

## **Optimization in the presence of bifurcations: A simple crash example**

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### BIFURCATIONS OR RESPONSE SURFACE DISCONTINUITIES ARE UBIQUITOUS IN CRASH OPTIMIZATION

#### We explore three approaches to the problem of response surface discontinuities in crash problems

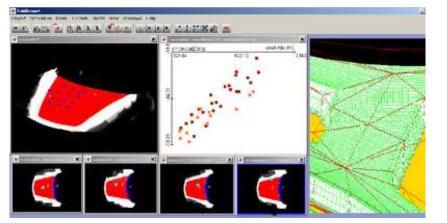
#### We can modify the design space to the failure modes sought

This was the approach we used in the first crash optimization project in France (2001), where we introduced geometrical and assembly design variables to control failure modes



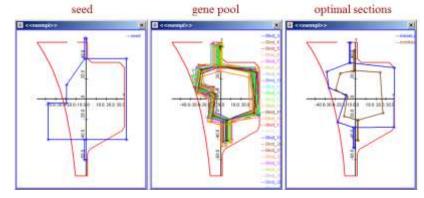
#### We can modify the problem formulation to avoid the discontinuity

This was the approach we used for bonnet design for pedestrian safety (2007), where we introduced an additional constraint on stroke to model secondary impact



#### We can simplify the problem so that the design point runs very quickly

In the design of energy absorption components (2009) we had very short CPU times and we could use genetic algorithms

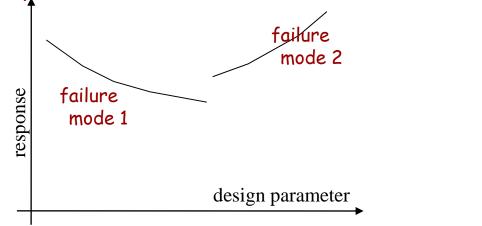


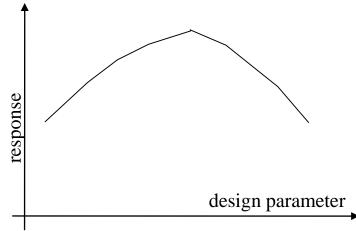




## WHAT DO WE MEAN BY BIFURCATION

In this presentation, we call bifurcation a discontinuity of the response surface in our optimization problem





A discontinuity in the response surface itself is associated to a physical bifurcation, *typically the change of a failure mode* 

A discontinuity in the first derivative of the response surface is much more common Examples are most assembly problems

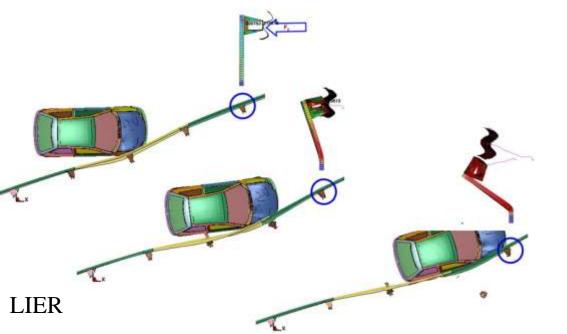
Bifurcations may cause failure of ALL optimization algorithms o Gradient or Surface Approximation algorithms are particularly affected

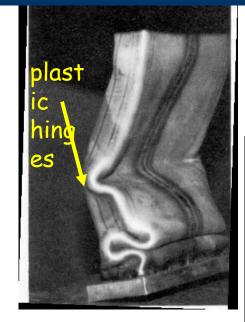


**Presenter Logo** 

## WHAT IS A FAILURE MODE ?

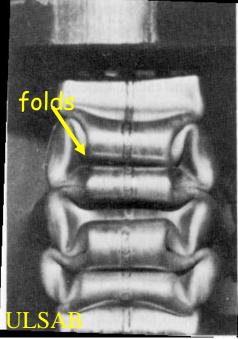
A failure mode is a sequence of events which activate a mechanism or dissipates energy in a component or device





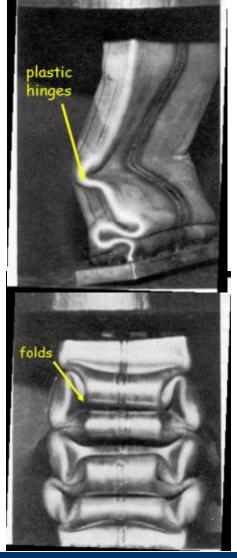
For a simple beam loaded in its axes, we we usually have two modes:

plastic hinge or axial crush









## OPTIMIZATION WITH MULTIPLE FAILURE MODES USING VRAND VISUALDOC

Our crushed beam must stop the impact of a 500 Kg mass at 5 m/sec The beam section is rectangular, thin walled We use a simplified model to switch from axial crushing to plastic hinges In our model, we have crushing when

What is the effect on optimization ?

 $ind_{min} < (a+b)/t < ind_{max}$ 

Optimization problem minimize beam mass deltaMax < 200 mm averAcc < 30g a < b (architecture constraint)

DOE analysis shows multiple failure modes

Discontinuities are graphically visible

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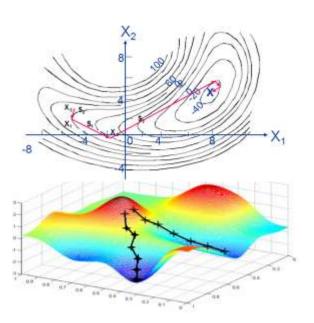
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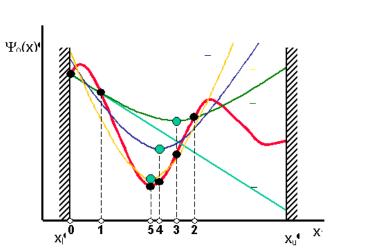
### FAMILIES OF OPTIMIZATION ALGORITHMS

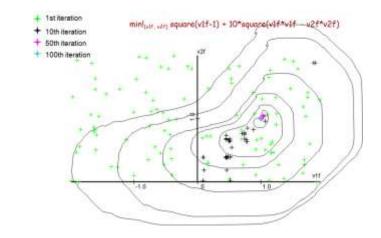
#### Gradient based



#### Response Surface Approximation

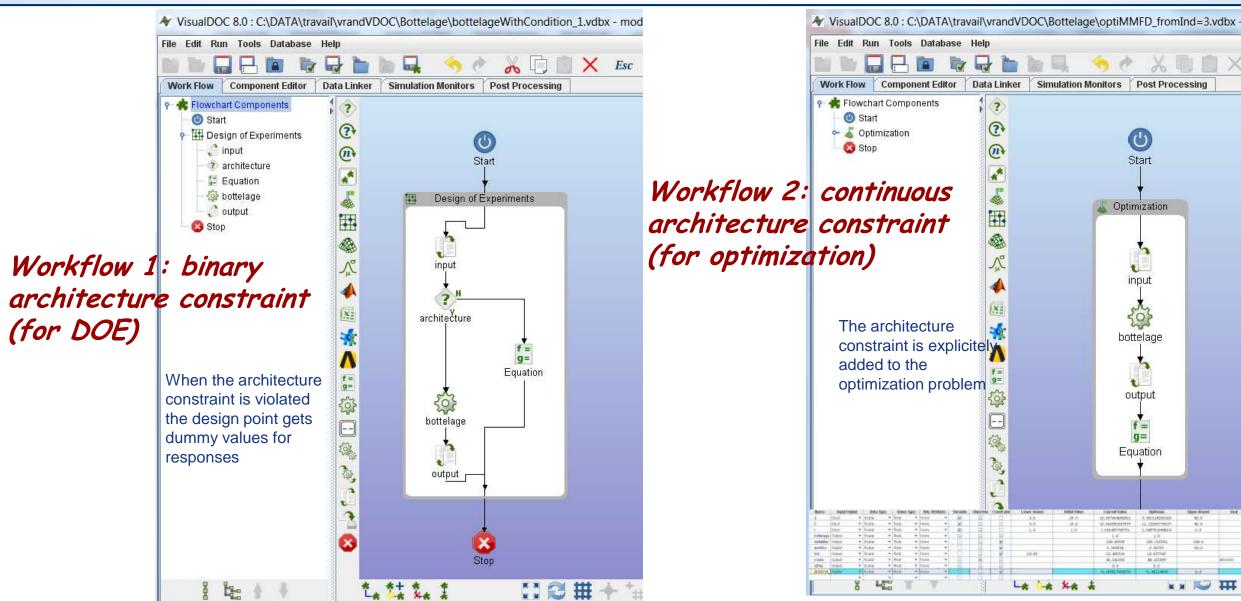
#### Genetic and evolutionary Strategies





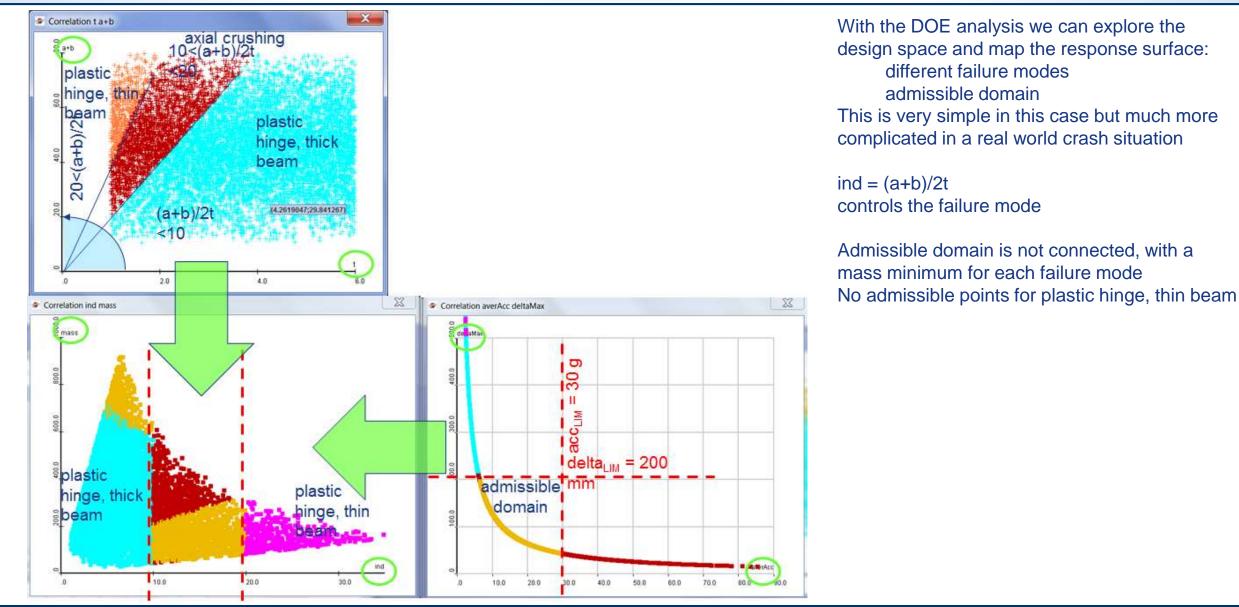














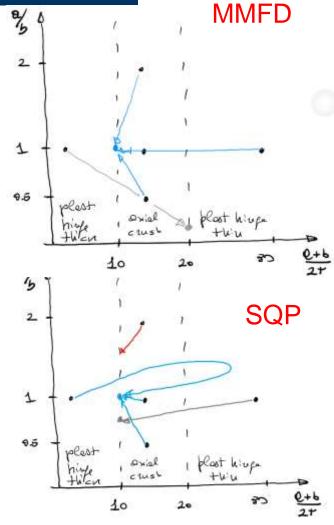


MMFD													
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	20	10	1	9,8	11,4	1,049	15	10,10486	60	2	0,859649	44,51	94
	10	20	1	8,3	13,22	1,05	15	10,24762	60	0,5	0,627837	44,97	120
	6	6	2	6,13	39,72	1,15	3	19,93478	48	1	0,15433	105,13	227
	30	30	1	10,54	10,54	1,05	30	10,0381	120	1	1	44,28	214
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	10	20	1	7,75	13,25	1,05	15	10	60	0,5	0,584906	44,09	32
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	30	30	1	35,12	40	3,82	30	9,832461	120	1	0,878	573,96	77

For all optimal designs displacement constraint is active

Two main gradient methods do not work all the time

Final point is ALWAYS on the border between two failure modes



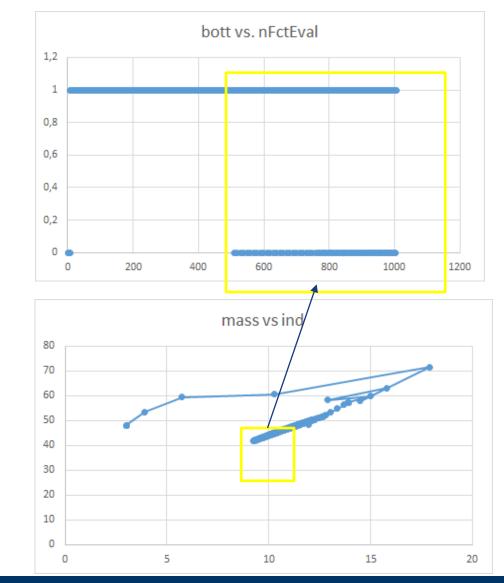
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#### What happens ?

- When the optimizer hits the border between two modes, the points evaluated can belong to one or the other.
- This introduces noise in ANY algorithm, even though the DOT algos perform very well, converging to the good optimal point most of the time.
- In the presence of such noise, results are unpredictable.
- Sometimes the optimizer converges to the right optimum, sometimes it does not.



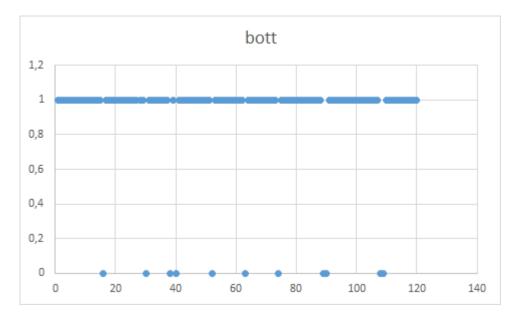




If we constraint the optimization to stay in the axial crushing area, putting a constraint on ind = (a+b)/2t , both MMFD and SQP have stable solutions.

MMFD														1		-	1			1		<b>n</b> /
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There are still some oscillations between the two failure modes, but the algo is good enough to filter them







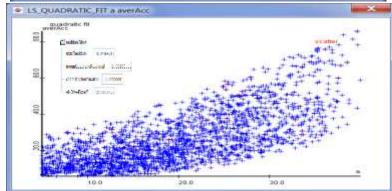
## OPTIMIZATION WITH RESPONSE SURFACE APPROXIMATION

RSA optimization performs very poorly on this problem.

This is normal because, due to the discontinuity in the response surface, a smooth approximation cannot work.

However, if we can constraint the problem to the axial crushing domain, the approximation is very good and the optimization should work.

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Using a change of variables, we can restrain the problem to the axial crushing failure mode.

The RSA methods reaches the best solution (in terms of mass) with the lowest number of function evaluations (23) of ALL methods studied.

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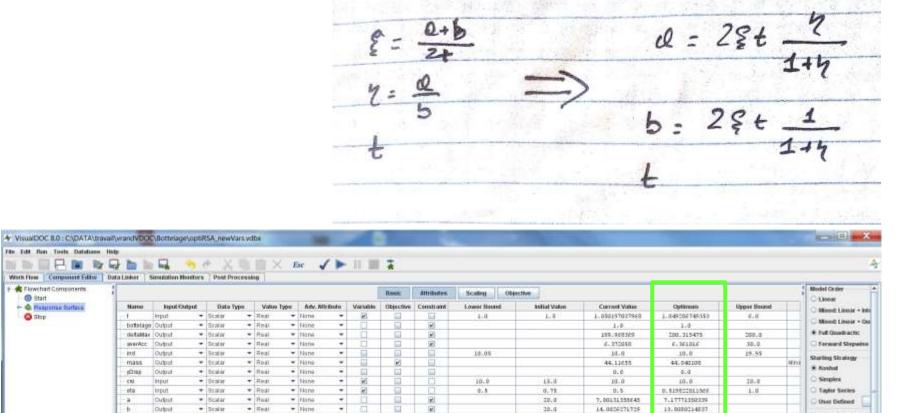
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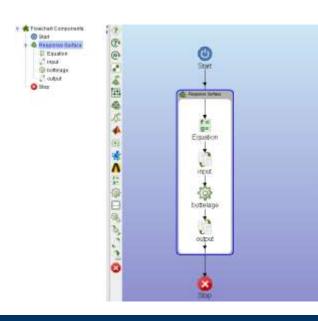
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GA and RSA converge to the 'good' solution, however the number of design points is very high



# **OPTIMIZATION - SUMMARY**



method	а			mass	nlter
Gradient - MMFD	5.18-10.99	10.64 - 16.40	1.04 - 1.05	44.16 - 45.91	55 - 142
Gradient - SQP	5.18 - 10.85	10.54 - 13.21	1.04 - 1.05	44.29 - 45.28	68 - 188
RSA	7,17	13,81	1,05	44,04	23
Genetic Algo	5.23	15.75	1.05	44.00	10000
Particle Swarm	5.24	15.75	1.05	44.00	1500

#### Only successful formulations are reported

All optimal point correspond to (a+b)/2t = 10