Optimization in the presence of bifurcations: A simple crash example

Edmondo Di Pasquale, SimTech
edmondo@simtech.fr

October 2, 2018 | Plymouth, MI

BIFURCATIONS OR RESPONSE SURFACE DISCONTINUITIES ARE UBIQUITOUS IN CRASH OPTIMIZATION

We explore three approaches to the problem of response surface discontinuities in crash problems

We can modify the design space to the failure modes sought

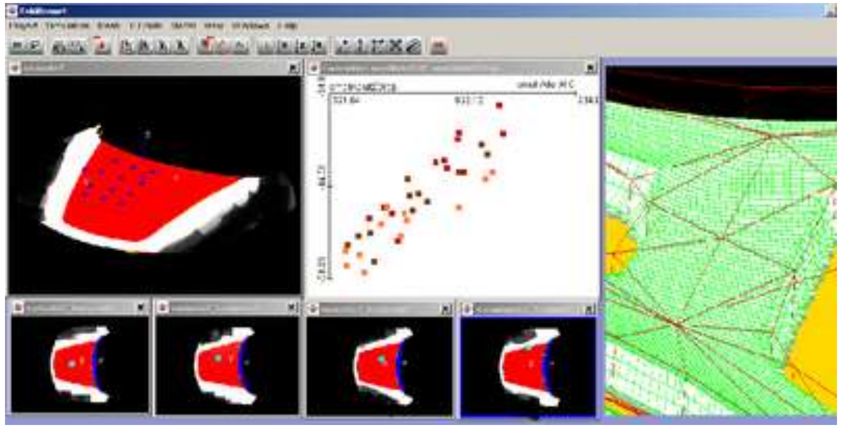
This was the approach we used in the first crash optimization project in France (2001), where we introduced geometrical and assembly design variables to control failure modes

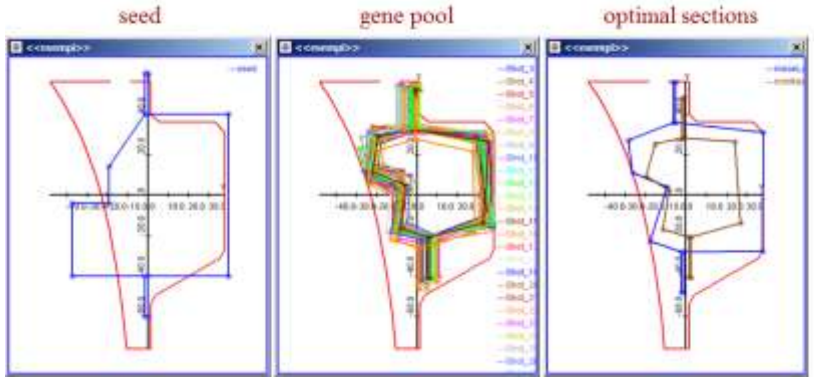
We can modify the problem formulation to avoid the discontinuity

This was the approach we used for bonnet design for pedestrian safety (2007), where we introduced an additional constraint on stroke to model secondary impact

We can simplify the problem so that the design point runs very quickly

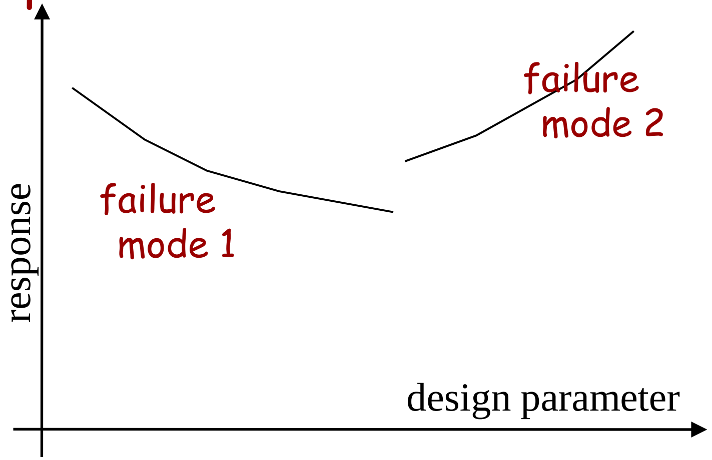
In the design of energy absorption components (2009) we had very short CPU times and we could use genetic algorithms



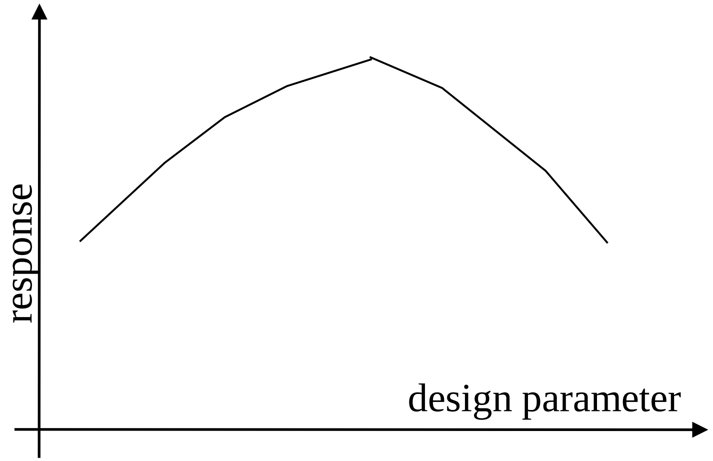


WHAT DO WE MEAN BY BIFURCATION

In this presentation, we call bifurcation a discontinuity of the response surface in our optimization problem



A discontinuity in the response surface itself is associated to a physical bifurcation, ***typically the change of a failure mode***

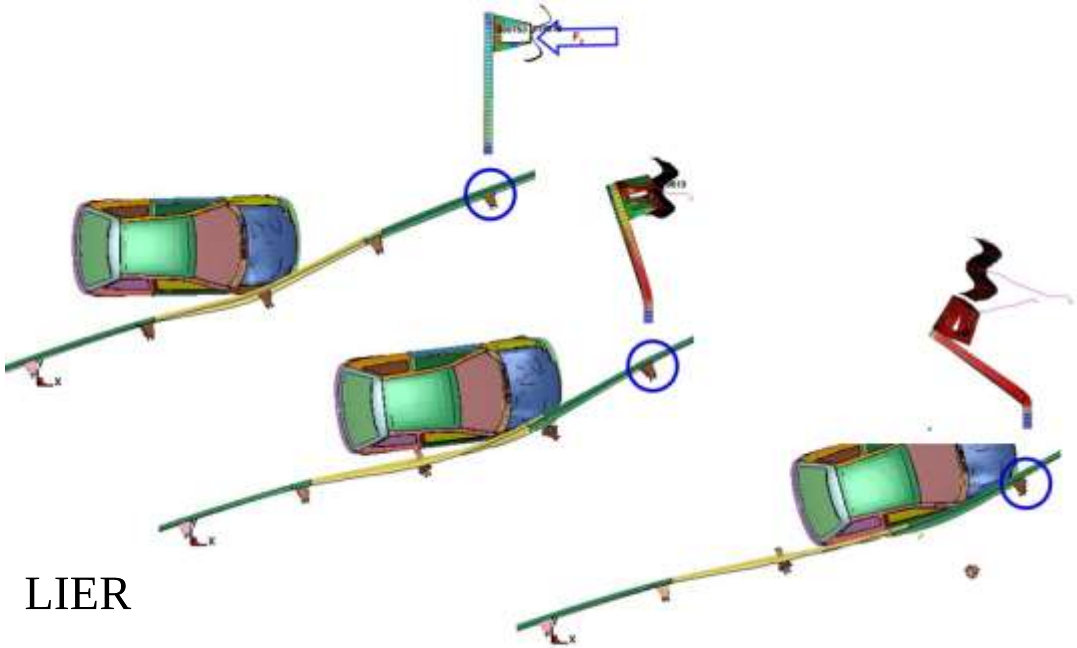


A discontinuity in the first derivative of the response surface is much more common Examples are most assembly problems

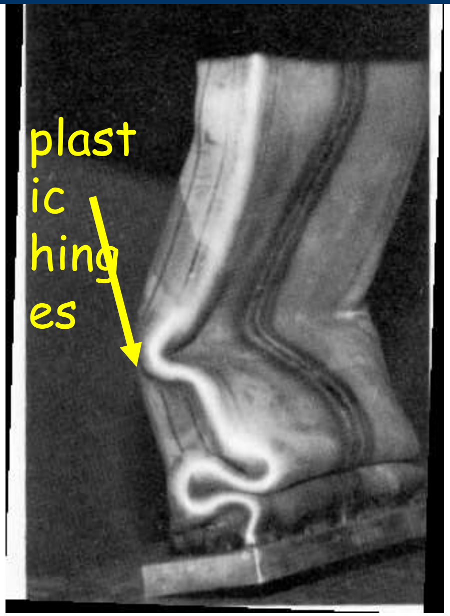
Bifurcations may cause failure of ALL optimization algorithms o Gradient or Surface Approximation algorithms are particularly affected

WHAT IS A FAILURE MODE ?

A failure mode is a sequence of events which activate a mechanism or dissipates energy in a component or device

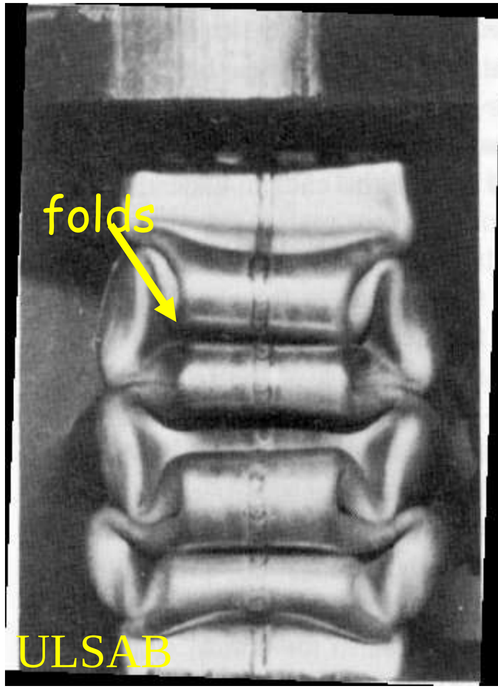


LIER

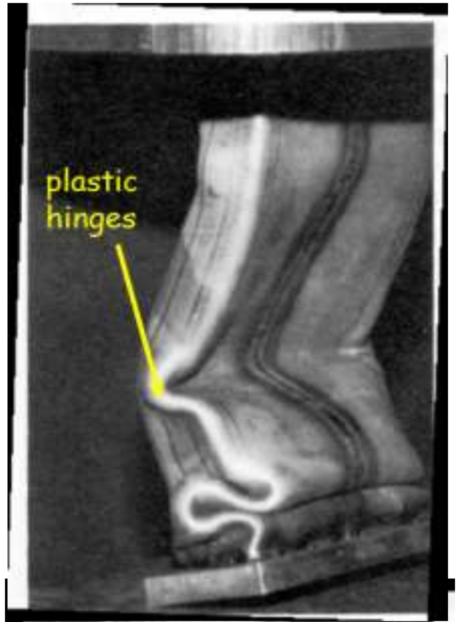


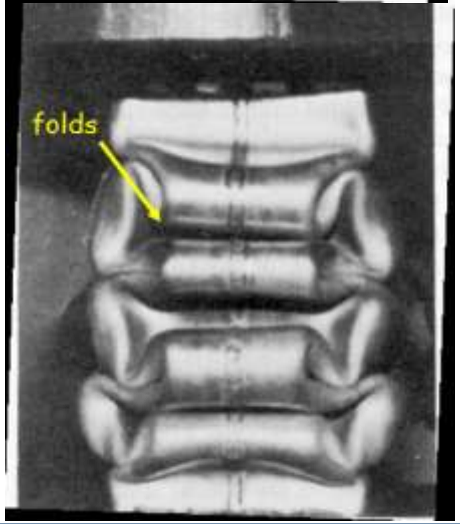
For a simple beam loaded in its axes, we we usually have two modes:

plastic hinge or
axial crush



OPTIMIZATION WITH MULTIPLE FAILURE MODES USING VRAND VISUALDOC





Our crushed beam must stop the impact of a 500 Kg mass at 5 m/sec

The beam section is rectangular, thin walled

We use a simplified model to switch from axial crushing to plastic hinges

In our model, we have crushing when

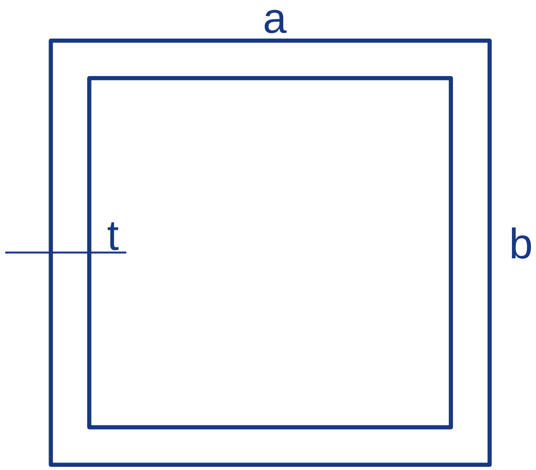
$$ind_{min} < (a+b)/t < ind_{max}$$

Optimization problem

- minimize beam mass
- deltaMax < 200 mm
- averAcc < 30g
- a < b (architecture constraint)

DOE analysis shows multiple failure modes

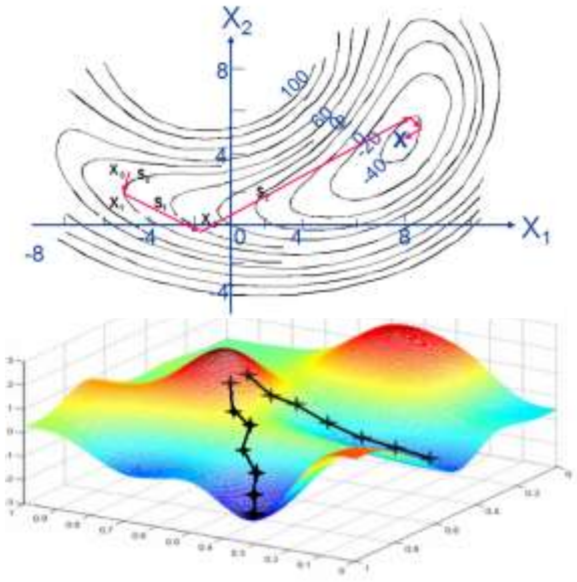
Discontinuities are graphically visible



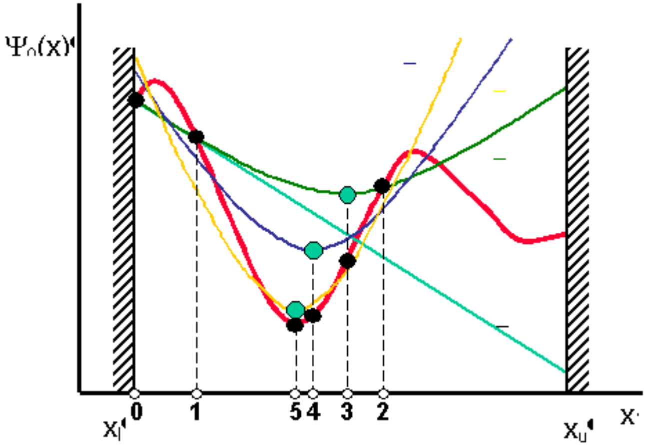
What is the effect on optimization ?

FAMILIES OF OPTIMIZATION ALGORITHMS

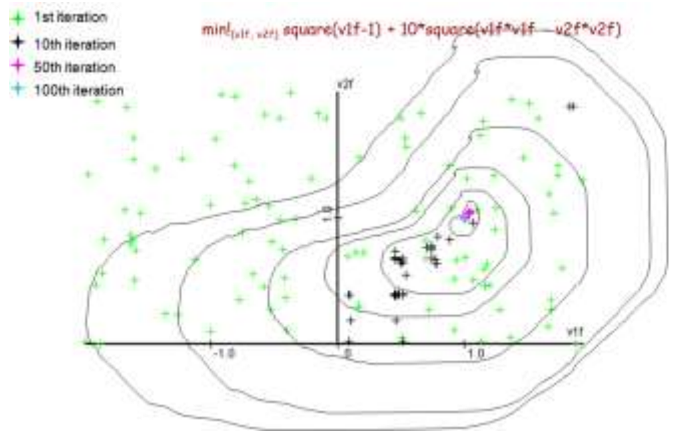
Gradient based

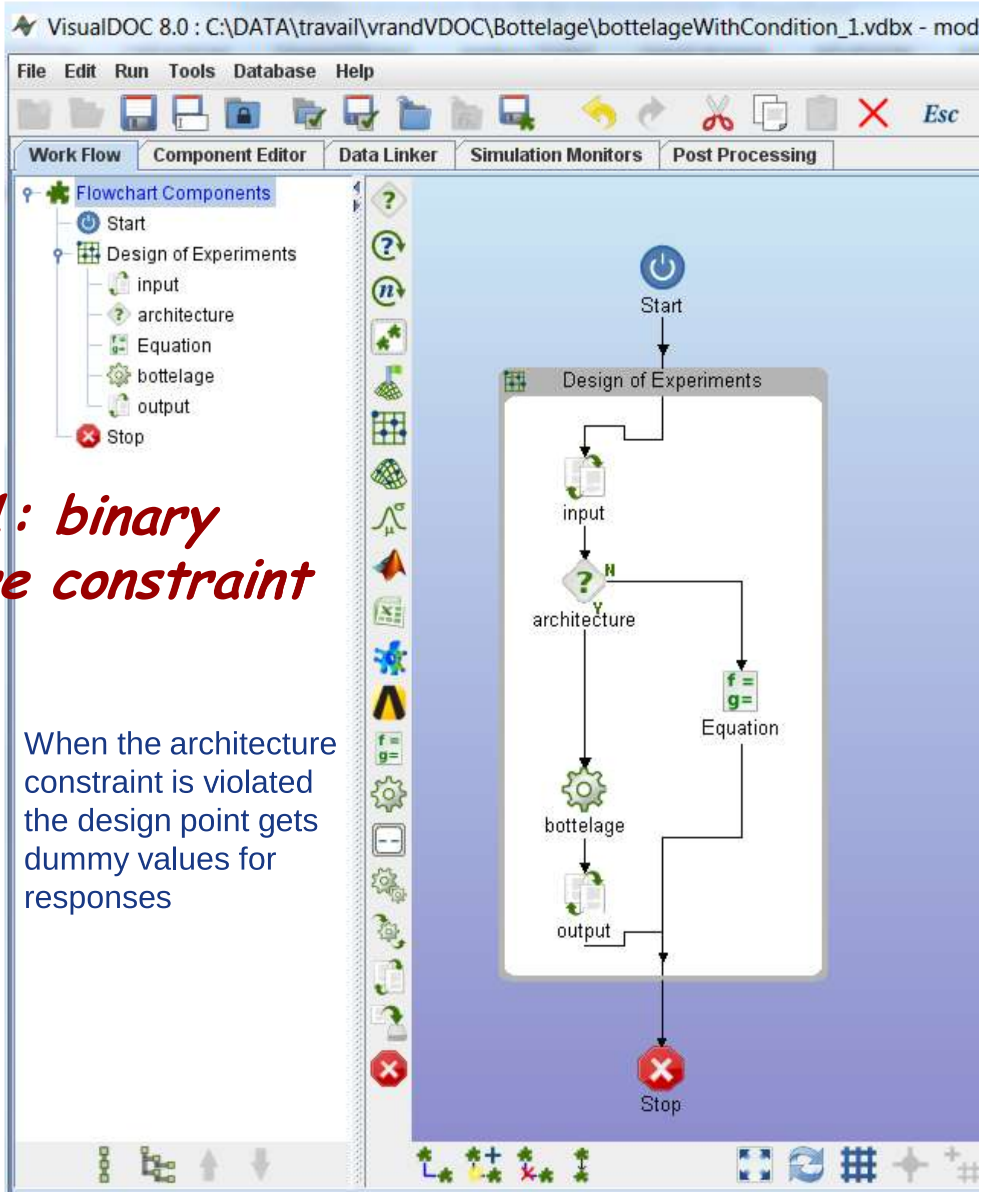


Response Surface Approximation



Genetic and evolutionary Strategies

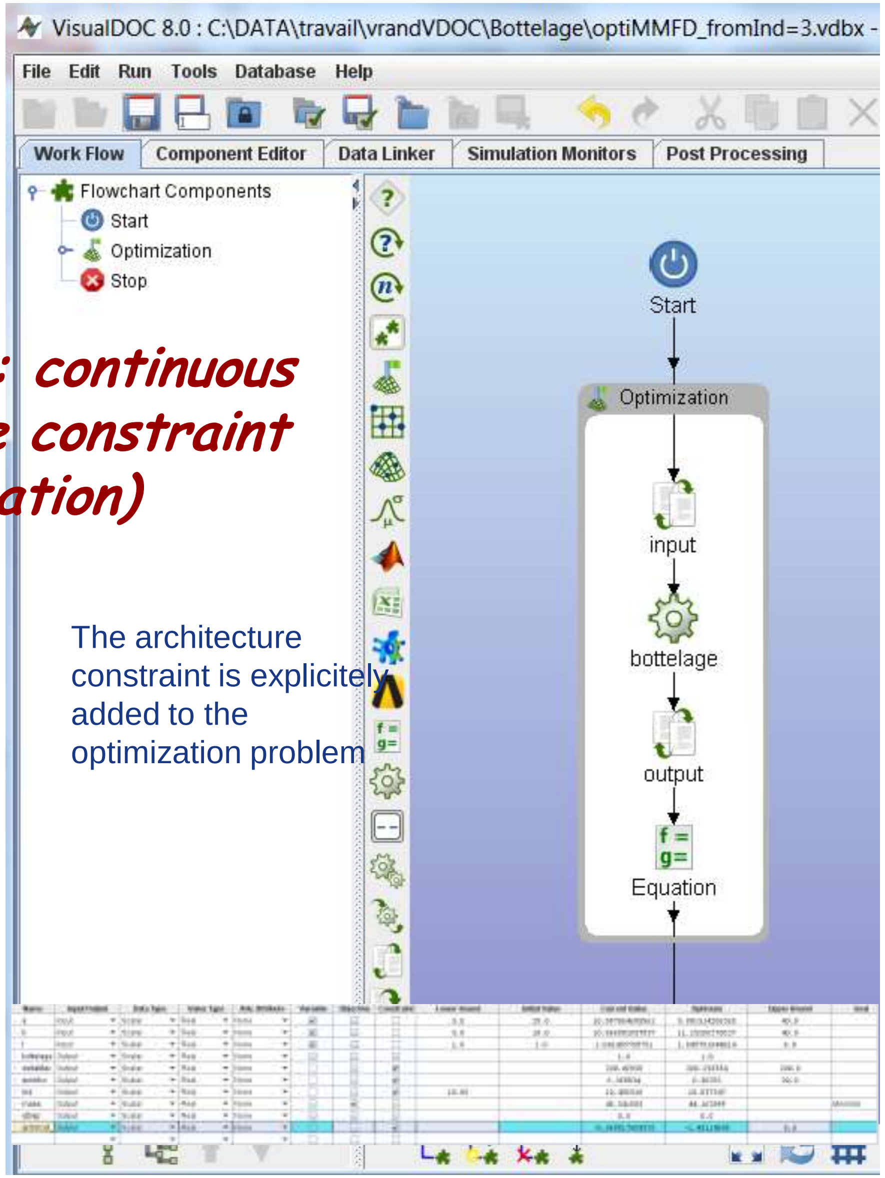




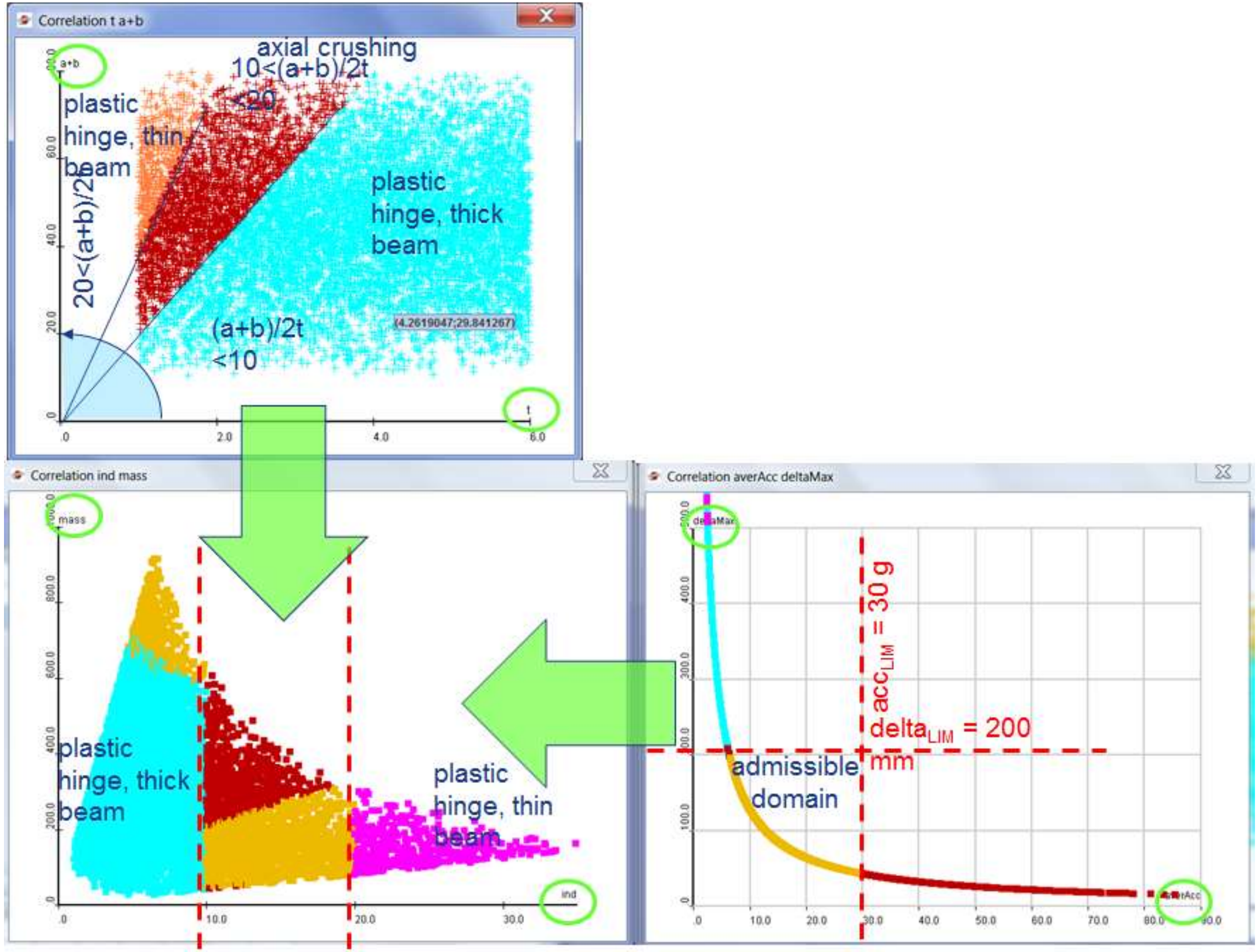
Workflow 1: binary architecture constraint (for DOE)

When the architecture constraint is violated the design point gets dummy values for responses

Workflow 2: continuous architecture constraint (for optimization)



The architecture constraint is explicitely added to the optimization problem



With the DOE analysis we can explore the design space and map the response surface:

- different failure modes
- admissible domain

This is very simple in this case but much more complicated in a real world crash situation

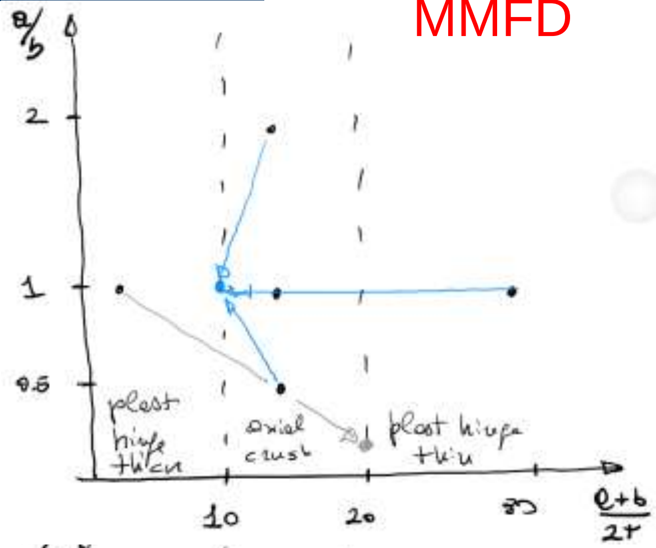
ind = (a+b)/2t
controls the failure mode

Admissible domain is not connected, with a mass minimum for each failure mode
No admissible points for plastic hinge, thin beam

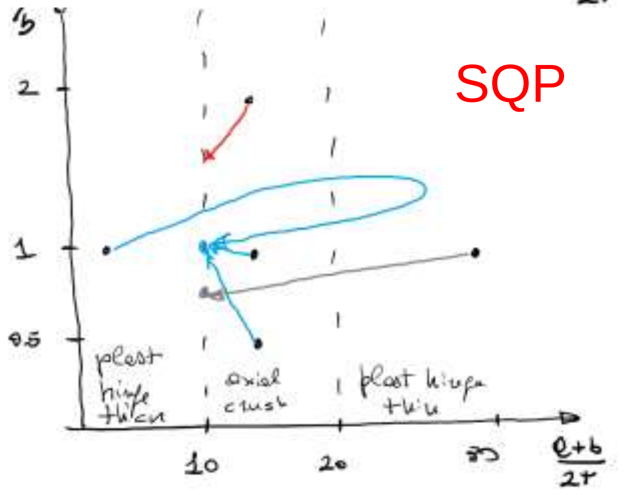
OPTIMIZATION WITH GRADIENT BASED METHODS

MMFD													
	a0	b0	t0	aOpt	bOpt	tOpt	ind0	indOpt	m0	(a/b)0	(a/b)Opt	mOpt	nFctEval
	15	15	1	10,85	10,85	1,04	15	10,43269	60	1	1	45,28	120
	20	10	1	9,8	11,4	1,049	15	10,10486	60	2	0,859649	44,51	94
	10	20	1	8,3	13,22	1,05	15	10,24762	60	0,5	0,627837	44,97	120
	6	6	2	6,13	39,72	1,15	3	19,93478	48	1	0,15433	105,13	227
	30	30	1	10,54	10,54	1,05	30	10,0381	120	1	1	44,28	214
SQP	a0	b0	t0	aOpt	bOpt	tOpt	ind0	indOpt		(a/b)0	(a/b)Opt	mOpt	nFctEval
	15	15	1	10,49	10,49	1,049	15	10	60	1	1	44,09	32
	20	10	1	13,24	7,76	1,05	15	10	60	2	1,706186	44,11	22
	10	20	1	7,75	13,25	1,05	15	10	60	0,5	0,584906	44,09	32
	6	6	2	5	15,99	1,049	3	10,00477	48	1	0,312695	44,06	1005
	30	30	1	35,12	40	3,82	30	9,832461	120	1	0,878	573,96	77

MMFD



SQP



For all optimal designs displacement constraint is active

Two main gradient methods do not work all the time

Final point is ALWAYS on the border between two failure modes

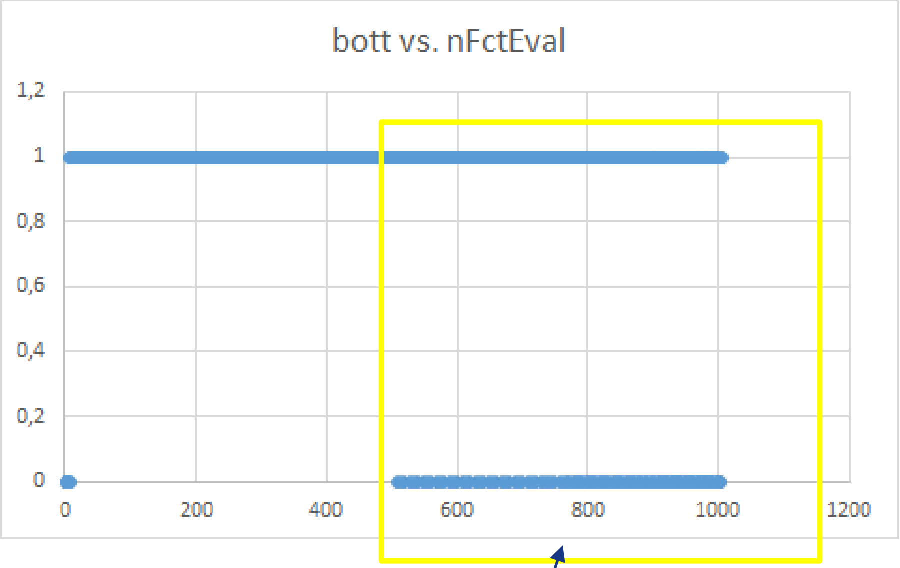
What happens ?

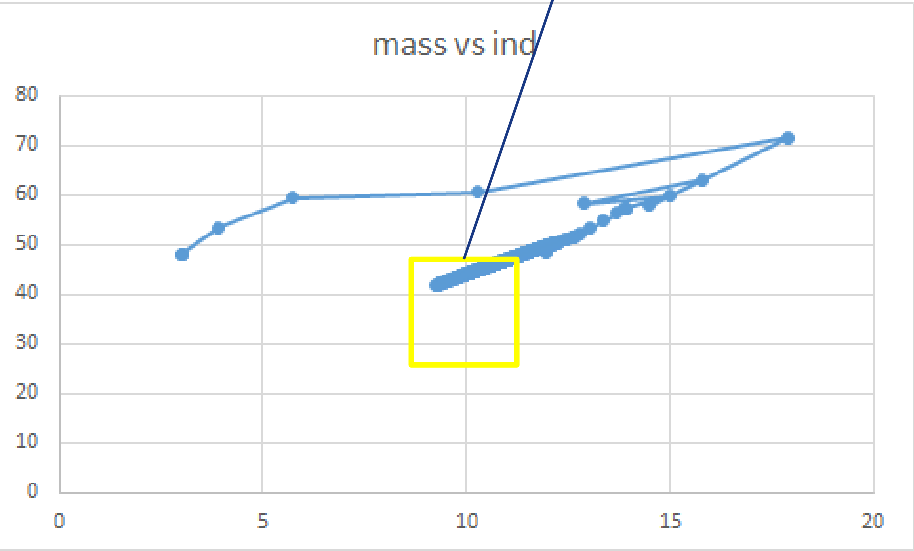
When the optimizer hits the border between two modes, the points evaluated can belong to one or the other.

This introduces noise in ANY algorithm, even though the DOT algos perform very well, converging to the good optimal point most of the time.

In the presence of such noise, results are unpredictable.

Sometimes the optimizer converges to the right optimum, sometimes it does not.



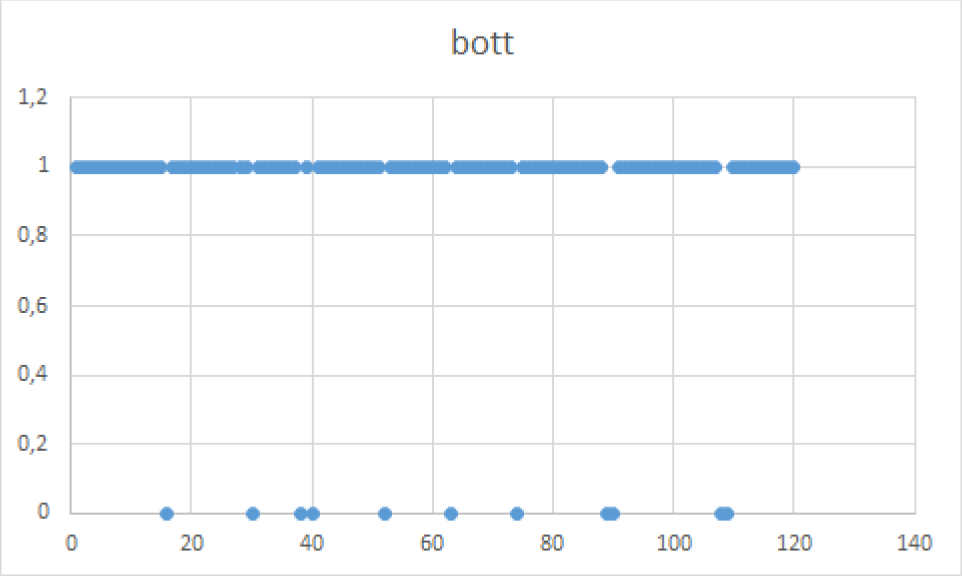


If we constraint the optimization to stay in the axial crushing area, putting a constraint on ind = (a+b)/2t , both MMFD and SQP have stable solutions.

MMFD	a0	b0	t0	aOpt	bOpt	tOpt	ind0	indOpt	m0	(a/b)0	(a/b)Opt	mOpt	nFctEval
	15	15	1	10,99	10,99	1,04	15	10,5673077	60	1		45,75	142
	20	10	1	9,8	11,25	1,048	15	10,0429389	60	2	0,8711111	44,16	55
	10	20	1	8,41	13,67	1,04	15	10,6153846	60	0,5	0,615215	45,91	102
	6	6	2	5,18	16,4	1,04	3	10,375	48	1	0,3158536	45,07	118
	30	30	1	10,54	10,64	1,05	30	10,0857143	120	1	0,990601	44,53	62

SQP	a0	b0	t0	aOpt	bOpt	tOpt	ind0	indOpt		(a/b)0	(a/b)Opt	mOpt	nFctEval
	15	15	1	10,85	10,85	1,04	15	10,4326923	60	1		45,28	120
	20	10	1	9,8	11,4	1,05	15	10,0952381	60	2	0,8596491	44,51	68
	10	20	1	8,31	13,21	1,05	15	10,247619	60	0,5	0,6290688	44,97	88
	6	6	2	16,65	10,65	1,05	3	13	48	1	1,5633802	44,61	165
	30	30	1	10,54	10,54	1,05	30	10,0380952	120	1		44,29	188

There are still some oscillations between the two failure modes, but the algo is good enough to filter them

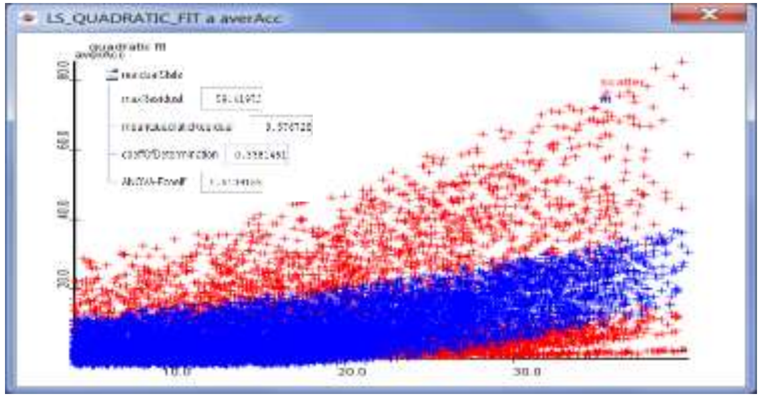


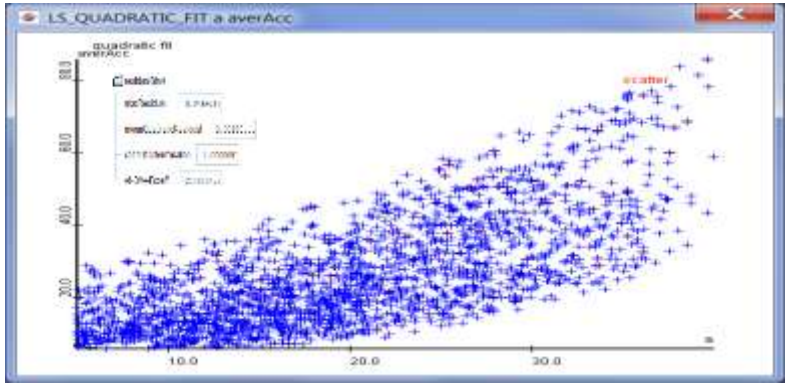
OPTIMIZATION WITH RESPONSE SURFACE APPROXIMATION

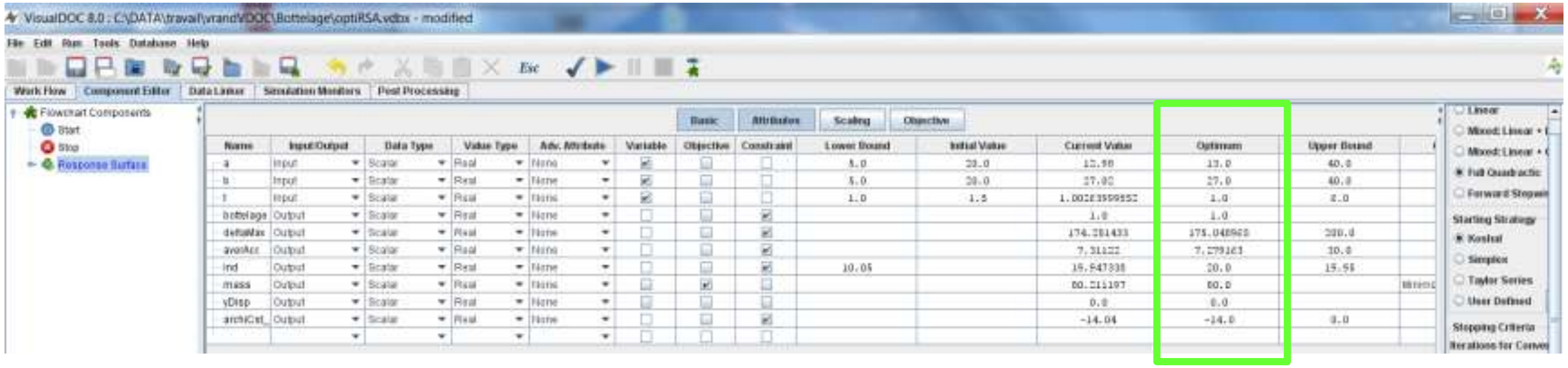
RSA optimization performs very poorly on this problem.

This is normal because, due to the discontinuity in the response surface, a smooth approximation cannot work.

However, if we can constraint the problem to the axial crushing domain, the approximation is very good and the optimization should work.



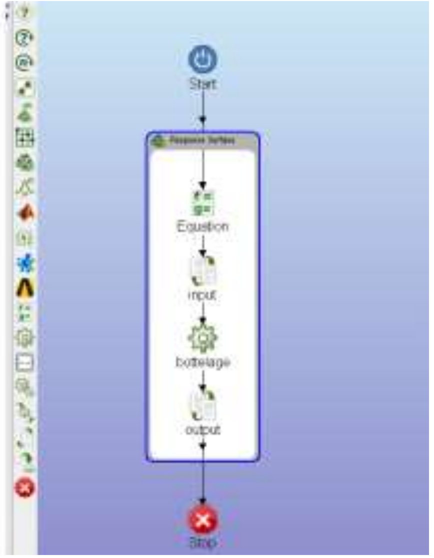


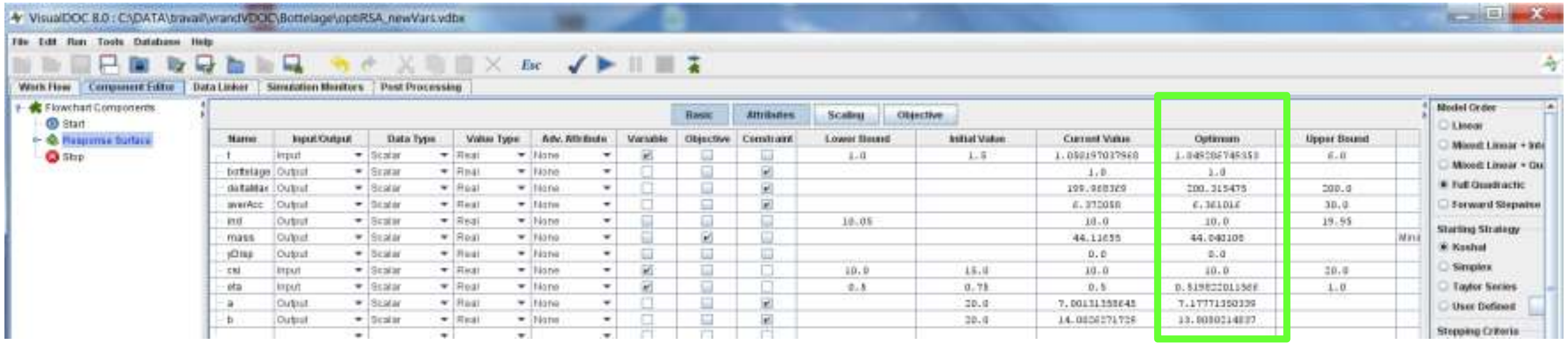


Using a change of variables, we can restrain the problem to the axial crushing failure mode.

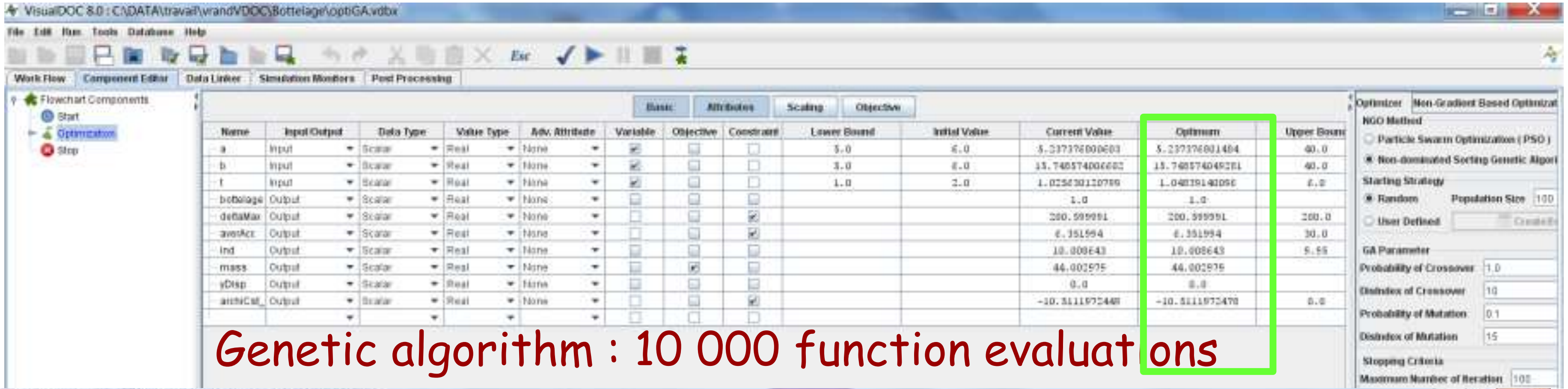
The RSA methods reaches the best solution (in terms of mass) with the lowest number of function evaluations (23) of ALL methods studied.

$$\xi = \frac{a+b}{2t} \qquad \eta = \frac{a}{b} \qquad t \quad \Longrightarrow \quad a = 2\xi t\,\frac{\eta}{1+\eta} \qquad b = 2\xi t\,\frac{1}{1+\eta} \qquad t$$

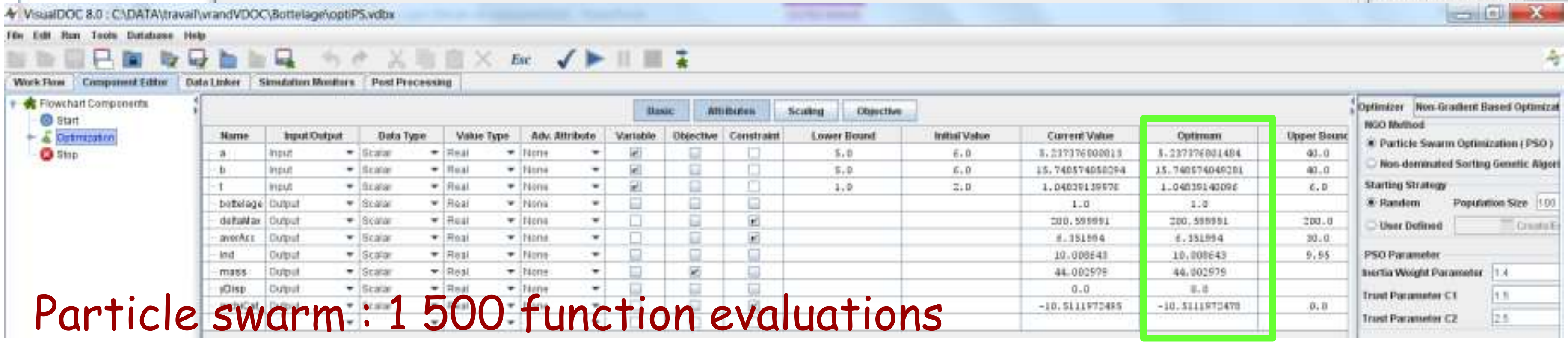




OPTIMIZATION WITH EVOLUTIONARY ALGORITHMS



Genetic algorithm : 10 000 function evaluations



Particle swarm : 1 500 function evaluations

GA and RSA converge to the 'good' solution, however the number of design points is very high

OPTIMIZATION - SUMMARY

Pres

method	a	b	t	mass	nIter
Gradient - MMFD	5.18-10.99	10.64 - 16.40	1.04 - 1.05	44.16 - 45.91	55 - 142
Gradient - SQP	5.18 - 10.85	10.54 - 13.21	1.04 - 1.05	44.29 - 45.28	68 - 188
RSA	7,17	13,81	1,05	44,04	23
Genetic Algo	5.23	15.75	1.05	44.00	10000
Particle Swarm	5.24	15.75	1.05	44.00	1500

Only successful formulations are reported

All optimal point correspond to $(a+b)/2t = 10$