## MULTIDISCIPLINE DESIGN OPTIMIZATION



**Garret N. Vanderplaats** Founder & Chief Executive Officer Vanderplaats Research & Development, Inc. Colorado Springs, CO The concept of optimization is intrinsically tied to our desire to excel. Though we may not consciously recognize it and though the optimization process takes different forms in different fields of endeavor, this drive to do better than before consumes much of our energy, whether we are athletes, artists, business-persons, or engineers. The field we are concerned with is that of engineering and, while the emphasis here is on mechanical, aeronautical and automotive engineering, this is by no means the limit of applicability; it simply reflects my own experience and interests.

We will consider the use of numerical optimization techniques to devise a rational, directed design procedure. Although these methods provide a computational tool for design, there is much more to be gained from this study. Indeed, numerical optimization provides us with a new design philosophy. It gives us an ordered approach to design decisions where before we relied heavily on intuition and experience. It is this order which makes the techniques presented here so very attractive, because it provides insight into the actual design process. However, this should not be construed to suggest that the design process can be reduced to a few computer runs or that our intuition and experience are unimportant. Rather, the computer can now be used to relieve us of the tedium of repetitive calculations, freeing us to spend time on the truly creative aspects of engineering design.

The methods presented here have gone through an extensive development period of about 60 years. The field has matured to the point where the techniques can be routinely applied by practicing engineers to a large percentage of design tasks. However, education in the concepts and applications of numerical optimization is not yet a principal part of most engineering curricula. Consequently, few practicing engineers are actually using this powerful tool in design. For this reason, there is strong motivation to provide a book which can be used for classroom education and also for self-study or as a continuing education aid. It is hoped that the concepts presented here are organized in a logical enough way and presented in plain enough language that many engineers can use this information. The purpose of this book is threefold. First, we wish to gain a basic knowledge of numerical optimization algorithms, together with their strengths and weaknesses. Second, the student is encouraged to gain computational experience by programming these algorithms for the computer, and finally, a variety of design applications are discussed to identify those design areas where numerical optimization techniques have been applied in the past and where they may be applied in the future.

This book is written to be a senior or graduate level textbook but is suited for self-study by practicing engineers. Also, the book will provide a ready reference for many of the best numerical optimization algorithms. The student should have a solid background in engineering fundamentals, matrix algebra. Computer programming is useful if the reader wishes to program one or more of the algorithms presented here. Appendix A provides a very brief review of matrix algebra for those who need a refresher in the mathematics used here.

Because the principal purpose here is to provide design tools for the engineer, we will avoid mathematical proofs and theoretical discussions of the various algorithms. Instead we will address such questions from a more pragmatic viewpoint of asking, does a given technique provide a good engineering answer in an efficient manner? To this end, we will emphasize those aspects of numerical optimization which attempt to model the thought processes of a good design engineer.

In the first chapter, we introduce the basic concepts of numerical optimization. We identify the characteristics of unconstrained and constrained optimization problems and offer the general mathematical problem statement. We describe the most common iterative approach to the solution of the optimization problem and very briefly discuss the necessary mathematical conditions for a solution to be the optimum among all possible solutions. These concepts provide the mathematical basis for understanding some of the more powerful optimization algorithms to be discussed in later chapters. Finally, we identify some of the advantages and limitations to the use of numerical optimization techniques in engineering design to provide a broad perspective on the application of this general design approach.

In Chapter 2, we begin to develop the numerical tools necessary for the creation of an efficient design capability. In this chapter we discuss the solution of optimization problems defined by only one variable. Here we consider two of the most popular and powerful techniques for solving this problem, polynomial approximations and the Golden Section method. Methods are presented for finding either the minimum or the zero value of a function of one variable. Considerable generality is maintained throughout this discussion so that the student gains a broad capability for the solution of

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this problem. This chapter contains a discussion of minimizing a function of one variable subject to constraints on other functions. The chapter concludes with a recommended algorithm for minimizing functions of one variable. The techniques discussed here are important because these tools will be used throughout the remainder of the book when dealing with more general multivariable optimization problems.

Chapter 3 extends the discussion to the minimization of multivariable unconstrained functions. This is a natural progression because this is the simplest multivariable problem to understand and yet the methods developed here are later used to solve constrained multivariable optimization problems.

Chapter 4 introduces the most extensively developed of the multivariable constrained optimization techniques. This is the solution of strictly linear analysis and design problems. The basic algorithm presented here is the standard Simplex method. While this is not the most efficient of the simplex methods, it is most easily developed and understood and therefore serves best to introduce the concepts. Linear problems are encountered in such diverse areas as limit analysis and design of structures and industrial resource allocation and, as such, are valuable in their own right. Also, linear programming techniques will be used later in solving nonlinear constrained minimization problems.

Chapters 5, 6, and 7 provide the fundamental capability we wish to develop for general engineering design. This gives the student the ability to solve multivariable nonlinear constrained optimization problems of the type most often encountered in practice.

Chapter 5 develops a technique for the solution of the constrained optimization problem by conversion to an equivalent unconstrained problem. The basic concept here is to convert the original constrained optimization problem to a sequence of unconstrained problems through the use of penalties for constraints. Three techniques are presented, the exterior, interior, and extended interior penalty functions. Each approach has its own attractions as well as drawbacks, and a good understanding of these techniques aids in determining which is most useful for solving specific design problems. A powerful sequential unconstrained minimization technique known as the Augmented Lagrange Multiplier method is presented in this chapter. Finally, a recent sequential unconstrained minimization technique is presented that is capable of solving very large nonlinear constrained optimization problems. Each of the methods developed in this chapter makes use of the algorithms developed in Chaps. 2 and 3 for unconstrained minimization. Chapter 6 is devoted to so-called direct methods. These methods attempt to incorporate information about the constraints directly into the optimization problem rather than converting the problem to an equivalent unconstrained one as was done in Chapter 5. Chapter 6 includes sequential linear programming, the method of feasible directions and the generalized reduced gradient method as well as a modified feasible directions method containing desirable features from the other techniques. The final section of Chapter 6 presents a sequential quadratic programming technique which is found to be a particularly powerful method.

Chapter 7 addresses many of the practical aspects of using numerical optimization in engineering design. These include design variable linking where one design variable in the optimization process controls more than one variable in the actual engineering system. The concept of a reduced basis for design is shown to be a generalization of design variable linking and is particularly powerful for including practical constraints into the mathematical design problem. The use of approximation techniques in design optimization is discussed. The principal motivation here is to develop methods which use whatever detailed analysis techniques we choose but formally approximate this problem in such a way that the efficiency of the optimization process is greatly improved, while retaining all of the features of the original problem. Chapter 7 concludes with the presentation of techniques whereby the sensitivity of the optimum design with respect to various changes in the problem statement is obtained. This provides us with the ability to predict the effect of changes in material properties and loading or constraint limits on the design, even after the optimization process has been completed.

Chapter 8 addresses problems where we may require that the design variables be chosen from sets of discrete values and where we may have more than one objective function. Discrete variable problems are encountered when we must choose the values of design variables from a manufacturer's catalog, when we want to determine the optimum number of teeth in a gear, or when we must determine the number of plies in a composite laminate, as examples. Multi-objective problems exist when we wish to minimize more than one objective simultaneously, for example, minimize weight and cost.

Chapter 9 is the first applications chapter and here the specific topic of structural optimization is addressed. A concerted effort is made to identify efficient means of approaching the problem and understanding lessons learned from past experience. It is a clear understanding of how numerical optimization has been applied in the past that gives us insight into how it may be applied in the future.

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Chapter 10 discusses a variety of optimization applications. The purpose here, as in Chapter 9, is to identify problems where optimization has been used effectively. While a comprehensive list is not possible, this discussion does identify typical applications and provides further insight into problem formulation and possible future applications.

Chapter 11 concludes the text with a discussion of Multidiscipline Design Optimization, or MDO. It is seen that MDO has been performed for nearly forty years. Some of the early applications are particularly useful for demonstrating proper formulation and how we can create optimization oriented analysis software. Formal mathematical formulations for MDO have been studied for nearly thirty years and these are referenced for further study. Finally, an engineering approach to MDO is presented that is being used successfully and examples of this approach are briefly described.

CDROMs which contain commercial optimization software are provided with this book. This is described in Appendix B. The reader may use this software for small demonstration problems, and can obtain a temporary evaluation license from the developer, with full program capabilities. Universities can obtain the full commercial capabilities of this software for teaching and research at nominal cost.

Throughout the book, the practicalities of making engineering design decisions on the computer are stressed. Theoretical detail is limited to that necessary to understand the concepts. Most of all, it is hoped that the practicing engineer of the future will be better equipped to use advanced optimization techniques to improve the quality of life for all.

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