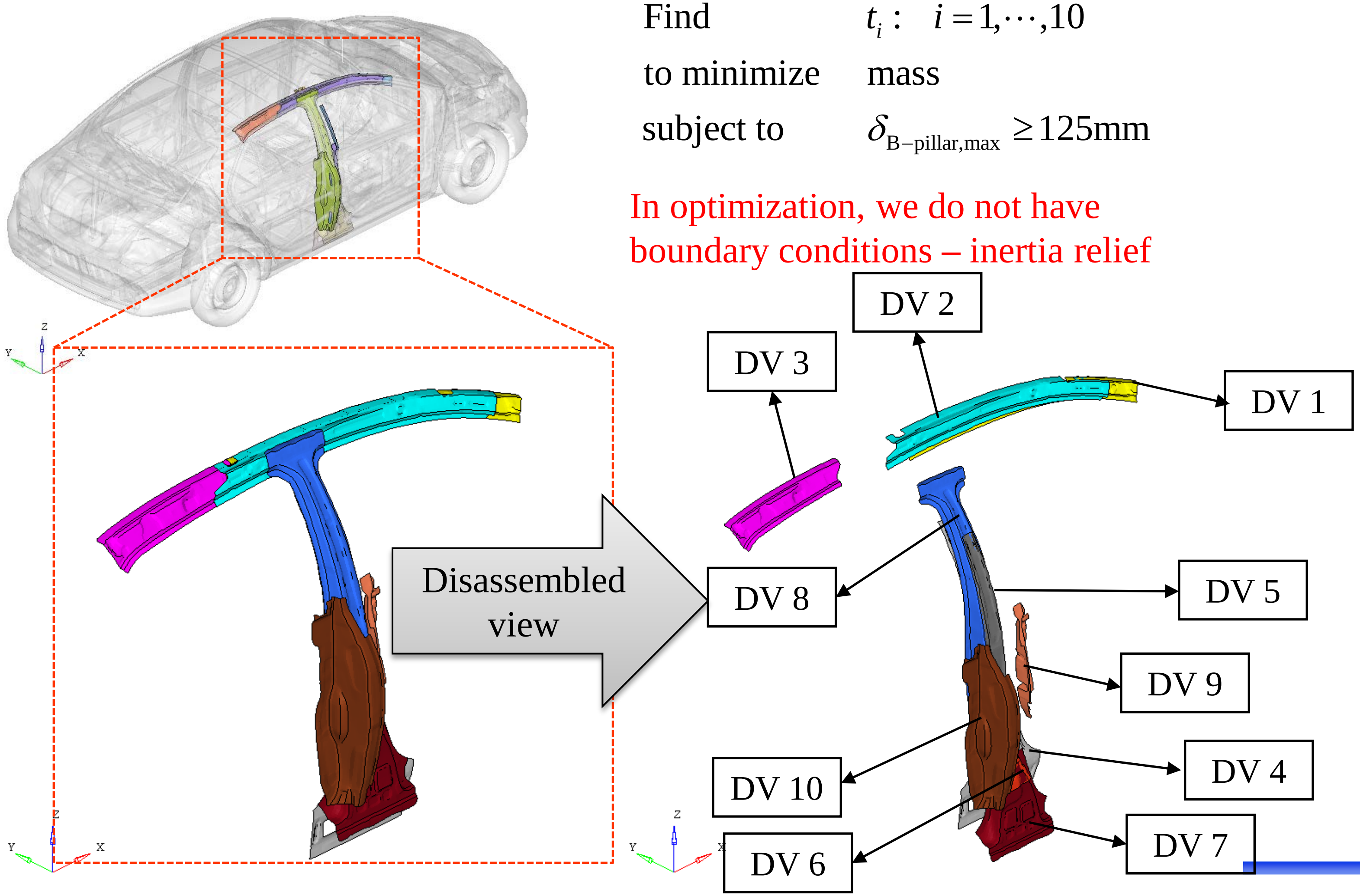
Automobile Crash Optimization

- **Side Impact Optimization**

• **Formulation (using ESLs)**

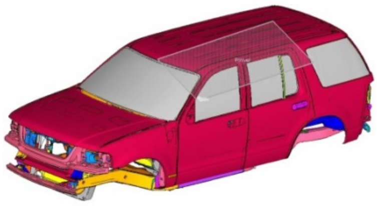
$$\begin{aligned} &\text{Find} && t_i : \quad i = 1, \cdots, 10 \\ &\text{to minimize} && \text{mass} \\ &\text{subject to} && \delta_{\text{B-pillar,max}} \geq 125\text{mm} \end{aligned}$$

In optimization, we do not have boundary conditions – inertia relief



Automobile Crash Optimization

- **Roof Crush Optimization**



  - **Formulation (FMVSS216 condition)**

Find t_1, t_2, t_3

to minimize mass

subject to $F_{rigidwall} \geq 1.5 \times \text{unloaded vehicle's weight (displacement}_{\text{roof}} = 127\text{mm)}$

Modify

Rigid wall force constraints

Displacement constraints

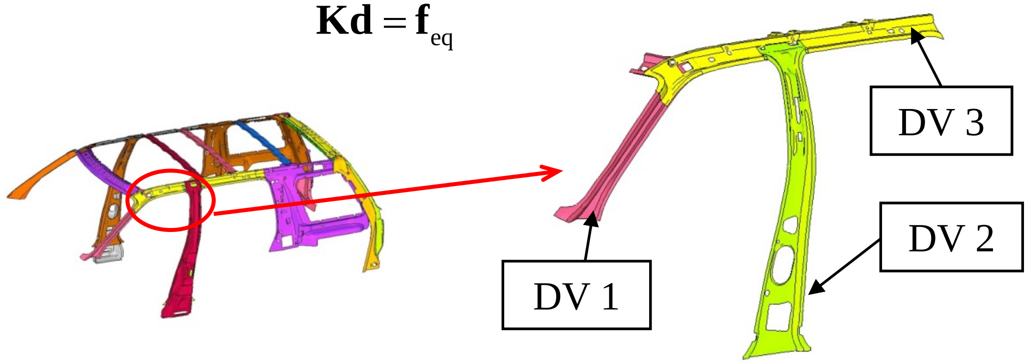
  - **Formulation (using ESLs) → conventional method**

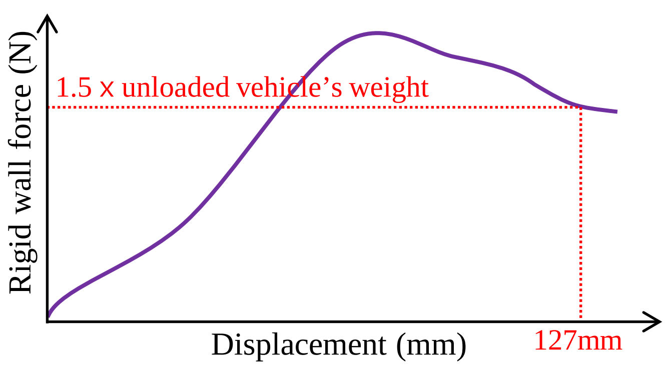
Find t_1, t_2, t_3 (thickness)

to minimize mass

subject to $d_{\max} \leq 127\text{mm} \;\; (\text{at } t_{\text{critical time step}})$

$$\mathbf{Kd} = \mathbf{f}_{eq}$$





Automobile Crash Optimization

- **Roof Crush Optimization**
  - **Formulation (using ESLs) → current method**

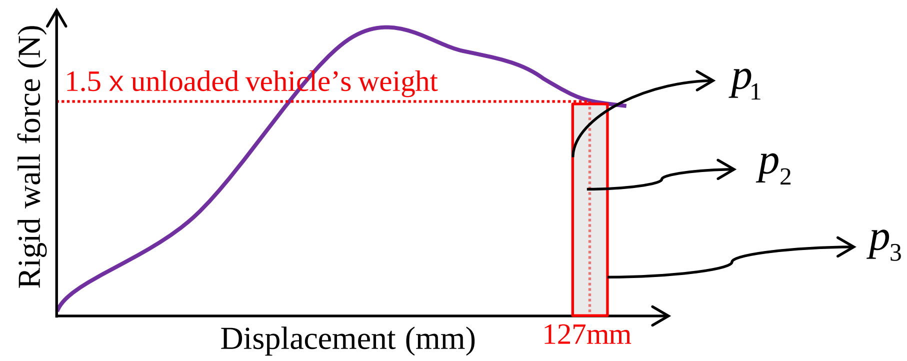
Find $\quad t_1, t_2, t_3, p_1, p_2, p_3 \quad$ (thickness, force)

to minimize $\quad$ mass

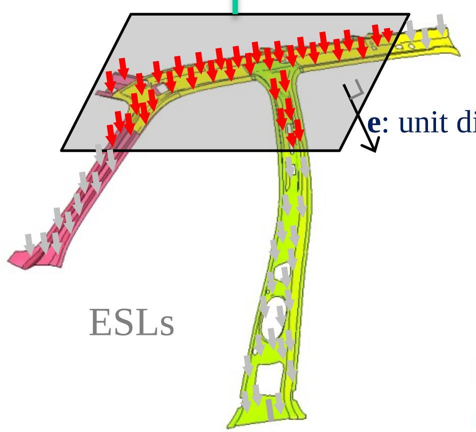
subject to $\quad d_{\max} \le 127\text{mm} \quad$ (at $t_{\text{critical time steps}}$)

$$\sum f_{\text{eq},\,j} \ge 1.5 \times \text{unloaded vehicle's weight}$$

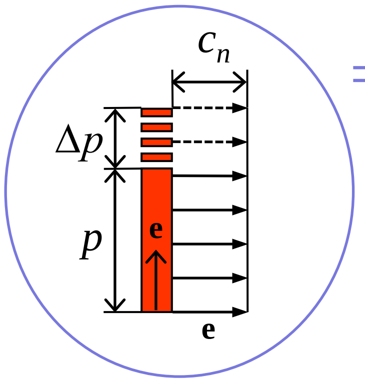
$$f_{\text{eq},\,j} \; (j = \text{node on the contact surface})$$



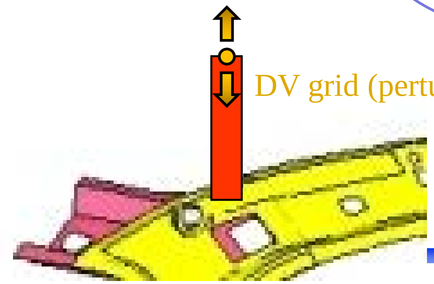
$$\sum f_{\text{eq},\,j} \ge 1.5 \times \text{unloaded vehicle's weight}$$



virtual beams (ESLs)



$= f_{\text{eq},\,j}$



$$\left.\begin{aligned} f_{\text{eq},\,j1} &= c_1 p_i \\ f_{\text{eq}\,j2} &= c_2 p_i \\ &\vdots \\ f_{\text{eq},\,jn} &= c_n p_i \end{aligned}\right\}$$

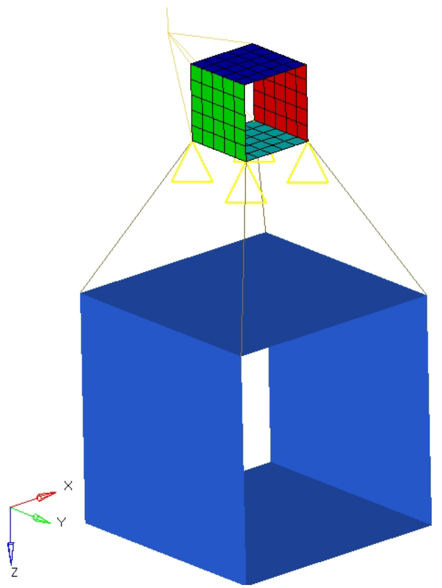
$f_{eq,j}$ is proportional to the size of p_i.

c_n: distributed force

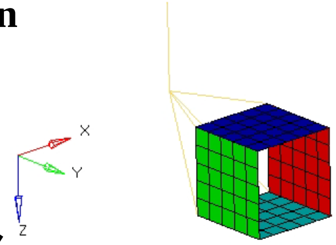
Substructure Method Problems

- Design optimization of the substructure (Superelement) method
  - Nonlinear dynamic response optimization
  - Generation of substructures: LS-DYNA, MSC.Nastran
  - Nonlinear dynamic analysis: LS-DYNA, MSC.Nastran
  - Linear static response optimization: MSC.Nastran

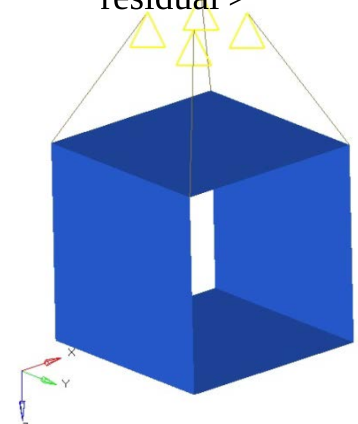
- Finite element model



< Finite element model >



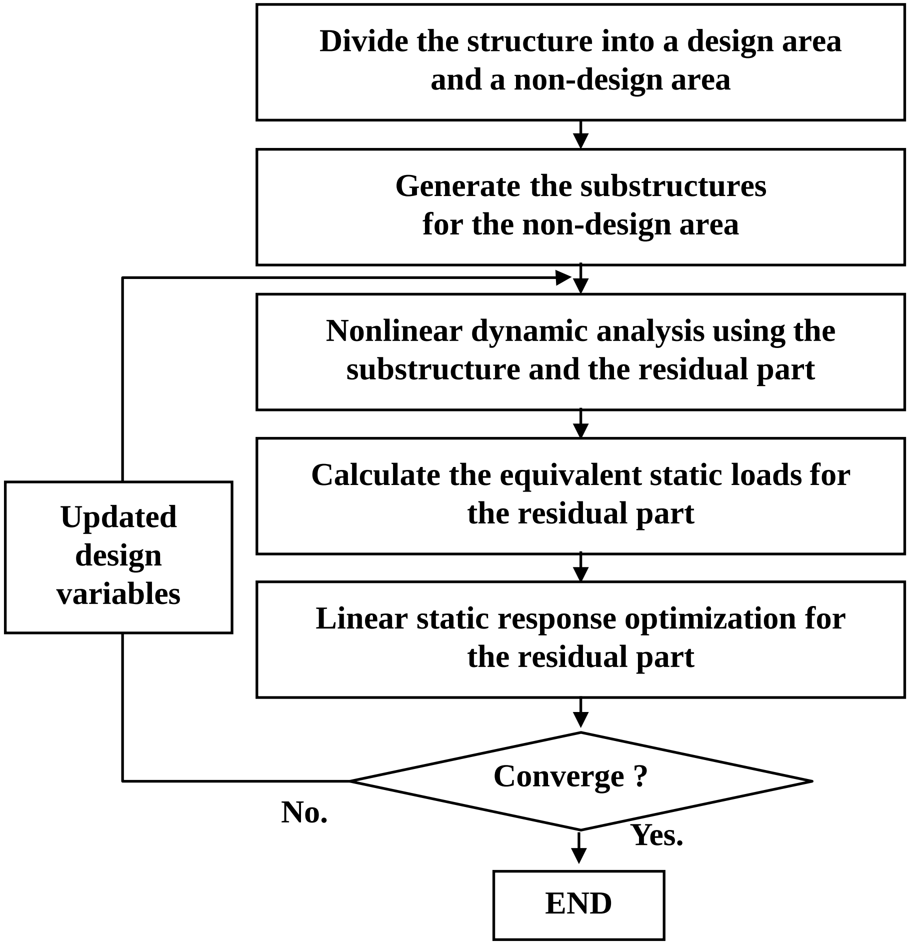
< Design area: residual >



< Non-design area: superelement >

Substructure Method Problems

- **Optimization process of the substructure method**



- **Formulation**

Find $\mathbf{b} \in R^n$

to minimize $f(\mathbf{b})$

subject to $\mathbf{M}\ddot{\mathbf{z}}_N(\mathbf{b},t) + \mathbf{C}\dot{\mathbf{z}}_N(\mathbf{b},t) + \mathbf{K}_N(\mathbf{z}_N(\mathbf{b},t))\mathbf{z}_N - \mathbf{F}(\mathbf{b},t) = 0$

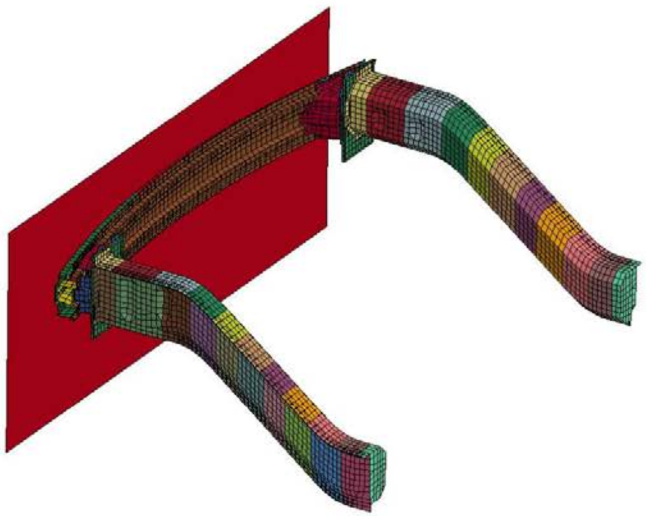
$h_i(\mathbf{b},\mathbf{z}_N) = 0; \quad i = 1,\cdots,p$

$g_j(\mathbf{b},\mathbf{z}_N) \le 0; \quad j = 1,\cdots,q$

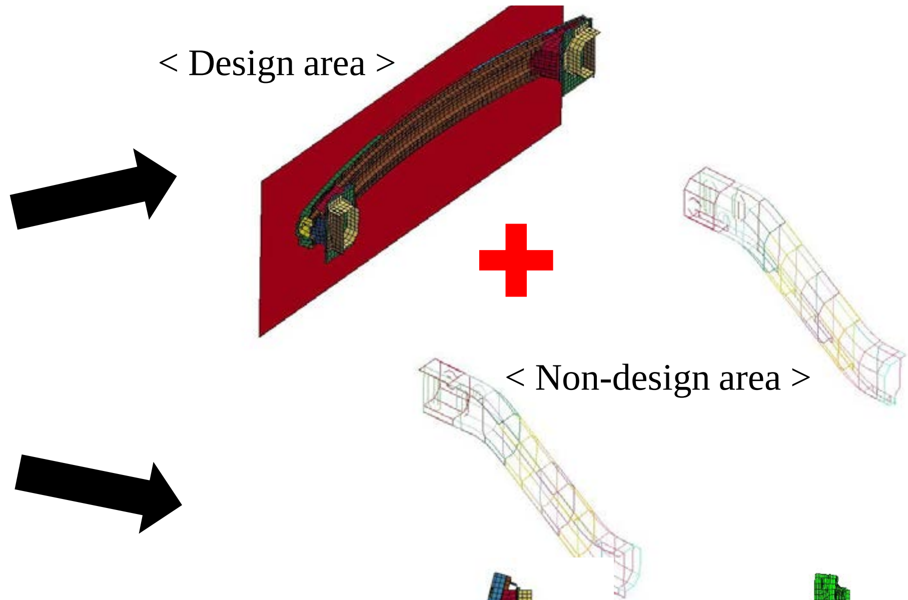
$\mathbf{b}_L \le \mathbf{b} \le \mathbf{b}_U$

Substructure Method Problems

- **If there is no boundary condition in the design area, the inertia relief can be utilized.**



< Finite element model >



- **Formulation**

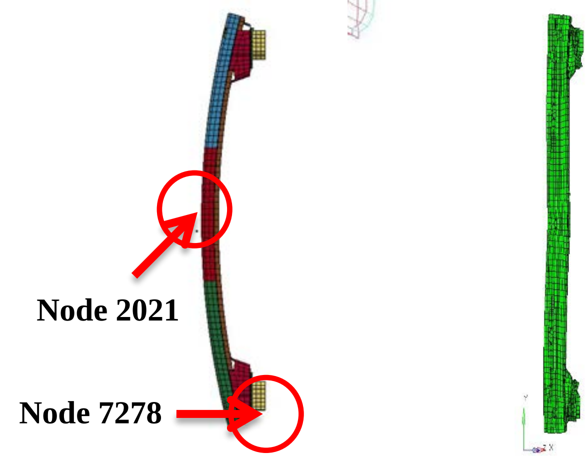
Find b_i (i = 1,2,…,10)

to minimize Weight [kg]

subject to $70 \leq \left|\delta_{x,Node\#7278} - \delta_{x,Node\#2021}\right| \leq 80$ [mm]

$1.2 \leq b_i \leq 3.5$ [mm] (i=1~6)

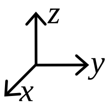
$1.6 \leq b_i \leq 3.5$ [mm] (i=7~10)

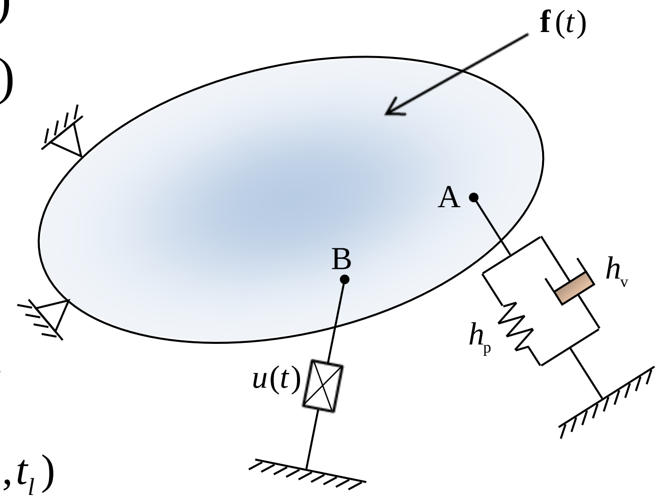


< Initial > < Deformed>

- **Formulation**

$$
\begin{aligned}
&\text{Find} && \mathbf{b}_{\text{st}} \in R^n, \mathbf{u}(t) \\
&\text{to minimize} && f(\mathbf{b}_{\text{st}}, \mathbf{u}(t), \mathbf{z}(t)) \\
&\text{subject to} && \dot{\mathbf{z}} = \mathbf{h}(\mathbf{b}_{\text{st}}, \mathbf{u}(t), \mathbf{z}(t)) \quad (t_0 < t < t_F) \\
& && g_j(\mathbf{b}_{\text{st}}, \mathbf{u}(t), \mathbf{z}(t)) \begin{cases} = 0 & (j = 1, 2, \cdots, m') \\ \leq 0 & (j = m', m'+1, \cdots, m) \end{cases} \\
& && b_{\text{st},q}^{\text{LB}} \leq b_{\text{st},q} \leq b_{\text{st},q}^{\text{UB}} \qquad (q = 1, 2, \cdots, n)
\end{aligned}
$$





$$
f = G_0(\mathbf{b}_{\text{st}}, \mathbf{z}(t_f), t_f) + \int_{t_0}^{t_f} F_0(\mathbf{b}_{\text{st}}, \mathbf{u}(t), \mathbf{z}(t), t)\,\mathrm{d}t
$$

$$
\mathbf{M}(\mathbf{b}_{\text{st}})\ddot{\mathbf{z}}(t) + (\mathbf{C}(\mathbf{b}_{\text{st}}) + \mathbf{H}_{\text{v}})\dot{\mathbf{z}}(t) + \mathbf{K}_{\text{A}}\mathbf{z}(t) = \mathbf{f}(t) + \mathbf{u}(t) \quad (t = t_0, t_1, \cdots, t_l)
$$

$$
\mathbf{K}_{\text{A}} = (\mathbf{K}(\mathbf{b}_{\text{st}}) + \mathbf{H}_{\text{p}})
$$

- **Since the conventional equivalent static loads method cannot handle the control forces, a new method is developed.**

Equivalent Static Loads Method

Analysis domain

Design domain

Equivalent static loads

$$\mathbf{M}(\mathbf{b}_{st})\ddot{\mathbf{z}}(t) + (\mathbf{C}(\mathbf{b}_{st}) + \mathbf{H}_v)\dot{\mathbf{z}}(t) + \mathbf{K}_A\mathbf{z}(t) = \mathbf{f}(t) + \mathbf{u}(t)$$
$$(t = t_0, t_1, \cdots, t_l)$$

$$\mathbf{f}_{eq}(s) = \mathbf{K}_A\mathbf{z}(t)$$
$$= \left[\mathbf{f}(t) - \left\{\mathbf{M}(\mathbf{b}_{st})\ddot{\mathbf{z}}(t) + \mathbf{C}(\mathbf{b}_{st})\dot{\mathbf{z}}(t)\right\}\right] + \left[-\mathbf{H}_v\dot{\mathbf{z}}(t)\right] + \left[\mathbf{u}(t)\right]$$
$$= \mathbf{f}_{eq1}(s) + \mathbf{f}_{eq2}(s) + \mathbf{f}_{eq3}(s)$$

Find $\mathbf{b} \in R^n$ where $\mathbf{b} = [\mathbf{b}_{st}{}^T, h_v, h_p, \mathbf{u}]^T$

to minimize $f(\mathbf{b}_{st}, h_v, h_p, \mathbf{u}, \mathbf{z}, \dot{\mathbf{z}}, \ddot{\mathbf{z}})$ where $\mathbf{u} = [u_1, u_2, \cdots, u_l]^T$

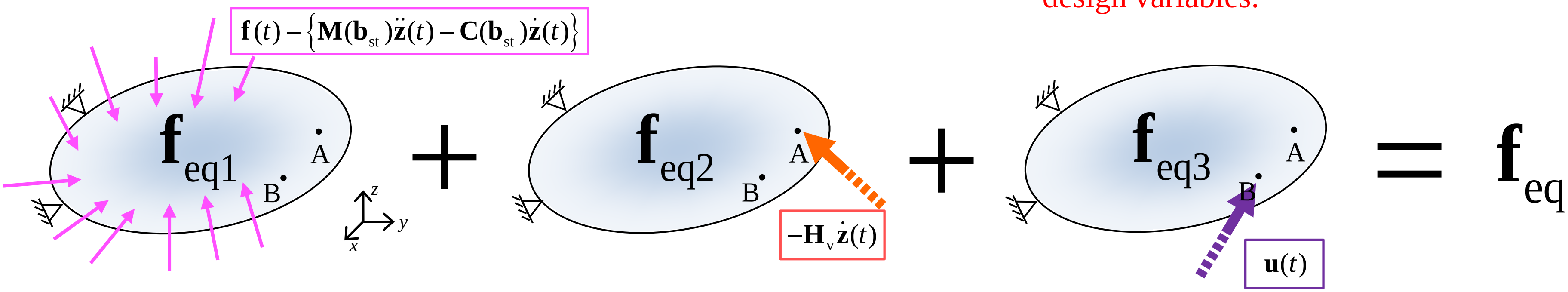
subject to $\mathbf{K}_A\mathbf{z} = \mathbf{f}_{eq1}(s) + \mathbf{f}_{eq2}(s) + \mathbf{f}_{eq3}(s) \quad (s = 0, 1, \cdots, l)$

$g_w(\mathbf{b}, h_v, h_p, \mathbf{u}, \mathbf{z}, \dot{\mathbf{z}}, \ddot{\mathbf{z}}) \le 0 \quad (w = 1, 2, \cdots, m)$

$\mathbf{b}_q^{LB} \le \mathbf{b}_q \le \mathbf{b}_q^{UB} \quad (q = 1, 2, \cdots, n)$

New design variables

The external loads are the functions of design variables.

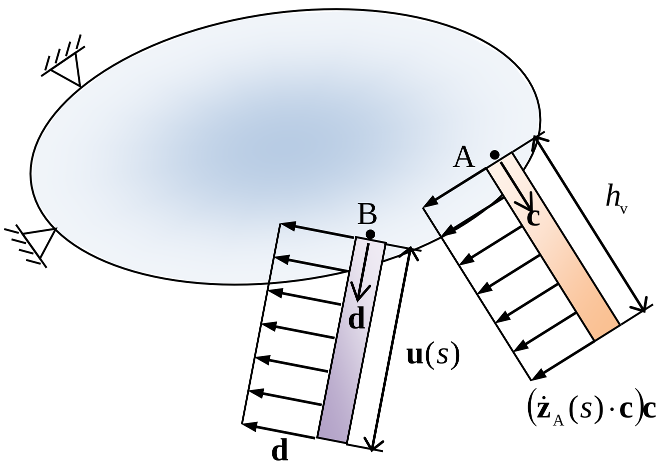


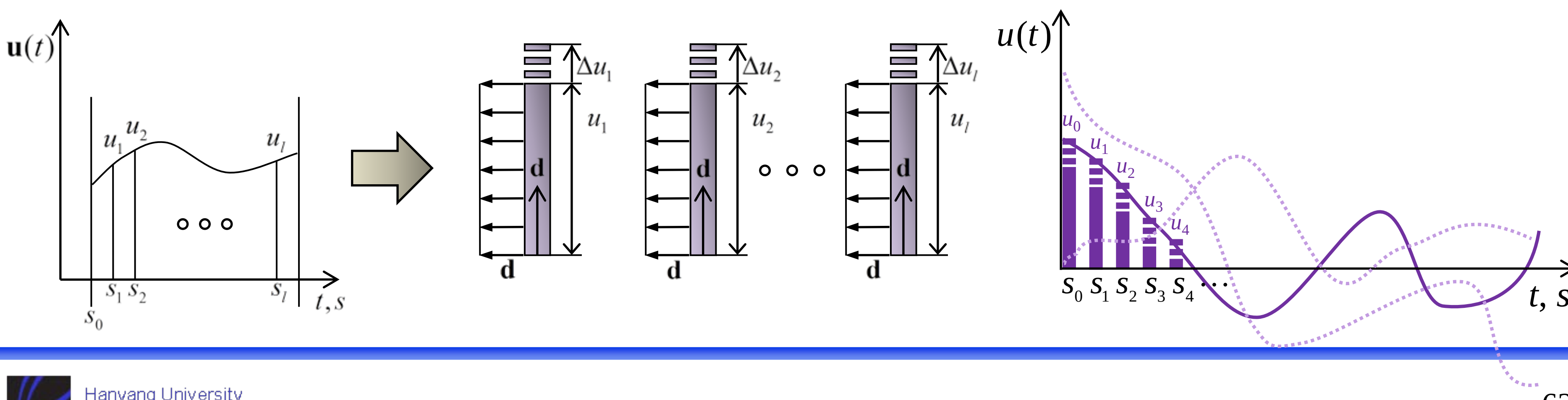
Simultaneous Optimization of Control and Structural Systems

- **An external load can be expressed by a shape variable of a virtual beam and a distributed force. It is the same as the hydroforming case.**

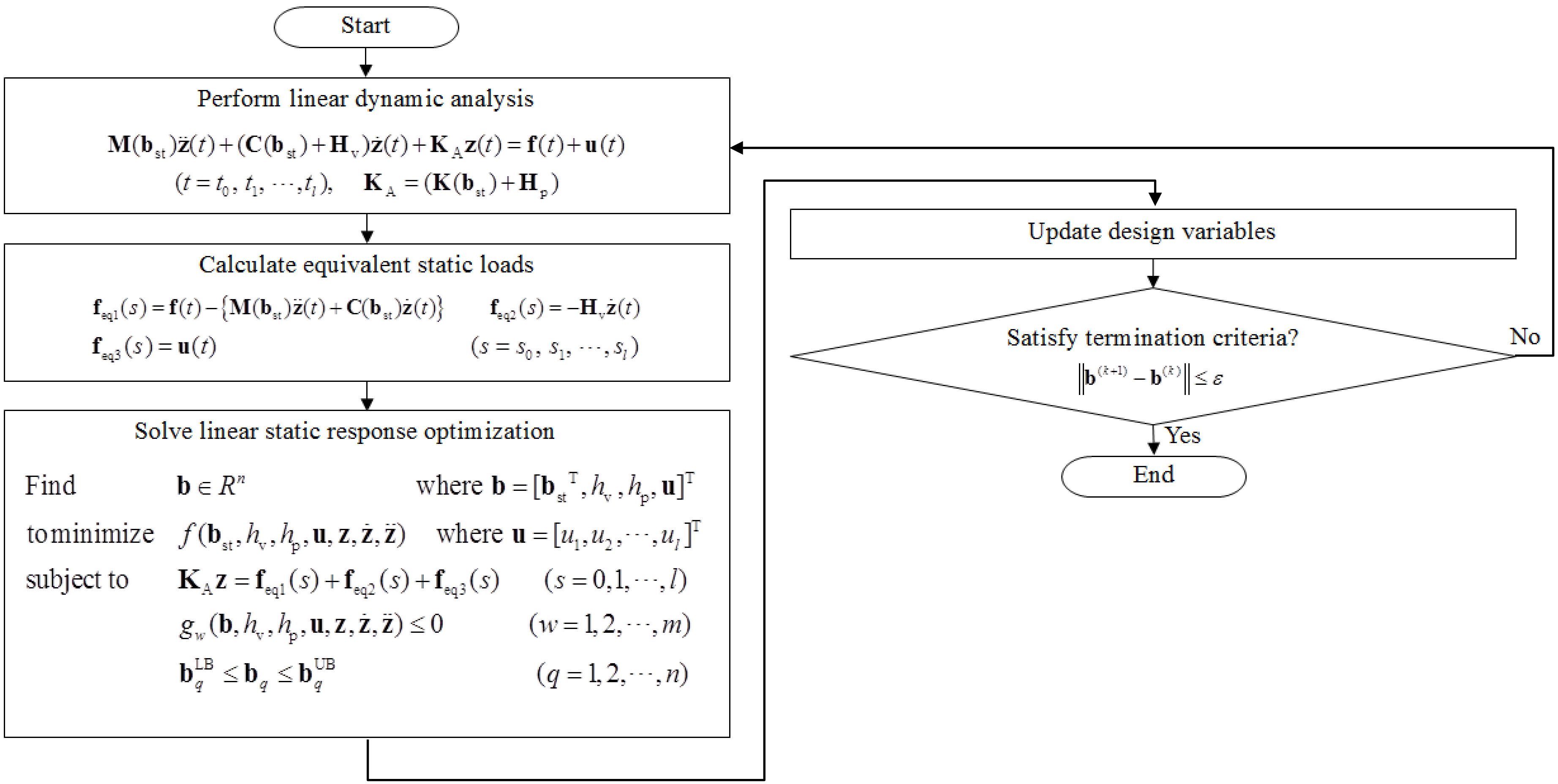
Design domain

$$
\begin{aligned}
&\text{Find} && \mathbf{b} \in R^{n} && \text{where } \mathbf{b} = [\mathbf{b}_{\mathrm{st}}{}^{\mathrm{T}}, h_{\mathrm{v}}, h_{\mathrm{p}}, \mathbf{u}]^{\mathrm{T}} \\
&\text{to minimize} && f(\mathbf{b}_{\mathrm{st}}, h_{\mathrm{v}}, h_{\mathrm{p}}, \mathbf{u}, \mathbf{z}, \dot{\mathbf{z}}, \ddot{\mathbf{z}}) && \text{where } \mathbf{u} = [u_1, u_2, \cdots, u_l]^{\mathrm{T}} \\
&\text{subject to} && \mathbf{K}_{\mathrm{A}}\mathbf{z} = \mathbf{f}_{\mathrm{eq1}}(s) + \mathbf{f}_{\mathrm{eq2}}(s) + \mathbf{f}_{\mathrm{eq3}}(s) && (s = 0, 1, \cdots, l) \\
& && g_w(\mathbf{b}, h_{\mathrm{v}}, h_{\mathrm{p}}, \mathbf{u}, \mathbf{z}, \dot{\mathbf{z}}, \ddot{\mathbf{z}}) \le 0 && (w = 1, 2, \cdots, m) \\
& && \mathbf{b}_q^{\mathrm{LB}} \le \mathbf{b}_q \le \mathbf{b}_q^{\mathrm{UB}} && (q = 1, 2, \cdots, n)
\end{aligned}
$$



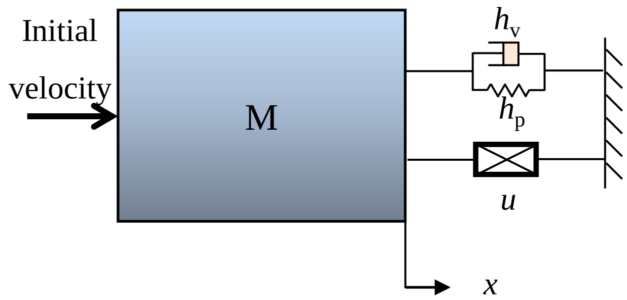


- **Optimization process using equivalent static loads for structural and control systems**



Simultaneous Optimization of Control and Structural Systems

- **Example: Single degree of freedom linear impact absorber**



Find $h_p, h_v, u_i \quad (i = 1, 2, \cdots, 100)$

to minimize $J = \int_{0.0}^{12.0} \{100.0\ddot{x}^2 + 1.0x^2 + 0.005u^2\} dt$

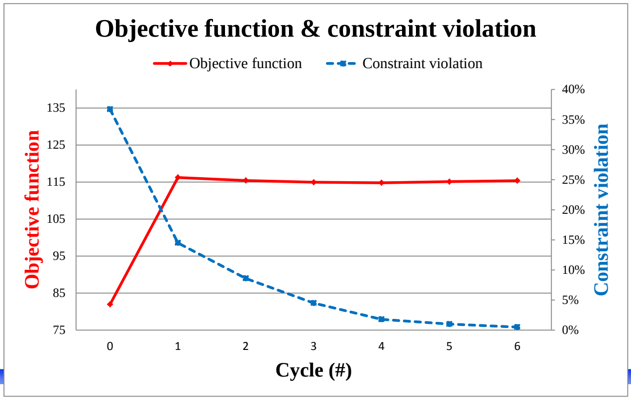
subject to $-0.6\text{mm} \le x \le 0.6\text{mm}$

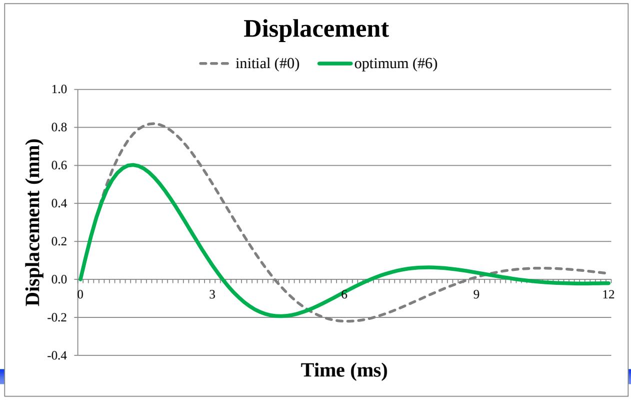
$0.100\text{kN/mm} \le h_p \le 1.000\text{kN/mm}$

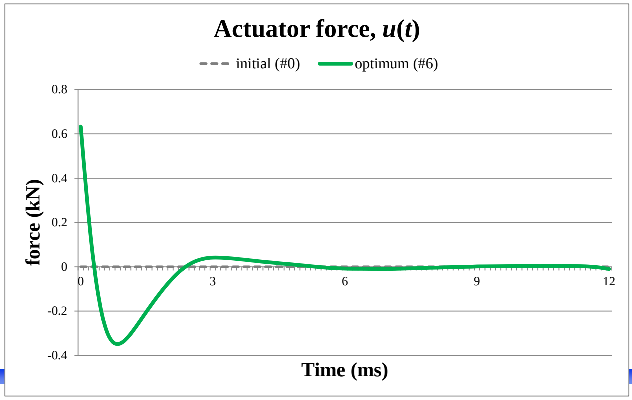
$0.300\text{kN} \cdot \text{ms/mm} \le h_v \le 1.194\text{kN} \cdot \text{ms/mm}$

$0 \le |u_i| \le 2.000\text{kN}$

Results: h_p 0.597→1.000kN/mm, h_v 0.597→0.686kN·ms/mm, objective function 82.9→115.3

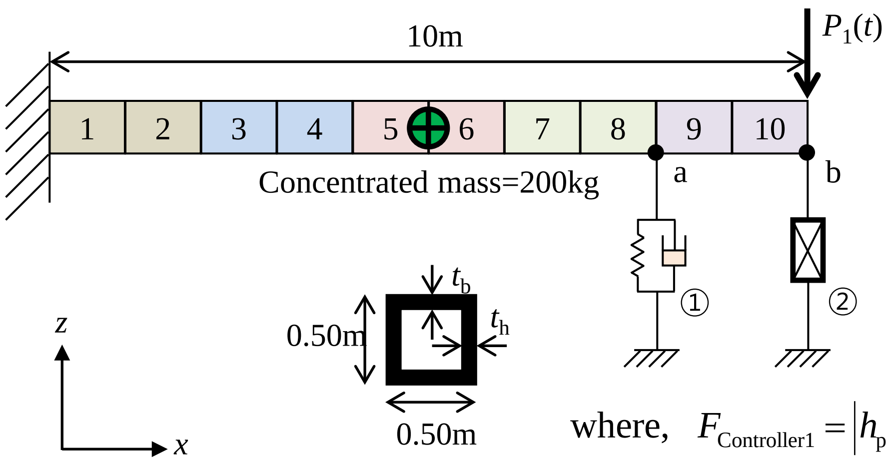






Simultaneous Optimization of Control and Structural Systems

▪ Example: Cantilevered beam (30-DOF): Mass minimization



Find $t_{b1}, t_{b2}, t_{b3}, t_{b4}, t_{b5}, t_h, h_p, h_v, u_i \; (i = 1, 2, \cdots 100)$

to minimize mass

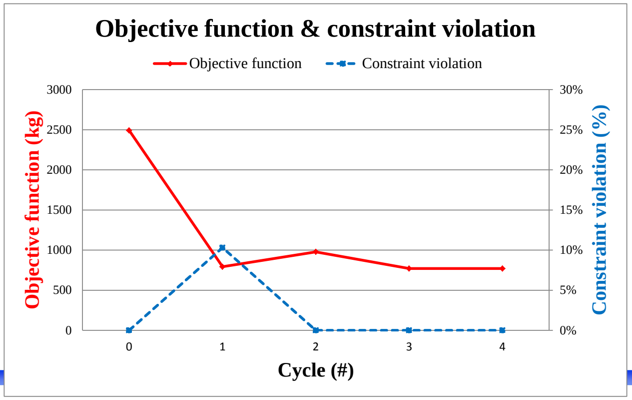
subject to

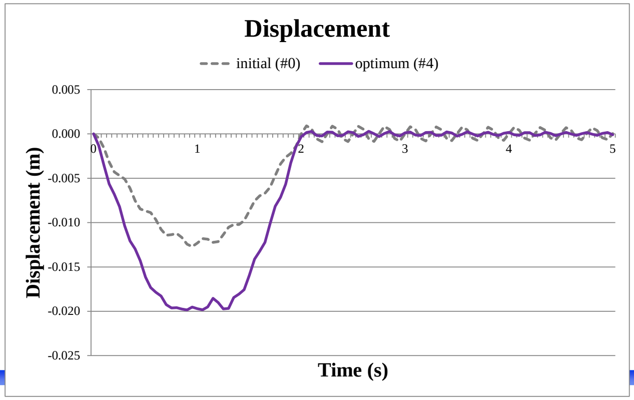
$$-0.02\text{m} \le \delta_{z,\text{tip}} \le 0.02\text{m}$$
$$f_1 \ge 6\text{Hz}$$
$$F_{\text{Controller1}} \le 30.0\text{N}$$
$$F_{\text{Controller2}} \le 1000.0\text{N}$$
$$F_{\text{Controller1}} + F_{\text{Controller2}} \le 1000.0\text{N}$$
$$0.005\text{m} \le t_b, t_h \le 0.1\text{m}$$
$$5.0\text{N/m} \le h_p \le 10000.0\text{N/m}$$
$$120.0\text{N}\cdot\text{s/m} \le h_v \le 1000.0\text{N}\cdot\text{s/m}$$

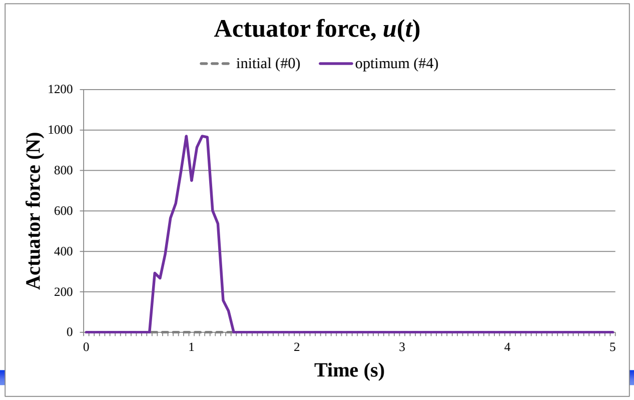
where, $F_{\text{Controller1}} = |h_p \cdot z_a| + |h_v \cdot \dot{z}_a|$

$F_{\text{Controller2}} = |u_i|$

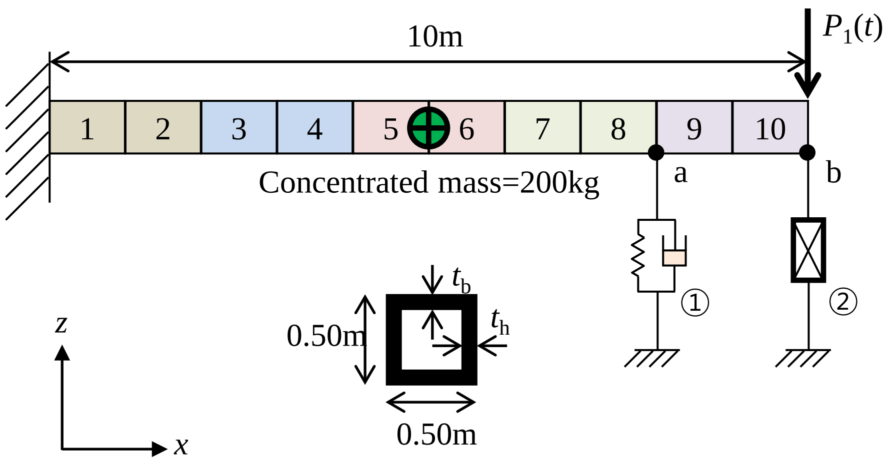
Results: h_p 2000→1912N/m, h_v 500→398N·s/m, objective function 2491→770kg







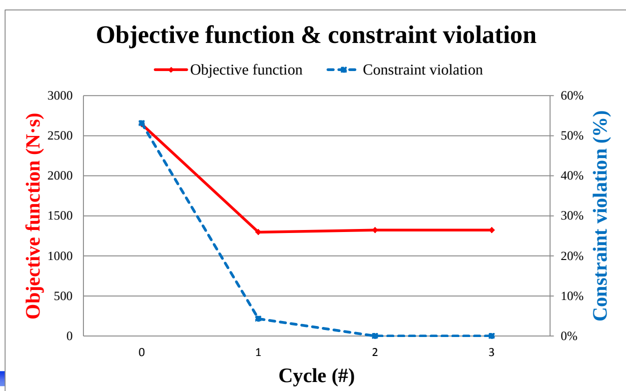
Example: Cantilevered beam (30-DOF): Control energy minimization

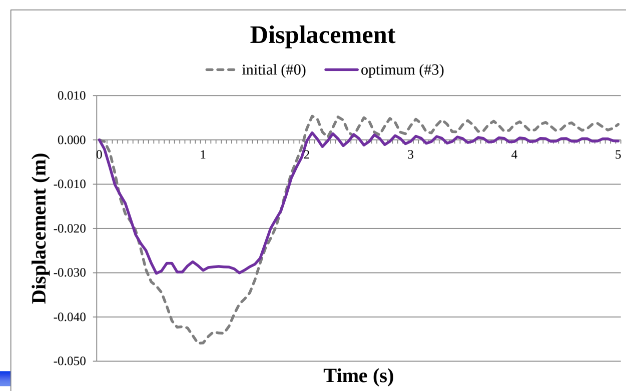


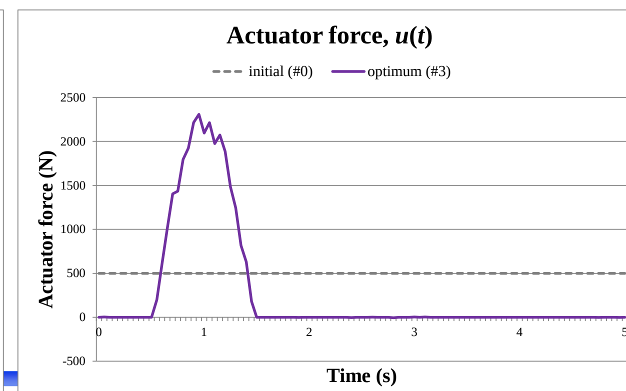
$$
\begin{aligned}
&\text{Find} && t_{b1},\ t_{b2},\ t_{b3},\ t_{b4},\ t_{b5},\ t_h,\ h_p,\ h_v,\ u_i\ (i=1,2,\cdots 100)\\
&\text{to minimize} && \text{control energy}\ (\mathrm{N\cdot s})\\
&\text{subject to} && -0.03\mathrm{m} \le \delta_{tip} \le 0.03\mathrm{m}\\
& && f_1 \ge 6\mathrm{Hz}\\
& && \text{mass} \le 650\mathrm{kg}\\
& && 0.005\mathrm{m} \le t_b,\ t_h \le 0.1\mathrm{m}\\
& && 5.0\mathrm{N/m} \le h_p \le 10000.0\mathrm{N/m}\\
& && 120.0\mathrm{N\cdot s/m} \le h_v \le 1000.0\mathrm{N\cdot s/m}
\end{aligned}
$$

$$\text{control energy} = \int_0^5 \left\{ \left| h_p \cdot z_a(t) \right| + \left| h_v \cdot \dot{z}_a(t) \right| + \left| u(t) \right| \right\} dt$$

Results: h_p 2000→998N/m, h_v 500→591N·s/m, objective function 2644→1322N·s



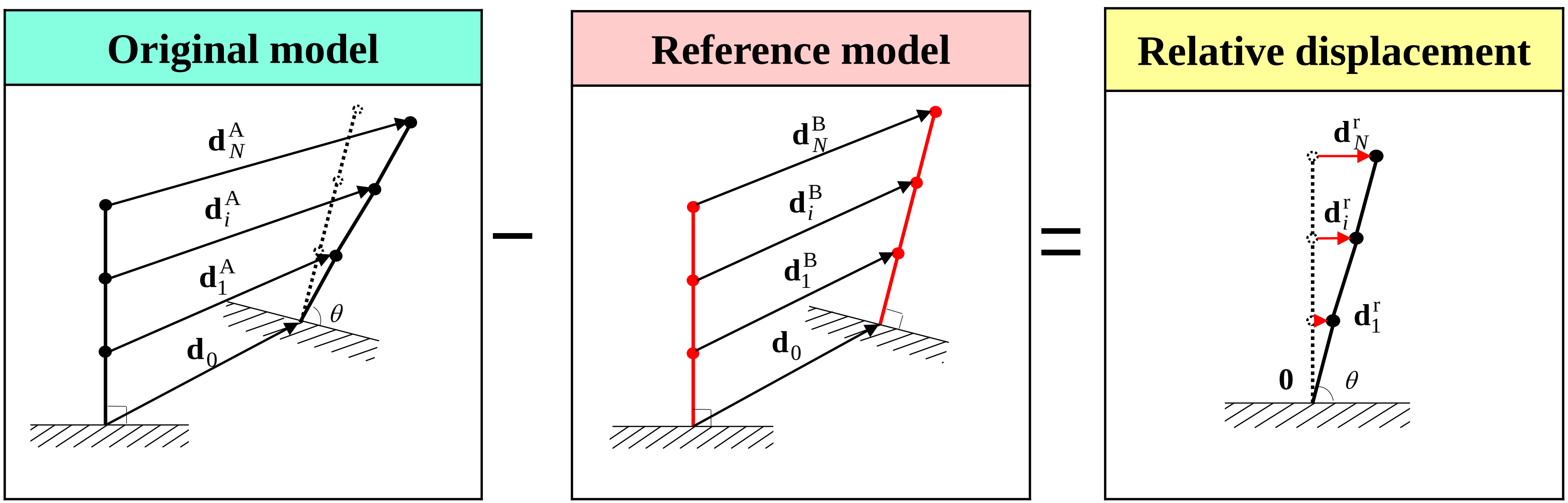




Earthquake Problems

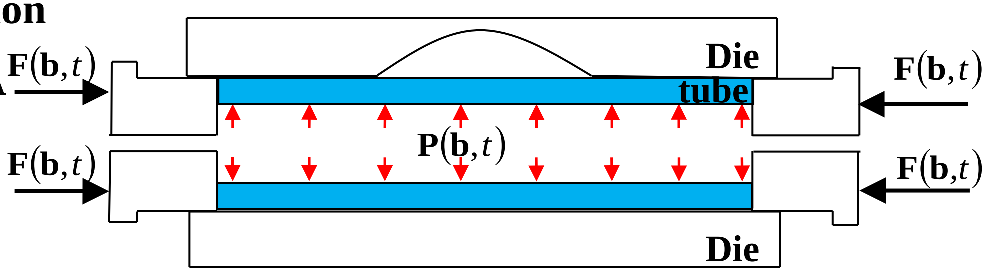
- **Optimization of a structure that has moving boundary conditions**

  1. Define the reference model that has rigid body motion.
  2. The relative displacement can be calculated from the dynamic analyses of the original model and the reference model.
  3. The ESLs for displacement are generated from relative displacement: structural deformation.



Hydroforming Problems

- Optimization of the tube hydroforming process
  - Nonlinear dynamic response optimization
  - Nonlinear dynamic analysis: LS-DYNA
  - Linear static analysis: GENESIS
  - Linear static optimization: GENESIS



- Formulation

Find	$\mathbf{b} \in R^n$
to minimize	$f\left(\mathbf{h}(\mathbf{b}),\mathbf{h}_0\right)$
subject to	$\mathbf{M}\ddot{\mathbf{z}}_{\mathrm{N}}(\mathbf{b},t)+\mathbf{C}\dot{\mathbf{z}}_{\mathrm{N}}(\mathbf{b},t)+\mathbf{K}_{\mathrm{N}}\left(\mathbf{z}_{\mathrm{N}}(\mathbf{b},t)\right)\mathbf{z}_{\mathrm{N}}(\mathbf{b},t)$ $-\mathbf{F}(\mathbf{b},t)-\mathbf{P}(\mathbf{b},t)=0$
	$g_l(\mathbf{b},\mathbf{z}_{\mathrm{N}}(\mathbf{b},t)) \le 0; \quad l=1,\cdots,q$
	$\mathbf{b}_{\mathrm{L}} \le \mathbf{b} \le \mathbf{b}_{\mathrm{U}}$

- $\mathbf{F}(\mathbf{b},t)$: axial force
- $\mathbf{h}_0$: initial thickness
- $\mathbf{P}(\mathbf{b},t)$: pressure
- $\mathbf{h}(\mathbf{b})$: thickness after forming

- Tube hydroforming
  - Metal-forming process
  - Uses pressurized fluid
  - The pressures and axial forces are defined in the time domain.
  - The quality of the formed material is determined by the external forces.

Hydroforming Problems

- Definition of ESLs for tube hydroforming

  - Governing equation of tube hydroforming analysis

$$\mathbf{M}\ddot{\mathbf{z}}_{\mathrm{N}}(\mathbf{b},t)+\mathbf{C}\dot{\mathbf{z}}_{\mathrm{N}}(\mathbf{b},t)+\mathbf{K}_{\mathrm{N}}(\mathbf{z}_{\mathrm{N}}(\mathbf{b},t))\mathbf{z}_{\mathrm{N}}(\mathbf{b},t)=\mathbf{F}(\mathbf{b},t)+\mathbf{P}(\mathbf{b},t)$$

  - Virtual model (using the virtual Young's modulus)

$$\mathrm{E}_i^{f*}\equiv\left|\frac{h_{N,i}^{f,\mathrm{dim}ensionless}}{\sigma_{L\_von,i}^{f,\mathrm{dim}ensionless}}\right|\mathrm{E}_i$$

where i : element number

E_i : Young's modulus of the FE model

$\sigma_{L\_von,i}^{f,\mathrm{dim}ensionless}$: dimensionless form of the von Mises stress from linear analysis with ESLs

$h_{N,i}^{f,\mathrm{dim}ensionless}$: dimensionless form of the thickness from nonlinear analysis

f : final time step

  - Equivalent static loads

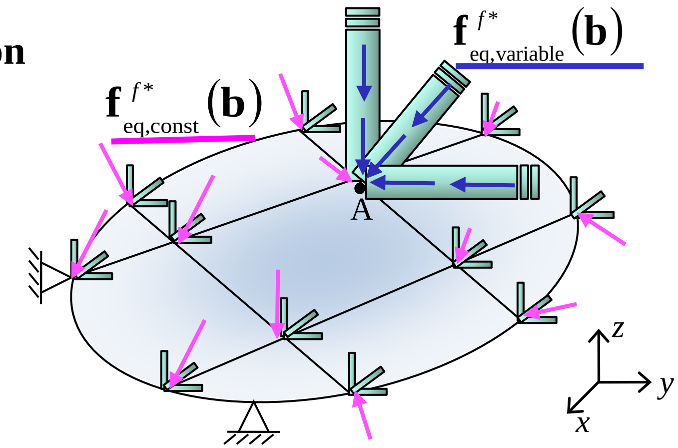
$$\mathbf{f}_{\mathrm{eq}}^{f*}(\mathbf{b})\equiv\mathbf{f}_{\mathrm{eq,const}}^{f*}(\mathbf{b})+\mathbf{f}_{\mathrm{eq,variable}}^{f*}(\mathbf{b})$$

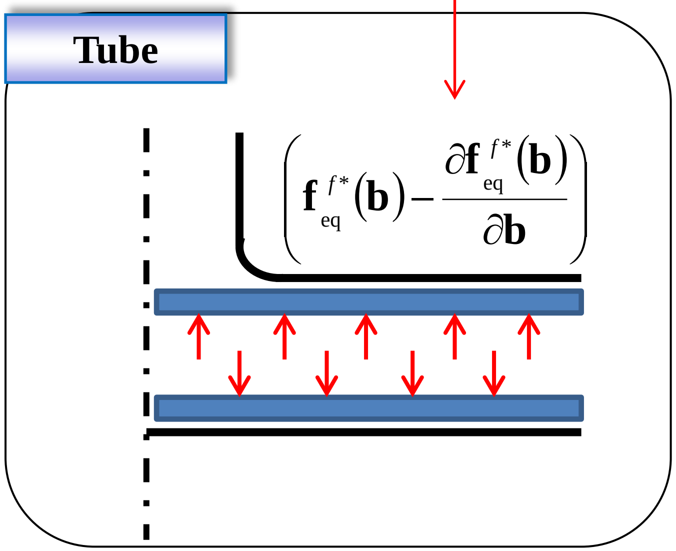
$$=\left(\mathbf{f}_{\mathrm{eq}}^{f*}(\mathbf{b})-\frac{\partial\mathbf{f}_{\mathrm{eq}}^{f*}(\mathbf{b})}{\partial\mathbf{b}}\right)+\left(\frac{\partial\mathbf{f}_{\mathrm{eq}}^{f*}(\mathbf{b})}{\partial\mathbf{b}}\right)=\left(\mathbf{K}_{\mathrm{L}}^{*}\mathbf{z}_{\mathrm{N}}^{f}(\mathbf{b})-\frac{\partial(\mathbf{K}_{\mathrm{L}}^{*}\mathbf{z}_{\mathrm{N}}^{f}(\mathbf{b}))}{\partial\mathbf{b}}\right)+\left(\frac{\partial(\mathbf{K}_{\mathrm{L}}^{*}\mathbf{z}_{\mathrm{N}}^{f}(\mathbf{b}))}{\partial\mathbf{b}}\right)$$

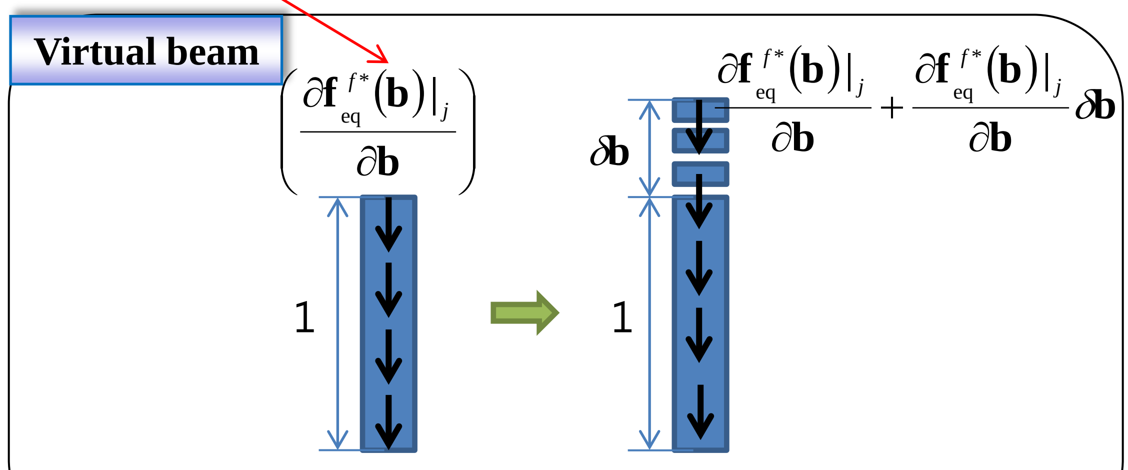
Hydroforming Problems

- **Definition of ESLs for tube hydroforming**
  - **Equivalent static loads in linear static response optimization**

$$
\begin{aligned}
\mathbf{f}_{\text{eq}}^{f*}(\mathbf{b}) &\equiv \mathbf{f}_{\text{eq,const}}^{f*}(\mathbf{b}) + \mathbf{f}_{\text{eq,variable}}^{f*}(\mathbf{b}) \\
&= \left(\mathbf{f}_{\text{eq}}^{f*}(\mathbf{b}) - \frac{\partial \mathbf{f}_{\text{eq}}^{f*}(\mathbf{b})}{\partial \mathbf{b}} \right) + \left(\frac{\partial \mathbf{f}_{\text{eq}}^{f*}(\mathbf{b})}{\partial \mathbf{b}} \right) \\
&= \left(\mathbf{K}_{\text{L}}^{*} \mathbf{z}_{\text{N}}^{f}(\mathbf{b}) - \frac{\partial \left(\mathbf{K}_{\text{L}}^{*} \mathbf{z}_{\text{N}}^{f}(\mathbf{b}) \right)}{\partial \mathbf{b}} \right) + \left(\frac{\partial \left(\mathbf{K}_{\text{L}}^{*} \mathbf{z}_{\text{N}}^{f}(\mathbf{b}) \right)}{\partial \mathbf{b}} \right)
\end{aligned}
$$







<The distributed axial load; j-d.o.f >

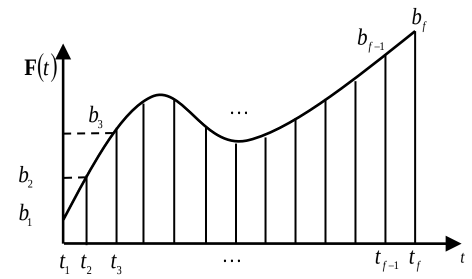
Hydroforming Problems

- **Definition of ESLs for tube hydroforming**
  - **Linear static response optimization**

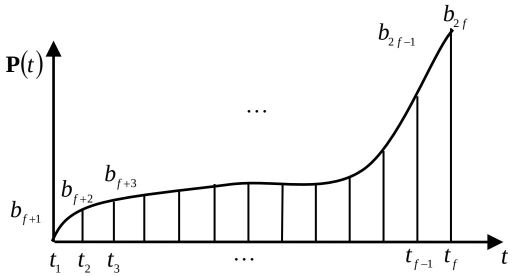
    - Formulation: The external forces are functions of design variables.

    - The points of the profiles are design variables.

$$
\begin{aligned}
&\text{Find} && \mathbf{b} \in R^n \\
&\text{to minimize} && f\left(\mathbf{h}(\mathbf{b}),\mathbf{h}_0\right) \\
&\text{subject to} && \mathbf{K}_\mathrm{L}^{*}\mathbf{z}_\mathrm{L}^{f*}(\mathbf{b}) - \mathbf{f}_\mathrm{eq}^{\,f*}(\mathbf{b}) = 0 \\
& && g_l\left(\mathbf{b},\mathbf{z}_\mathrm{L}^{f*}(\mathbf{b})\right) \le 0; \qquad l = 1,\cdots,q \\
& && \mathbf{b}_\mathrm{L} \le \mathbf{b} \le \mathbf{b}_\mathrm{U}
\end{aligned}
$$



<Force profile>



<Pressure profile>

- Basis functions can be used for the expression of the axial forces and pressures.

$$
\mathbf{F}(u) = \sum_{k=1}^{nf} \mathbf{F}_\mathrm{i} B_{i,d}(u) \qquad \mathbf{F}_\mathrm{i} = \begin{Bmatrix} t_i \\ a_i \end{Bmatrix} \qquad \mathbf{P}_\mathrm{i} = \begin{Bmatrix} t_i \\ c_i \end{Bmatrix}
$$

$$
\mathbf{P}(u) = \sum_{k=1}^{np} \mathbf{P}_\mathrm{i} B_{i,d}(u)
$$

$$
B_{i,1}(u) = \begin{cases} 1, & \text{if } u_i \le u < u_{i+1} \\ 0, & \text{otherwise} \end{cases}
$$

$$
B_{i,d}(u) = \frac{u - u_i}{u_{i+d-1} - u_i} B_{i,d-1}(u) + \frac{u_{i+d} - u}{u_{i+d} - u_{i+1}} B_{i+1,d-1}(u)
$$

- a_i and c_i are design variables.

- The number of design variables can be reduced.