

Michell Truss : Shape Optimization vs. Analytical Solution

Shiro YOSHIDA^{*1}

^{*1} SHONAN, R&D, Inc.
#402 Shonan Homes, 3-19-17 Katase-Kaigan, Fujisawa, 251-0035 Japan

The ultimate goal of the shape optimization is that standard deviation of the stress distribution becomes zero (each members in a structure has the stress of the same level). The shape optimization software is developed towards this goal and popularized steadily. Meanwhile, Michell truss is, because of its nature, an appropriate subject for the code verification and calculation verification. In this context, Michell truss is modeled by BAR, SHELL, 3/D SOLID elements, and trend of stress distribution is investigated.

Key Words : Michell Truss Shape Optimization Morphing

1. Preface

According to Herbert Simon (1978 Nobel Prize in Economics), decision-making is the act of trying to find the best solution in certain circumstances, in order to achieve a particular goal, from multiple alternatives [1].

The goal of decision-making is, making a rational choice, (i) comparing the collection of necessary information, (ii) recognition of the right goal, (iii) the strategy proposal for the goals, (iv) planning of the execution plan selection, and (v) the best course of action.

(a) Limit of rationality:

Simon assumes Human as decision maker to try to decide as rationally as possible. But, due to the limit in the rationality, the decision-making cannot be perfectly reasonable.

The limits of rationality include:

- i) Limit in decision-making : It is impossible to collect all the necessary information (limit in the information gathering capabilities)
- ii) Limit in judgment ability : It is impossible to prospect completely all the results of the action based on the decision-making

(b) Decision-making process:

According to Simon, decision-making process includes:

- i) Information collection : Information analysis follows Problem definition.
- ii) Searching alternatives : Explore alternatives for problem solving that was discovered in the previous step. But, because there is a limit in the information gathering capability, all alternatives cannot be sought.
- iii) Alternative evaluations : Predict the outcome of each alternative that has been searched, which follows the comparison and evaluation. But, due to the limit in the evaluation capability, it is impossible to completely forecast and evaluate all outcomes.
- iv) Alternative selection : After anticipating the results of each alternative, by comparison and evaluation, select relatively best alternative which meet a certain level of goal, on the basis of the evaluation. However, due to the limit in the rationality, a relatively best alternative must be selected among the limited number of choices.

^{*1} SHONAN R&D, Inc. (#1201 SHONAN Towers, 1-2-9 Katase-Kaigan, 251-0035 JAPAN) , E-mail: Sir_Yoshida@nifty.com

- v) Alternative implementations : Execute the selected alternative, and ensure a certain achievement.
- vi) Feedback : Evaluate and analyze the results of the execution, and feedback their lesson to the next decision-making.

By applying the above-mentioned procedure to the actual new product developing process, the following activities can be found:

- i) Information gathering : Case study of the product of competing companies.
- ii) Information collection : The performance improvement through the size and shape change.
- iii) Alternative evaluations : Evaluation by experiments and analysis.
- iv) Alternative selection : Select the relatively best alternative according to the level of evaluation indicators.
- v) Alternative implementations : implementation of prototyping, testing, simulation.
- vi) Feedback : Design changes such as alternative exchange.

(3) Prerequisites of alternative evaluation

In order to evaluate the alternatives reasonably, the following prerequisites are required:

- i) There is more than one alternative.
- ii) Sufficient knowledgeable to predict the effect of each alternative.
- iii) There exists value system that can evaluate the effect of alternative.

(4) Evaluation criteria of alternative

An alternative is optimal when the following condition is satisfied:

- i) all the available alternatives available has been found.
- ii) a set of comparable various scale (utility function) exists.
- iii) An alternative is superior to all the other alternatives in view of various scales.

However, the field of new product development, in many cases, is under the following circumstances:

- i) It is impossible to identify all the alternatives that are feasible.
- ii) It is difficult to set the suitable scale necessary to compare alternatives.

Meanwhile, it is difficult to reproduce the environment such as the "zero gravity", "cryogenic", "vacuum" on the ground, but CAE, based on the finite element method, plays an important role in providing a comparable scale.

Therefore, CAE can verify the various design patterns in the initial design stage, where the design specification has not yet solidified. However CAE, based on the finite element method, is not useful for alternatives search.

On the contrary, in order to automate the alternatives search phase, shape optimization software is developed.

2. Shape Optimization : BAR Element Model

Historically speaking, the phase and shape optimization software of the structure was originated by the presentation of Schmit of 1960. Though the software is supported by a rigorous mathematical theory, there was leap of resultant shape, when compared to conventional mechanical parts. As a result, when viewed from the eyes of design engineers with long experience, the results were reluctantly accepted. Because of this historical background, phase and shape optimization software has not spreaded easily. But as a result of long engineering effort, in recent years, the software begins to collect the attention in a real manufacturing field.

(1) Part shape optimization

In 1904, Michell formulated the stress distribution of cantilever structure as shown in Fig.1, which is composed of tensile members and compressive members, intersecting at right angles each other[2][3].

This formulation corresponds to the slip line, where the maximum shear stress is generated, in 2-dimensional press forming process. Namely, slip lines and shape optimization are mathematically equivalent, and can be expressed in the same formulation.

On the other hand, Bendsoe proposed the process of optimization to maximize stiffness, varying the density, which generates a similar shape with Michell's theory[4].

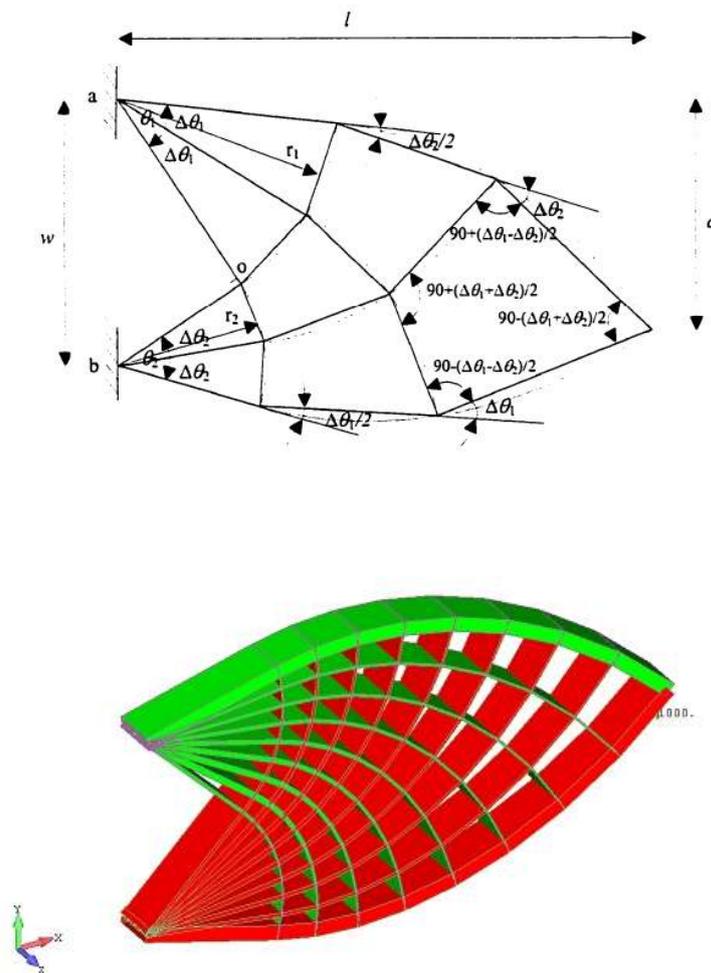
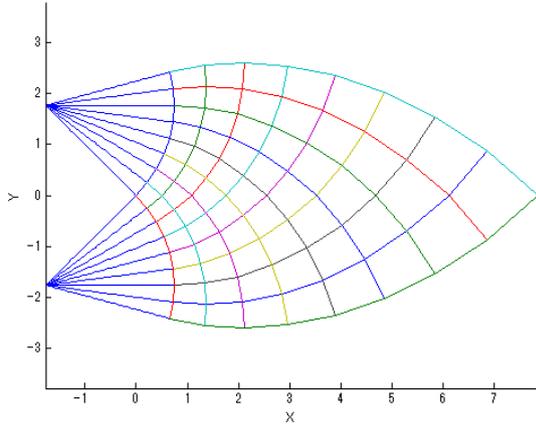


Fig.1 Michell Truss

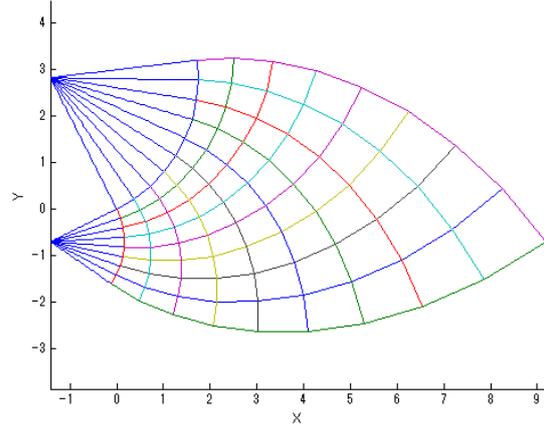
With the procedure as shown in Fig.1, members are equally placed as shown in Fig.2;

BASIC CANTILEVER, upper angle = 60, upper radius = 2.5; lower angle = 60; lower radius = 2.5



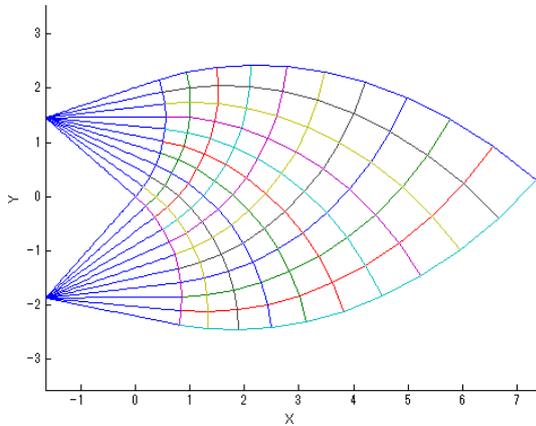
8/8 Equal Division / Fan Radius : 1 to 1

BASIC CANTILEVER, upper angle = 70.32, upper radius = 3.162; lower angle = 61.45; lower radius = 1.581



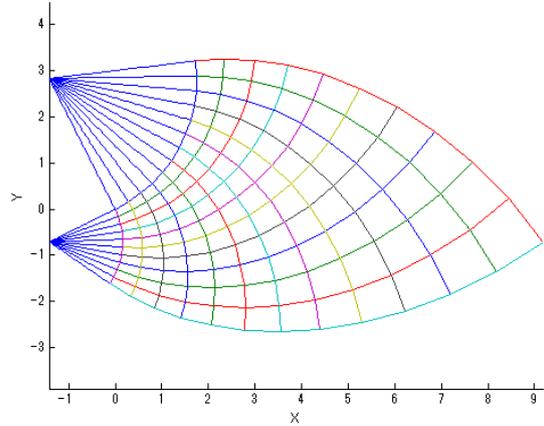
8/9 Equal Division / Fan Radius : 2 to 1

BASIC CANTILEVER, upper angle = 60, upper radius = 2.2; lower angle = 60; lower radius = 2.5



10/10 Equal Division / Fan Radius : 1 to 1

BASIC CANTILEVER, upper angle = 70.32, upper radius = 3.162; lower angle = 61.45; lower radius = 1.581



10/12 Equal Division / Fan Radius : 2 to 1

Fig.2 Michell Truss : Members Placement

In this report, geometrical parameters are specified as follows;

Angular Aperture of Leftmost Fans : $\theta = 60(\text{deg})$

Angular Aperture of Fan Members : $\Delta\theta = 7.5(\text{deg})$

Radial Length of Fan Members : $R = 63.5(\text{mm})$

Overall Length : $L = 244(\text{mm})$

Overall Breadth : $B = 132(\text{mm})$

The notation of each member is shown in Fig.3, where the uppermost members have 1(one) as the first digit, 9(nine) as the second digit, and 0 thru 8 as the third digit.

The section area of each member is tabulated in Table 1 and Table 2, according to the second and the third digits.

Meanwhile, the lowermost members have 2(two) as the first digit.

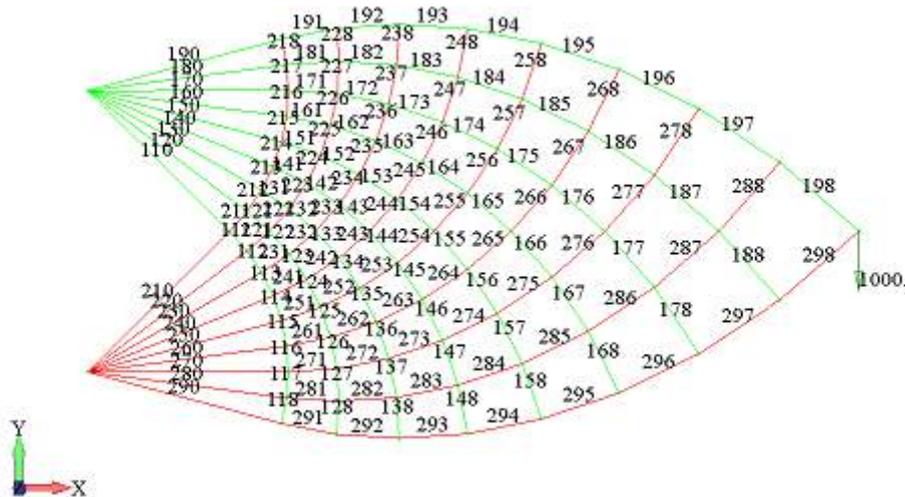


Fig.3 Notation of each Member

Table 1 Section Area Ratio of Each Member : Before Optimization

	1	2	3	4	5	6	7	8	9
0	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	2.500
1	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	2.500
2	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	2.500
3	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	2.500
4	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	2.500
5	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	2.500
6	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	2.500
7	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	2.500
8	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	2.500

Table 2 Section Area Ratio of Each Member : After Optimization

	1	2	3	4	5	6	7	8	9
0	0.814	1.547	1.425	1.311	1.201	1.097	1.000	0.908	3.487
1	0.765	1.457	1.346	1.243	1.143	1.049	0.961	0.877	3.479
2	0.669	1.278	1.190	1.106	1.028	0.950	0.879	0.810	3.450
3	0.581	1.112	1.045	0.979	0.917	0.856	0.799	0.744	3.421
4	0.499	0.958	0.908	0.858	0.811	0.765	0.722	0.680	3.392
5	0.424	0.815	0.781	0.745	0.713	0.678	0.647	0.617	3.364
6	0.354	0.685	0.663	0.639	0.618	0.594	0.575	0.554	3.336
7	0.290	0.564	0.552	0.539	0.528	0.516	0.504	0.492	3.308
8	0.231	0.454	0.453	0.446	0.442	0.438	0.436	0.432	3.281

It is shown in Fig.4 and Fig.5, the FEM solid model geometry before and after optimization respectively.

In this case, the weight minimization option of the phase and shape optimization software GENESIS (VR & D, Inc, Colorado, USA) is applied to the cantilever[5].

The comparison of the results of calculation with Bar element is described here in after.

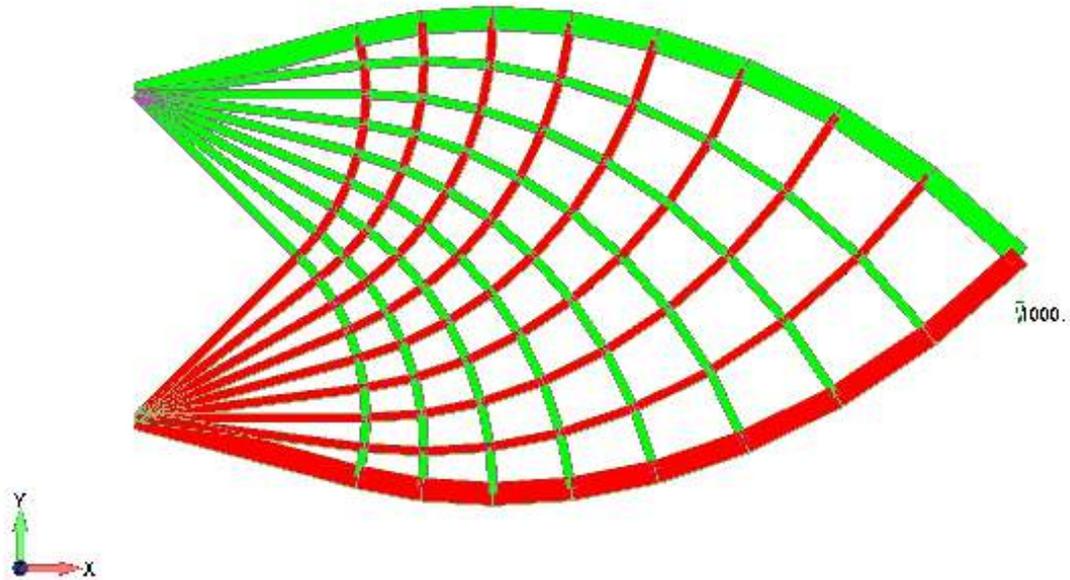


Fig.4 Michell Truss (Before Optimization)

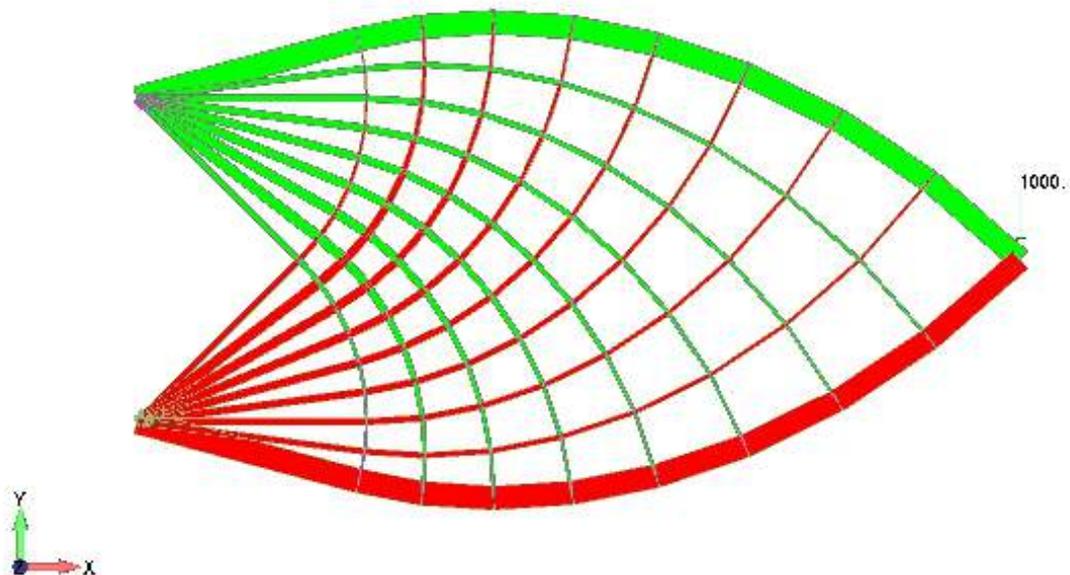


Fig.5 Michell Truss (After Optimization)

Synthetic Function for Axial Stiffness is defined with the following relation (See APPENDIX : A-1):

$$F(\text{AREA}, \text{FORCE}) = \text{FORCE} / \text{AREA}$$

Optimization conditions in the GENESIS are as follows,

- (1) design variable (Bar element cross-sectional area)
 - Outer edge member: Initial / 130 (mm²) => Upper bound / 130 (mm²) Lower bound 120 (mm²)
 - Inner member : Initial / 50 (mm²) => Upper bound / 58 (mm²) Lower bound 9 (mm²)
- (2) objective variable : Mass minimization
- (3) constraints (axial stress / maximum principal stress)
 - Upper bound / 8 (N / mm²) Lower bound -8 (N / mm²) Min

Table 3 shows quantitative evaluation of uniformness of axial stress, by comparing standard deviation.

Before optimization:

Mean tensile stress = 3.913 (N / mm²) standard deviation = 0.242

Mean compressive stress = -3.913 (N / mm²) standard deviation = 0.242

Maximum displacement = 2.519 (mm) Mass = 0.261 (kg)

After optimization:

Mean tensile stress = 5.926 (N / mm²) standard deviation = 0.031

Mean compressive stress = -5.926 (N / mm²) standard deviation = 0.031

Maximum displacement = 2.998 (mm) Mass = 0.202 (kg)

The results show a significant reduction of the standard deviation from 0.242 to 0.031, with 51 (%) increases in the mean stress, 19 (%) increases in the maximum displacement, 23 (%) decreases in mass.

Table3 Michell Truss :Before and After Optimization

Before Optimization			After Optimization		
	Average	S.D.(σ)		Average	S.D.(σ)
Upper	3.913	0.242	Upper	5.926	0.031
Lower	-3.913	0.242	Lower	-5.926	0.031
	δ max	2.519		δ max	2.999
	Mass	0.261		Mass	0.202

Fig.6 and Fig.7 show the axial stress distribution of initial shape, and after weight minimization respectively. In both diagrams, tensile member bears axial stress with positive values, and compressive member bears the axial stress with negative values. By comparison, the axial stress value of each member can be found to be uniform respectively. That is, after optimizing the cross-sectional area of each member, each member loads uniform axial stress – tensile and compressive.

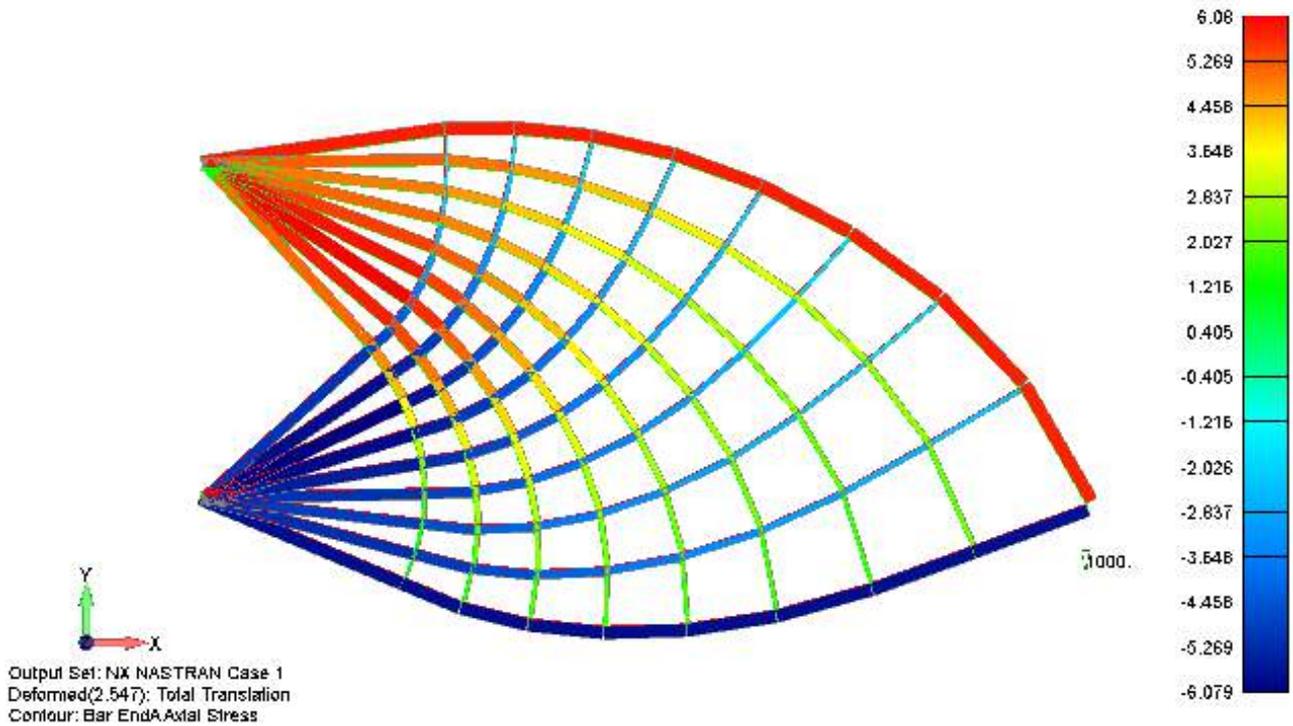


Fig.6 Michell Truss : Before Optimization

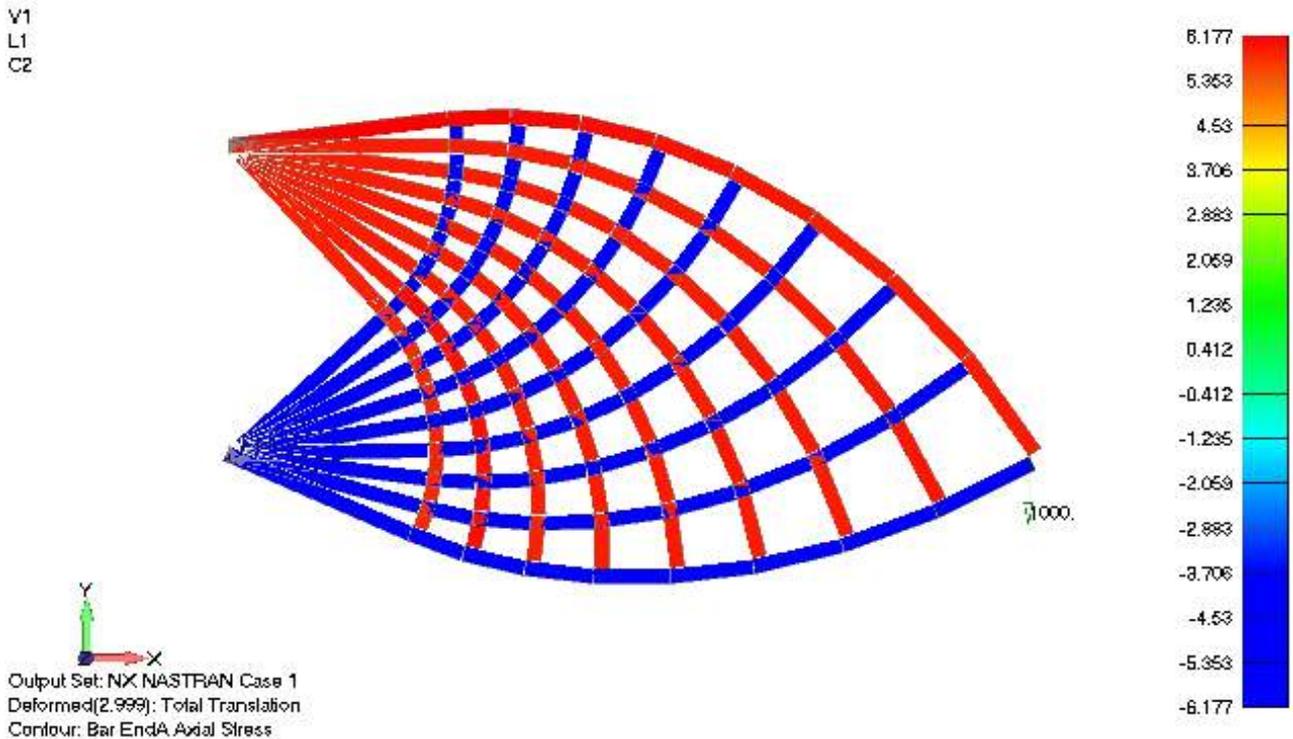


Fig.7 Michell Truss : After Optimization

As "Comparable various scale" described in the previous section, the standard deviation of the stress distribution are compared, and as far as the trend of the stress distribution is concerned, the comparison is seemed to be appropriate.

If the ultimate goal of shape optimization is changing all members of a structure to have the same stress level, a standard deviation of the stress distribution can be appropriate, as an evaluation measure.

3. Shape Optimization : SHELL Element Model

Following the previous section, the Michell Truss is modeled with SHELL elements as shown in Fig.8 and Fig.9.

In this case, the thickness of members and fillet radius of the member intersections are optimized by changing the area of SHELL elements.

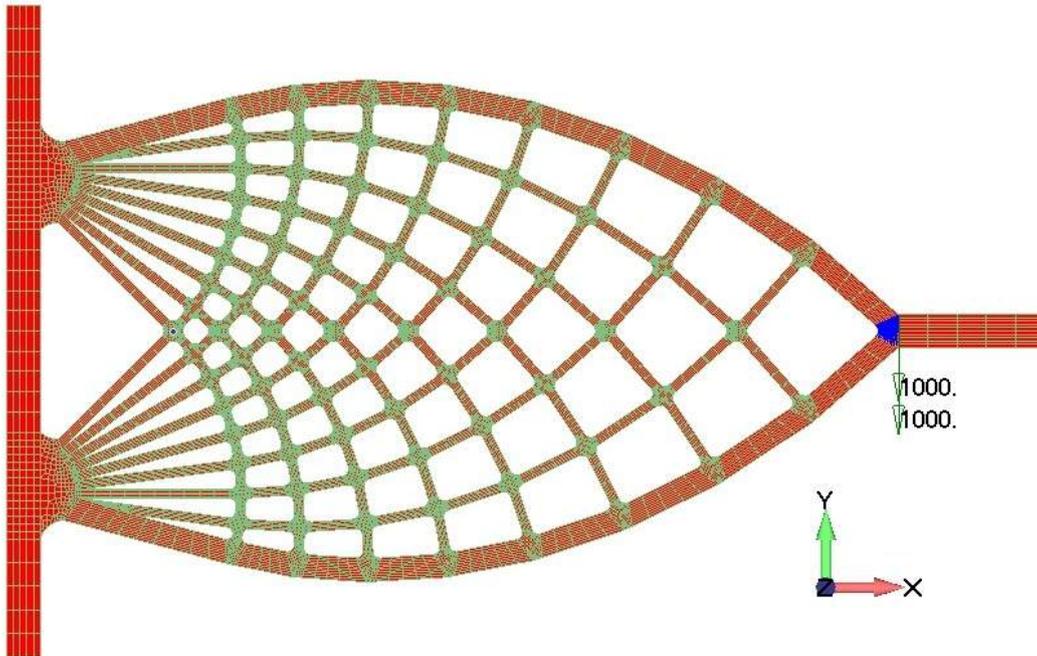


Fig. 8 Michell Truss : SHELL Elements Before Optimization

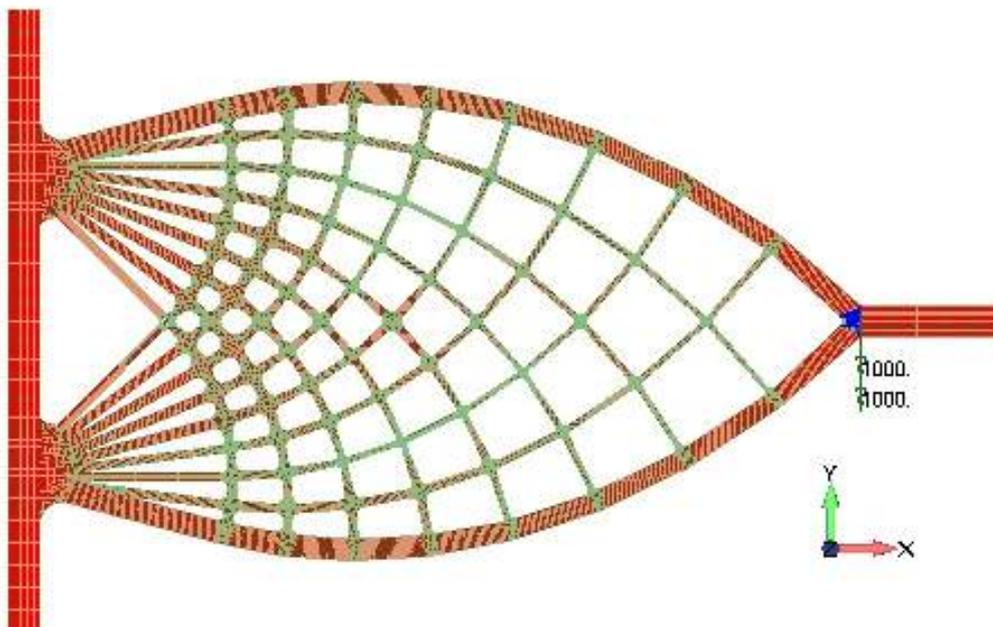


Fig. 9 Michell Truss : SHELL Elements After Optimization

In this context, Morphing Domains are specified by quadrilaterals with 4(four) corner nodes at the center of fillet arc on each intersection, as shown in Fig.10.
In addition, Morphing Action is specified by vectors from the center to the end of fillet arc, as shown in Fig.11.

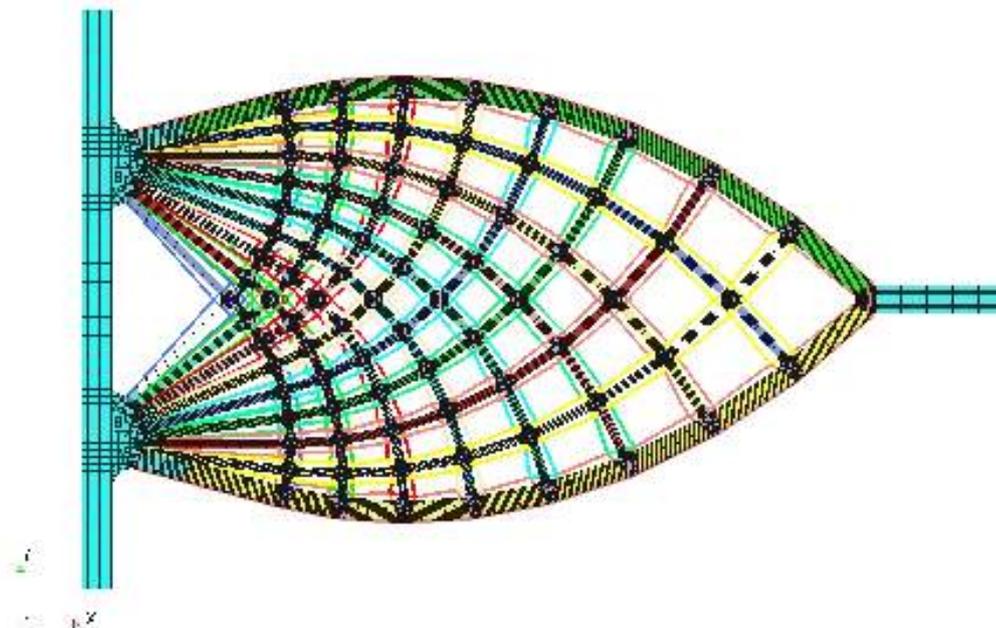


Fig. 10 Michell Truss : Morphing Domains

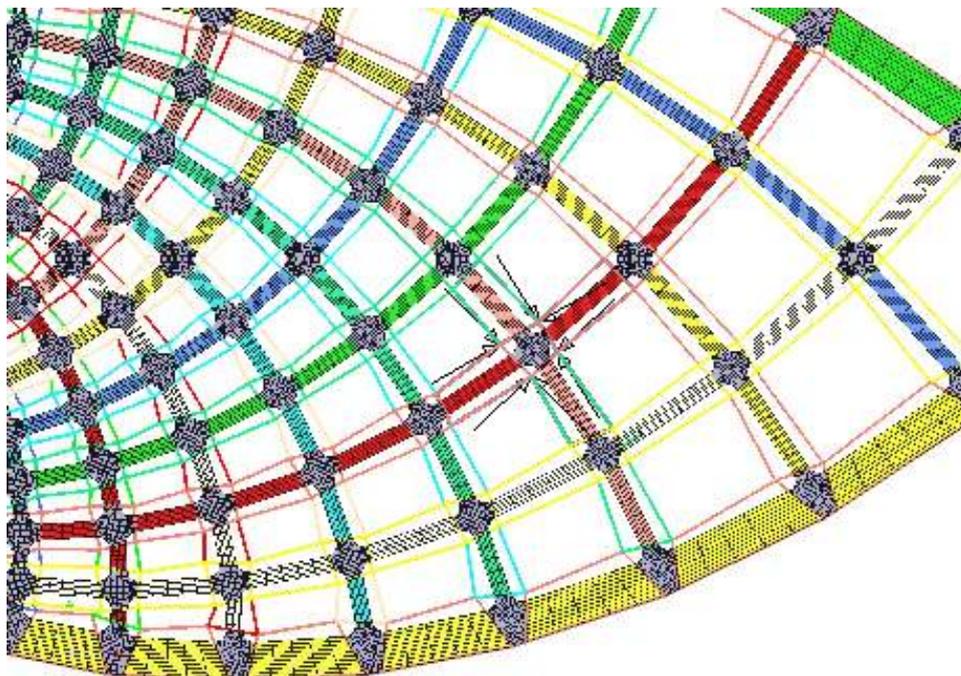


Fig. 11 Michell Truss : Morphing Action

As a result of the above-mentioned morphing actions, geometrical shape of each member and fillet are obtained, as shown in Fig.12.

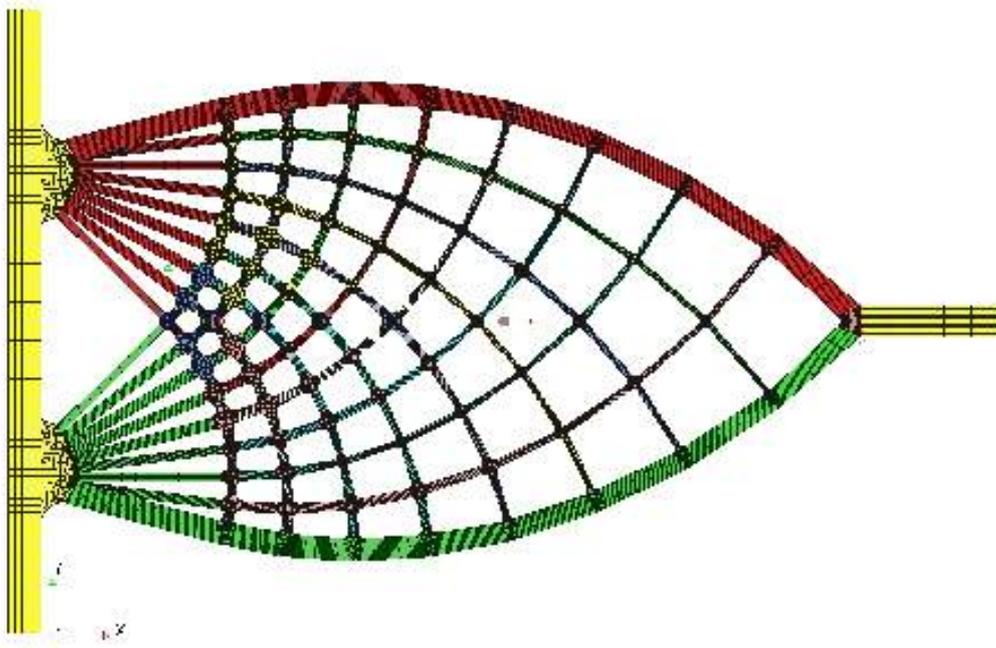


Fig. 12 Michell Truss : Morphing Result

Optimization conditions in the GENESIS are as follows (See APPENDIX : A-2);

(1) objective variable : Mass minimization

(2) constraints (maximum principal stress)

Outer edge member : Upper bound / 16 (N / mm²) Lower bound -16 (N / mm²) Min

Fan-shaped member : Upper bound / 13 (N / mm²) Lower bound -13 (N / mm²) Min

(3) design variable : Morphing action / Upper bound / 12.5 (mm) Lower bound -2.5 (mm) Min

As shown in Fig.13, in the thick beam at the top edge, the stress level is excessively higher than in other members. It is because of the in-plane bending moment generated at this portion.

By definition, Michell Truss is formulated on the equilibrium of the axial force of each member at their intersection.

Therefore, if it is necessary to model with SHELL elements, the stress component due to the in-plane bending moment must be included in the optimization parameters.

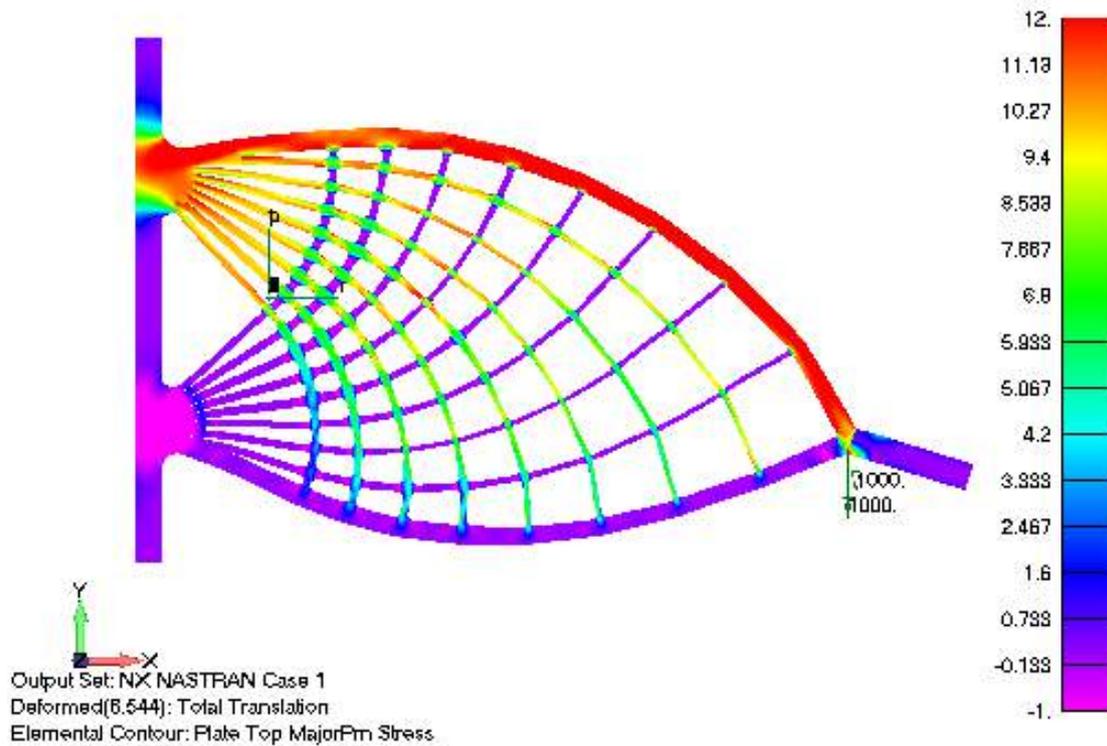


Fig. 13 Michell Truss : SHELL Elements after Optimization

In this context, the optimization procedure is divided into the following 2(two) steps;

- (1) Thickness of members and fillet radius of the member intersection are optimized by changing the area of elements.
- (2) Thickness of members is optimized by changing the thickness of elements.

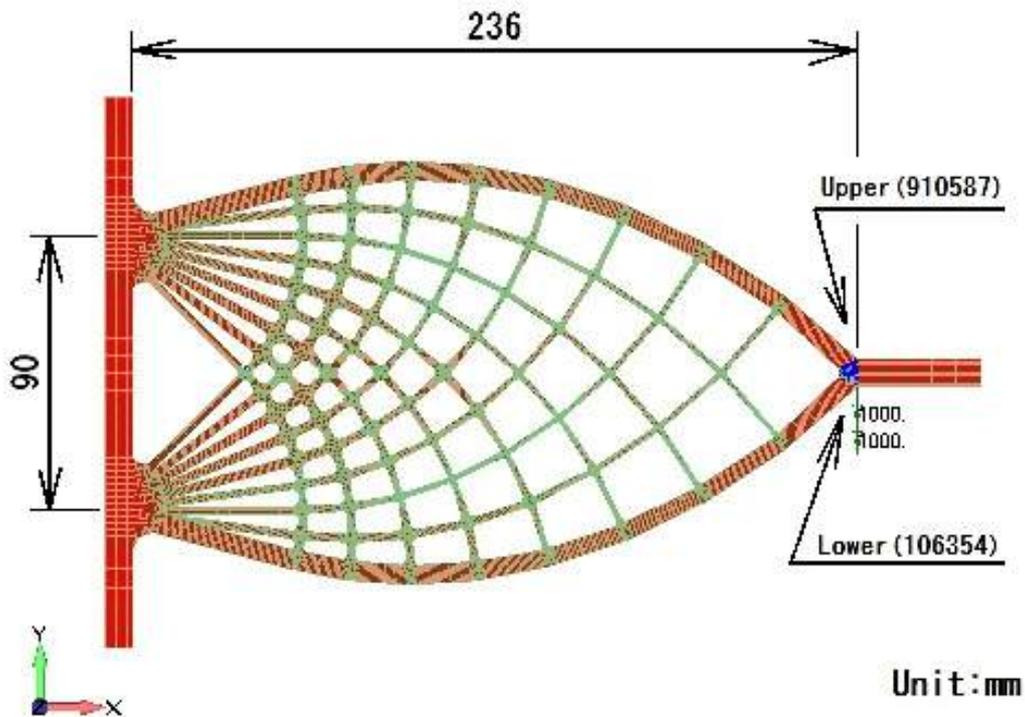


Fig. 14 Michell Truss : Dimension for Optimization Parameter

With the geometrical relations, as shown in Fig.14, Synthetic Function for Bending Stiffness is defined with the following relation (See APPENDIX : A-3):

Concentrated Load : Upper = 1000(N) / #910587

Concentrated Load : Lower = 1000(N) / #106354

$F(\text{Upper,Lower})=2000.*100./((\text{ATAN}(\text{ABS}(\text{UPPER})+\text{ABS}(\text{LOWER}))/2/100.)*(180.0/3.14))$

(1) design variable (SHELL element thickness)

Outer edge member: Initial / T20 (mm) => Upper bound / T30 (mm) Lower bound T10 (mm)

FAN-shaped member : Initial / T20 (mm) => Upper bound / T30 (mm) Lower bound T10 (mm)

(2) objective variable : Mass minimization

(3) constraints (axial stress / maximum principal stress)

Upper bound / 12 (N / mm²) Lower bound -1 (N / mm²) Min

As a result, the thickness of SHELL elements located on the outer edge members and FAN-shaped members is changed, as shown in Fig.15 and Fig.16.

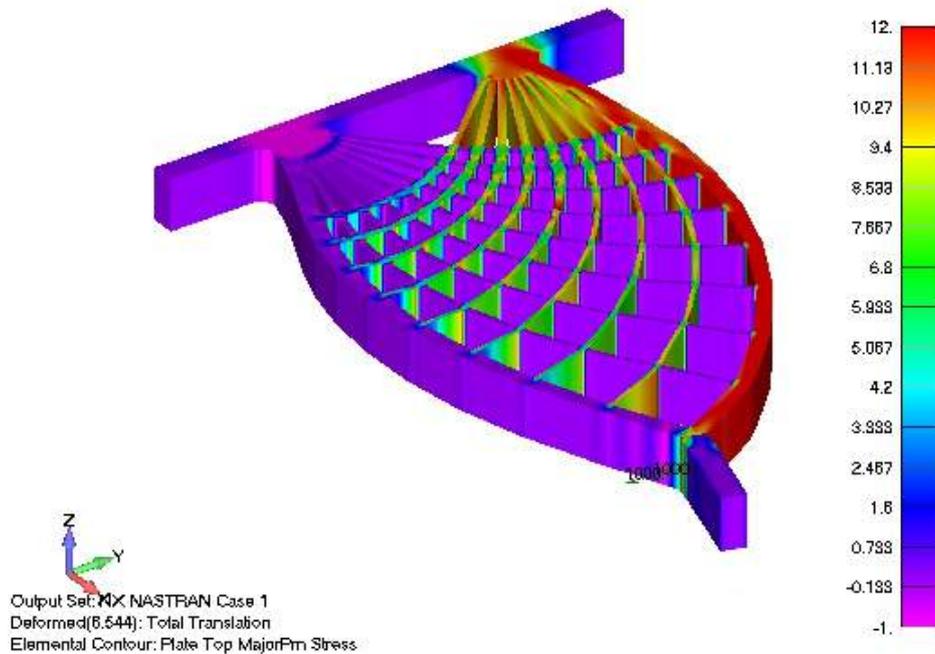


Fig. 15 Michell Truss : SHELL Elements before Thickness Optimization

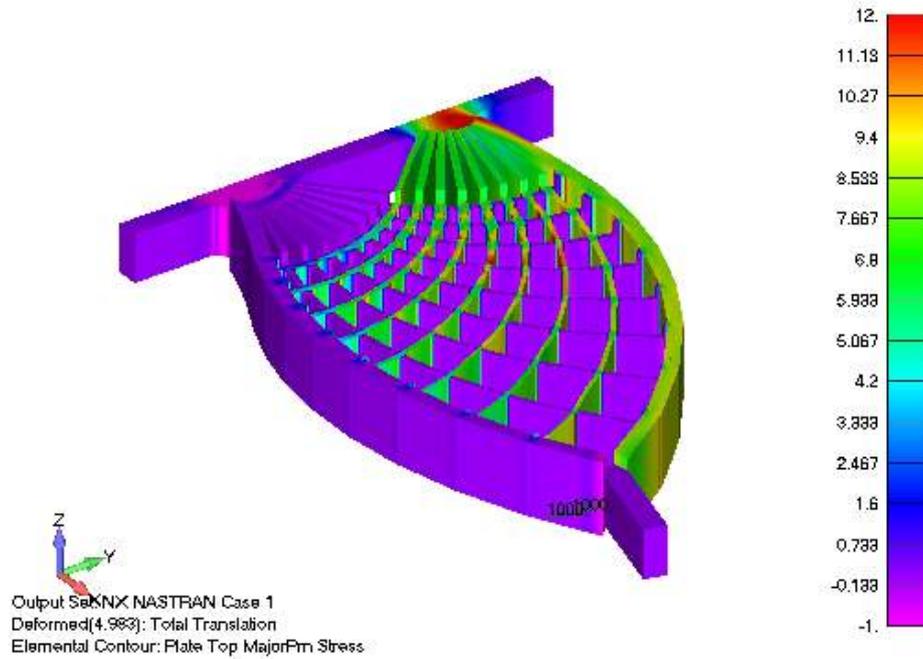


Fig. 16 Michell Truss : SHELL Elements after Thickness Optimization

Consequently, relatively leveled stress distribution is obtained as shown in Table 4 and Fig.17.

Table 4 Before and After Thickness Optimization

Constant Thickness			Optimized Thickness		
	Average	S.D.(σ)		Average	S.D.(σ)
Upper	9.068	2.640	Upper	7.727	1.840
Lower	-9.066	2.662	Lower	-7.754	1.805
	δ max	6.544		δ max	5.172
	Mass	0.268		Mass	0.319

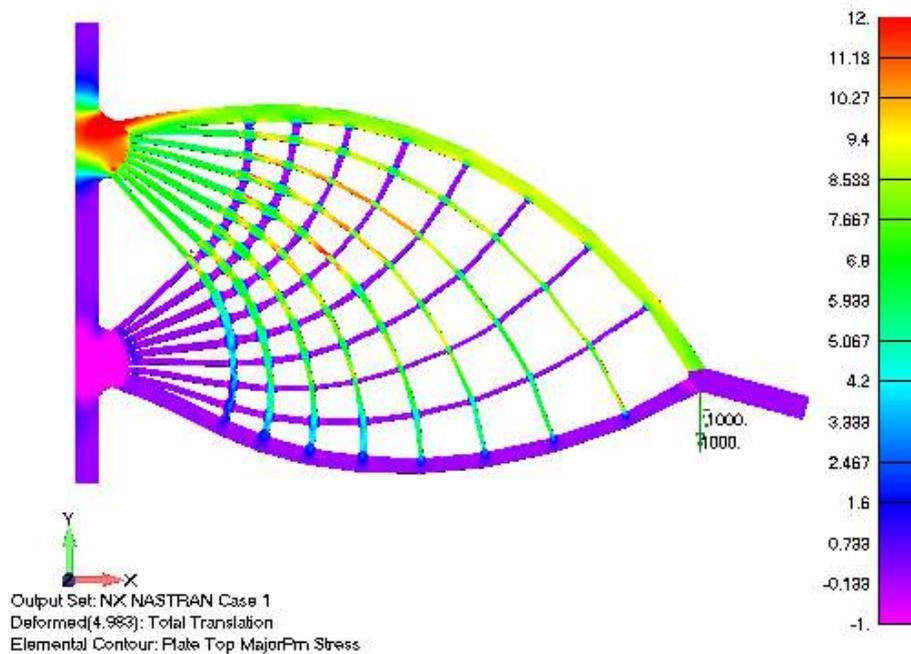


Fig. 17 SHELL Elements with Thickness Optimization

4. Considerations

(1) Michell Truss

Michell Truss is composed of tensile members and compressive members, intersecting at right angles each other.

It corresponds to the slip line, where the maximum shear stress is generated, in 2-dimensional press forming process. Namely, slip lines and shape optimization are mathematically equivalent, and can be expressed in the same formulation.

(2) BAR Element Model

When the Michell Truss is modeled by BAR elements, with the following parameters,

(a) design variable (Bar element cross-sectional area)

Outer edge member: Initial / 130 (mm²) => Upper bound / 130 (mm²) Lower bound 120 (mm²)

Inner member : Initial / 50 (mm²) => Upper bound / 58 (mm²) Lower bound 9 (mm²)

(b) objective variable : Mass minimization

(c) constraints (axial stress / maximum principal stress)

Upper bound / 10 (N / mm²) Lower bound -1 (N / mm²) Min

Then, the results show a significant reduction of the standard deviation of axial stress, from 0.242 to 0.003, with 52 (%) increases in the mean stress, 18 (%) increases in the maximum displacement, 23 (%) decreases in mass.

By definition, the Michell Truss is truss structure, which is composed of tensile members and compressive members, intersecting at right angles each other.

BAR element model directly corresponds to the structural nature of the Michell Truss, and the FEM results show this optimization effect clearly.

(3) SHELL Element Model (Changing the area of SHELL elements)

When the Michell Truss is modeled by SHELL elements, with the following parameters,

(a) objective variable : Mass minimization

(b) constraints (maximum principal stress)

Outer edge member: Upper bound / 16 (N / mm²) Lower bound -16 (N / mm²) Min

Fan-shaped member : Upper bound / 13 (N / mm²) Lower bound -13 (N / mm²) Min

(c) design variable : Morphing action / Upper bound / 12.5 (mm) Lower bound -2.5 (mm) Min

Then, in the thick beam at the top edge, the stress level is excessively higher than in other members.

It is because of the in-plane bending moment generated at this portion. By definition, Michell Truss is formulated on the equilibrium of the axial force of each member at their intersection.

Therefore, when it is modeled with SHELL elements, the stress component due to the in-plane bending moment must be included in the optimization parameters.

(4) SHELL Element Model (Changing the thickness of SHELL elements)

When the Michell Truss is modeled by SHELL elements, with the following parameters,

$F(\text{Upper}, \text{Lower}) = 2000 \cdot 100 / ((\text{ATAN}(\text{ABS}(\text{UPPER}) + \text{ABS}(\text{LOWER})) / 2 / 100) \cdot (180.0 / 3.14))$

(a) design variable (SHELL element thickness)

Outer edge member: Initial / T20 (mm) => Upper bound / T30 (mm) Lower bound T10 (mm)

FAN-shaped member : Initial / T20 (mm) => Upper bound / T30 (mm) Lower bound T10 (mm)

- (b) objective variable : Mass minimization
- (c) constraints (axial stress / maximum principal stress)
 - Upper bound / 12 (N / mm²) Lower bound -1 (N / mm²) Min

Then, the thickness of SHELL elements located on the outer edge members and FAN-shaped members is changed, and the standard deviation of principal stress is improved from 2.66 to 1.80. When this result are compared with that of the BAR element model, the effect of optimization is not so good.

Because the optimization parameters are defined as follows,

- (a) Synthetic Function is defined only by global bending stiffness
- (b) Design variable (Element Thickness) is intentionally defined only on the outer edge members and FAN-shaped members

Therefore, an ultimate optimization goal is not obtained yet.

It brings us to do a simultaneous optimization of

- (a) In-Plane optimization of the thickness of members and fillet radius of the member intersections
 - (b) Out-of-Plane thickness optimization of members
- using 3/D SOLID elements.

5. Summary

- (1) CAE can verify the various design patterns in the initial design stage, where the design specification has not yet solidified. However CAE, based on the finite element method, is not useful for alternatives search.
- (2) On the contrary, in order to automate the alternatives search phase, shape optimization software is developed.
- (3) When the Michell Truss is modeled by BAR elements, the results show a significant reduction of the standard deviation of axial stress.
- (4) It is because the BAR element model directly corresponds to the structural nature of the Michell Truss, and the FEM results show this optimization effect clearly.
- (5) When the Michell Truss is modeled by SHELL elements, in the thick beam at the top edge, the stress level is excessively higher than in other members,
- (6) It is because of the in-plane bending moment generated at this portion.
- (7) By definition, Michell Truss is formulated on the equilibrium of the axial force of each member at their intersection.
- (8) When the Michell Truss is modeled by SHELL elements, the thickness of SHELL elements located on the outer edge members and FAN-shaped members are changed, and the standard deviation of principal stress is improved, but the effect of optimization is not so good.
- (9) It brings us to do a simultaneous optimization of In-Plane optimization of the thickness of members and fillet radius of the member intersections Out-of-Plane thickness optimization of members using 3/D SOLID elements.

References

- [1] "Decision Making Process Detail",
[online] Available at:
<http://www2.toyo.ac.jp/~fujita-h/UT/keieisoshiki_03.pdf>
[Accessed Mar.14 2014].
- [2] T. P. Kicher, W. M. Morrow, L. A. Schmit
"Structural synthesis capability for integrally stiffened waffle plates",
AIAA Journal, Vol.1, No.12, 1963.
- [3] The Design of Michell Optimum Structures,
[online] Available at:
<<http://naca.central.cranfield.ac.uk/reports/arc/rm/3303.pdf>>
[Accessed Oct. 17 2013].
- [4] M. P. Bendsoe, O. Sigmund,
"Topology Optimization", Springer, 2003.
- [5] Software for Structural Analysis and Optimization,
[online] Available at:<<https://www.vrand.com/Genesis.html>>
[Accessed Sep. 04 2013].