Integrating Topology with Sizing and Shape Optimization Using the Approximation Concepts Approach

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Summary

This paper describes the integration of topology optimization with sizing, shape, and other types of optimization. Engineers and designers who use multiple types of optimization, have typically used topology optimization for preliminary designs and then used sizing and/or shape optimization to further refine their designs. Considering that usage, we originally implemented topology as a separate type of optimization. In the last decade we have implemented more advanced types of sizing and shape optimization. These types are topometry, topography, and freeform which like topology optimization have also been used for preliminary designs. Some of the users of these types of optimizations have found the need for a merged solution where topology and the other types of optimization can be used simultaneously. In response to this need, we have integrated topology with the other types of optimization. This paper describes the use of the approximation concepts approach to efficiently solve the mixed topology, shape, sizing, topometry, topography, and freeform optimization problem. This work has already been implemented in the structural optimization program GENESIS. Several examples, that show the benefits of the integration, are presented. One example will demonstrate the integration of topology and size optimization. Another example will demonstrate the integration of topology and shape optimization. Finally, an example that shows the integration of topology and freeform is presented.

Keywords

Structural optimization, topology, sizing, shape, topometry, topography, and freeform

Introduction

In this paper we will discuss the use of the approximation concepts approach to solve simultaneously topology optimization with other types of structural optimization. But before doing that, we will first describe what general optimization is, then we will describe what structural optimization is and what the common types of structural optimization are.

The Optimization Problem

The general optimization problem can be stated as:

$$
\begin{aligned}
&\min_{x_i} F(x_1, x_2, \ldots, x_n) \\
&\text{such that:} \\
&g_j(x_1, x_2, \ldots, x_n) \le 0; \quad j = 1, m \\
&xl_i \le x_i \le xu_i; \quad i = 1, n
\end{aligned}
$$

Figure 1: Optimization Problem Statement

In the above equations *m* is the number of constraint functions, *n* is the number of design variables, *F* is the objective function, g_j are the constraints, x_i are the design variables, and xl_i and xb_i are the side constraints applied to design variables [1].

Structural Optimization

Structural optimization is a class of optimization used to improve structures. In structural optimization, the responses typically come from the finite element results and the design variables correspond to parameters that describe the structure. The structural optimization problem can be solved using the approximation concepts approach [2].

Next, we will describe the typical objective functions, constraints, and design variables that can be used in formulating and solving structural optimization problems.

Objective Function

In our implementation, practically any of the finite element responses or geometric characteristics of the model can be used as the objective function for minimization or maximization. Often mass, strain energy, displacement, and natural frequencies are used as objective functions.

Constraints

As with the objective function, in our implementation, practically any of the finite element responses or geometric characteristics of the model can be used as constraint. Often mass, displacements, velocities, accelerations, stresses, strains, natural frequencies, bucking load factors, and/or temperatures are used as constraints.

Design Variables

Design variables are parameters that can change directly or indirectly the dimension of elements, grid locations and/or material properties. The type of design variables can be used to define the structural optimization types. In the next section, we will describe the optimization types that we use in our work.

Structural Optimization Types

We can classify structural optimization by the type of design variables used. A list of the different types in GENESIS is: topology, sizing, topometry, shape, topography, and freeform [3].

Topology Optimization

In topology optimization, the design variables correspond to parameters that can change the material properties of each element. A common variable used is the element density. This type of variable takes a value between 0.0 and 1.0. A value close to 0.0 corresponds to an element that should be discarded while a value close to 1.0 corresponds to an elements that should be kept [4]. Topology optimization is typically used for preliminary designs because it can give a good idea of the load path and allows the user to define the basic layout of the design. Details on the implementation of topology optimization in GENESIS can be found in Ref. [5].

Sizing Optimization

The design variables in sizing optimization usually represent physical dimensions. These design variables are typically linked to the properties of the elements. There are 17 types of properties that can be designed in our implementation. An example of a property type is PBAR. With PBAR, the user can design cross section properties such as areas and moments of inertia of bar elements. Another example of property type is PCOMP. For this type of property, the user can design individual thicknesses and individual angles that characterize the cross section of the composite elements. Sizing optimization can be used to find the optimal dimensions of components or parts of structures.

Topometry Optimization

Topometry optimization is a special case of sizing optimization. Topometry optimization can be considered an element by element size optimization capability [6]. When sizing is used, typically multiple elements are designed using the same set of design variables. For topometry, on the other hand, each element can be designed with its own set of design variables.

Shape Optimization

In shape optimization, scale factors of perturbation vectors are the design variables. The perturbation vectors are input directly or by providing basis vectors. Basis vectors contain alternative grid locations that represent candidate designs. When the user provides basis vectors, they can be internally converted into perturbation vectors by performing a vectorial difference between the provided basis vector and the original grid locations. Currently, GENESIS contains three methods to automate the creation of basis or perturbation vectors: the GRID basis vector method, the natural basis vector method, and the DOMAIN method [7].

Topography Optimization

Topography optimization is a special case of shape optimization where multiple shape perturbations are automatically created and used. The perturbations used in topography are typically normal to the surface of shell elements. Topography optimization is typically used in early stages of the design. It is commonly used to find optimal locations of bead patterns [8].

Freeform Optimization

Freeform optimization, like topography optimization, is also a special case of shape optimization. In this case the user has to create a basic perturbation vector that the program will split into individual perturbations and associated them with their own independent design variables [9]. Freeform can be used to find optimal location and dimension of rib patterns. In addition, freeform, like topography, optimization can be used to find the optimal location of bead patterns.

The Approximate Problem Approach

The structural optimization problem is solved using the approximation concepts approach. In this approach, an approximate analysis model is created and optimized at each design cycle. The design solution of the approximate optimization is then used to update the finite element model and a full system analysis is performed to create the next approximate analysis model. The sequence of design cycles continues until the approximate optimum design converges to the actual optimum design. In the mid-seventies Schmit et al. introduced approximation concepts for traditional structural optimization [2]. These concepts, in the eighties and early nineties, were refined to improve the quality of approximations [10-11]. We solve the approximate problem using either the BIGDOT [12-13] or DOT [14] or DSCDOT optimizers. The purpose of using the approximation concepts approach is to reduce the number of design cycles to reduce time. With these approximations a good engineering answer can be typically found in 10 to 15 design cycles. These approximations are implemented in the GENESIS software. To be able to simultaneously optimize topology with the other types of optimization, the design variable vector is now constructed so that it includes the topology design variables, as well as the other design variables. Also, the sensitivities are calculated with respect to all intermediate design variables.

Program Flow

Fig. 2, below, shows the flowchart of the optimization process using the approximate problem approach.

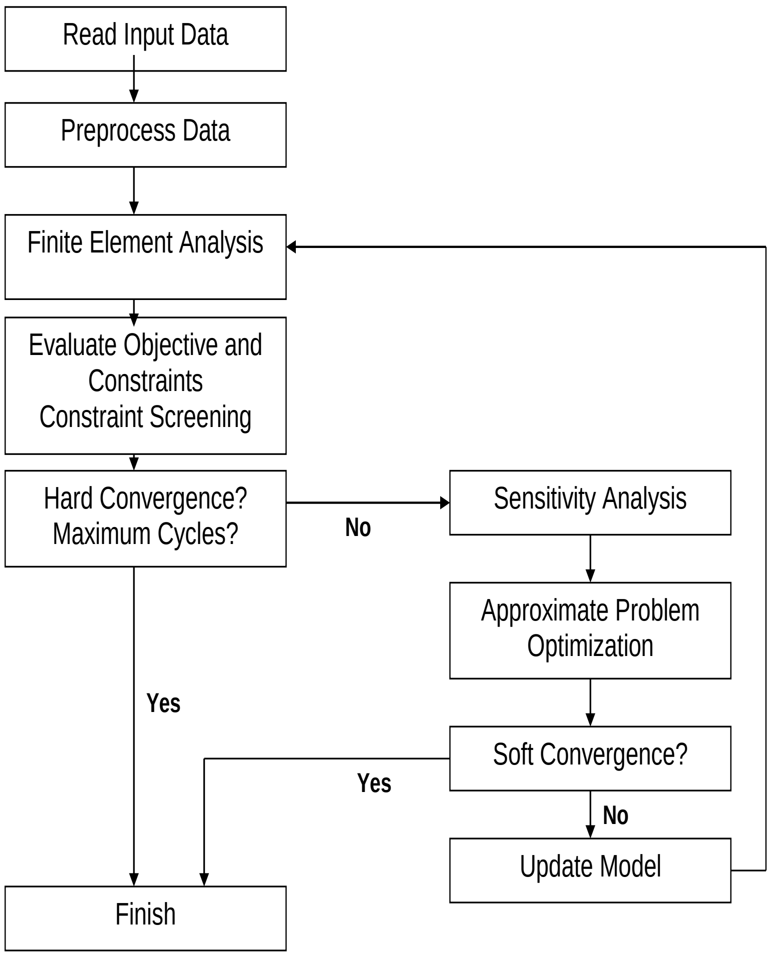

Figure 2: Structural Optimization Flowchart

Intermediate Design Variables and Intermediate Responses

To generate more accurate approximations GENESIS uses intermediate design variables and intermediate responses. Examples of intermediate design variables are: E and ρ for topology and Area and Iyy for sizing. Examples of intermediate responses are forces for stress approximations and modal kinetic energy and model potential energy for natural frequency approximations. The use of intermediate responses can be considered a second generation approximation and were introduced by Vanderplaats in the mid-eighties. A good intermediate response is a response that changes less than the response of interest. Please note that in statically determinate truss structure forces do not change when cross sectional areas change, so the force approximation is exact in this case. The sensitivity analysis in the flowchart uses the intermediate design variables and the intermediate responses.

Constraint Screening

To speed up the process, the actual number of constraints used in the approximated problem is reduced by temporarily ignoring the constraints that are low. In addition, on certain regions, the number of constraints is further reduced.

Sensitivity Analysis

The sensitivity analysis is performed only for the intermediate responses associated to the retained responses. The sensitivities of the intermediate responses are with respect to all intermediate design variables. The program chooses automatically to use either the direct method or the adjoint method depending on which method is more efficient timewise. For large number of design variables with few constraints the adjoint method is usually very efficient and faster than the direct method.

Intermediate Response Approximations

For most of our approximations, we use the conservative approximation approach first developed by Starnes and Haftka (1979) [15] and later refined by Fleury and Braibant (1986).

$$G(X) = G(X_0) + \Sigma h_i(x_i)$$

where,

G(X) is the intermediate response function being approximated. X_0 is the vector of intermediate design variables where the approximation is based, x_i is the *i*-th intermediate design, x_{0i} is the base value of the *i*-th intermediate design variable.

$$h_i(x_i) = \begin{cases} \left.\dfrac{\partial G}{\partial x_i}\right|_{X=X_0} (x_i - x_{0i}) & \text{if} \quad x_i \left.\dfrac{\partial G}{\partial x_i}\right|_{X=X_0} > 0 \\ -\left.\dfrac{\partial G}{\partial x_i}\right|_{X=X_0} \left(\dfrac{1}{x_i} - \dfrac{1}{x_{0i}}\right) x_{0i}^2 & \text{if} \quad x_i \left.\dfrac{\partial G}{\partial x_i}\right|_{X=X_0} \leqslant 0 \end{cases}$$

Response Calculation

The actual responses needed are calculated using the approximated intermediate responses. For example, if a stress in a rod is needed. GENESIS approximates the forces first (using the equations above) and the stress is then calculated using the physical equation that related the stress with its force (Stress=Force/Area). If the stress is the von Mises stress, GENESIS approximates the intermediate tensor stresses and then the von Mises equation is used to calculate the von Mises stress.

Examples

Three examples that show the benefits of the integration are presented. The first example will demonstrate the integration of topology and sizing optimization. The second example will demonstrate the integration of topology and shape optimization. Finally, the third example, will demonstrate the integration of topology and freeform optimization.

Mixed Sizing and Topology Optimization of a Hat Structure

Description of the Example

The purpose of this example is to demonstrate a combined sizing and topology optimization problem. Five independent sizing design variables are used to design the thicknesses of five regions of the hat structure. In addition, the topology of the entire structure is simultaneously designed. The structure is clamped at the two short ends while a point load is applied at the center of the top surface of the structure.

Problem Statement

The objective of the example is to produce the stiffest structure possible. To achieve this goal, the global strain energy will be minimized. To make sure that topology optimization uses only 30% of the available mass a 30% mass fraction constraint is used (0.3) and to ensure that the final structure will not be heavier than the initial mass, a mass constraint is added. For the topology variables we will set an initial value of 0.3. The sizing design variable initial value will be set to match the initial thickness.

The following optimization problem will be created, solved, and post-processed:

Minimize Strain energy
Subject to:
Mass fraction < 0.3
Mass < Initial mass

Designable Region

All the elements of the structure.

Results

The combined sizing and topology optimization results are shown in Fig. 3. It can be noticed, while in the initial design, the thicknesses of all the elements were the same, but in the final design, the thicknesses are different as in this case the optimization process changed the variables unevenly.
In this figure, the grey area corresponds the material removed by topology optimization.

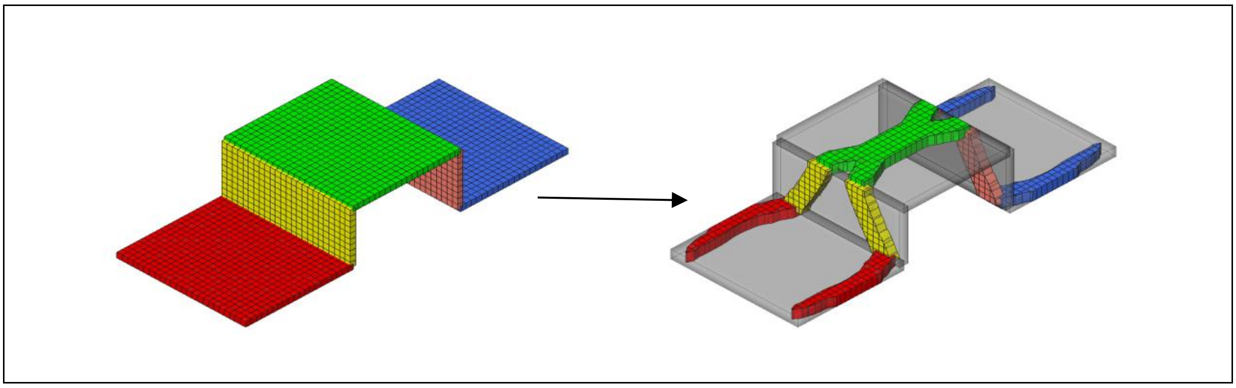

Figure 3: Initial and Final Designs

Mixed Shape and Topology Optimization of a Curved Shell

Description of Problem

The purpose of this example is to show a combined shape and topology optimization problem. Shape domains and shape morphing sets are used to define the shape optimization part of the problem. The topology regions include all shell elements.

This example will also show the final results of the combined shape and topology optimization.

Problem Statement

The objective is to simultaneously design the topology and shape of a curved shell structure so that the strain energy is minimized. The structure is clamped at the mid points of the four curved boundaries and subjected to five vertical forces. The loads are applied in the corners and in the top middle part. The initial shape is shown on the left part of the Fig. 4.

The following optimization problem is solved:

Minimize Strain energy
Subject to:
Mass fraction < 0.45
Mass < Initial mass:

Designable region

The topology design variables correspond to all elements' densities and the shape variables correspond to scale factors that control the shape of the structure.

Results

The combined shape and topology optimization results are shown in right part Fig. 4.

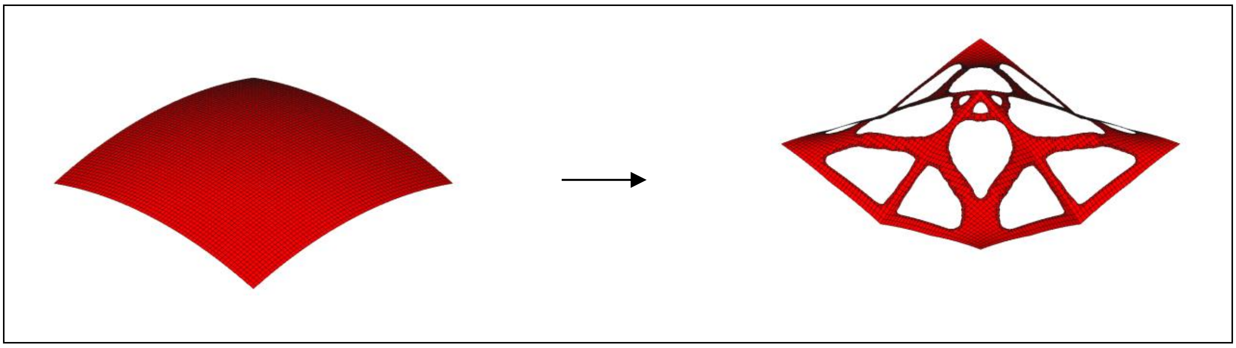

Figure 4: Initial and Final Designs

Topology optimization carved about 55% of the material while shape optimization changed the shape of the structure, it can be noticed that the initial design has only positive curvatures, while the final design has both positive and negative curvatures.

This problem used 4,121 independent design variable: 4,096 of the design variables were topology design variables, while the other 25 were shape design variables.

This problem converged in 13 design cycles and finished with hard convergence.

Mixed Freeform and Topology Optimization of a flat Plate

Description of the Example

This example demonstrates the solution of a combined freeform and topology optimization problem. A shape domain and one-shape morphing sets are used to define the shape optimization part of the problem. Freeform is used to get more variability in the design. The topology region includes all shell elements. Symmetries and grid fraction constraints are used to obtain a more buildable design.

Problem Statement

A flat rectangular plate is fixed at two of its corners in one of its short sides and is subjected to twisting loads at its opposite side. The objective of the problem is to minimize the displacement under the load by minimizing the total strain energy. The following optimization problem is solved and post-processed:

Minimize Strain energy
Subject to:
Mass fraction < 0.70
Grid Fraction ≤ 0.25

Designable Region

All the element densities and all interior grids location of the structure.

Results

The combined freeform and topology optimization results are shown in Fig. 5.

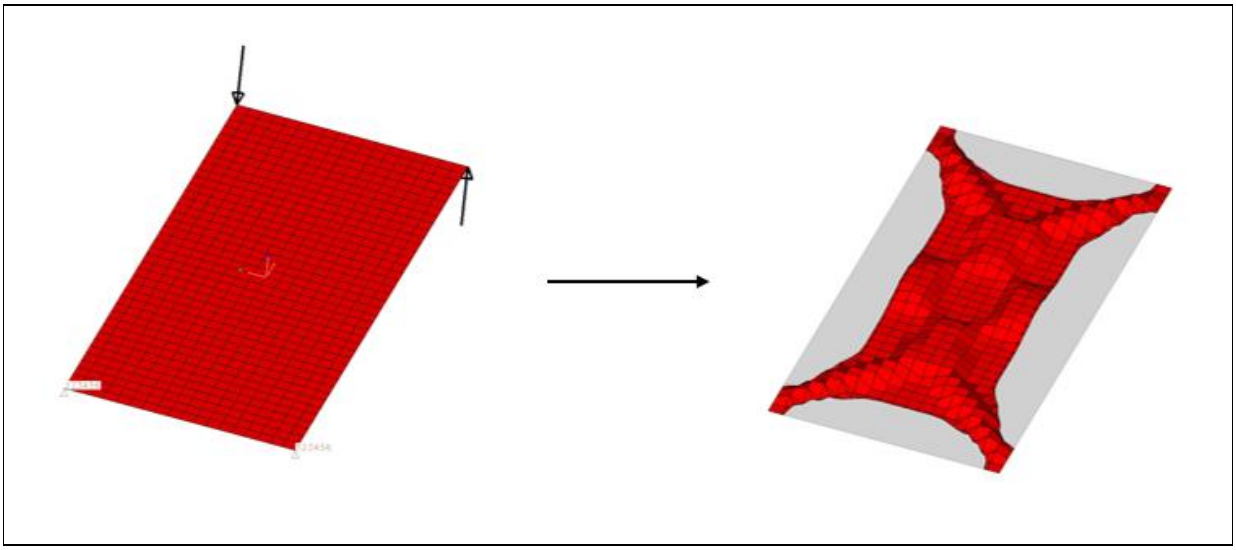

Figure 5: Initial and Final Designs

Topology optimization carved about 30% of the material while freeform optimization change the shape of the structure by creating beads. The answer is double symmetric as enforced.

This problem uses 390 independent design variables: 210 of the variables are topology design variables and the rest 180 are shape design variables.

The problem converged in 29 design cycles and finished with hard convergence.

Conclusions

The integration of topology with shape, sizing, and other types of structural optimization has been described. The use of approximation concepts allows to mix the types of optimization without problem. The implementation is in the GENESIS program and allows its users to obtain innovative designs.

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