A Topology Optimization Method to Extract Optimal Beam-Like, Plate-Like, or Shell-Like Structures from a Solid Finite Element Mesh

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1. Abstract
A new topology optimization method to extract optimal beam-like, plate-like, or shell-like structures from a solid finite element mesh is presented. The development of this method was motivated because traditional optimization methods tend to produce truss-like structures that are not necessarily optimal from the manufacturing point of view. Buildings, for example, are typically designed with beams and plates and not with trusses. Car bodies are designed using shell meshes because stampings are needed for enclosing the structure and for aesthetic reasons. Dams are designed using walls (plate-like) to reinforce the main shell. The presented method parameterizes the design domain assembled with finite elements. The method is suitable for general irregular meshes that are typically created by mesh generators. The method is very efficient as it uses a small number of design variables and it does not add additional constraints to impose the manufacturing requirements. The manufacturing requirements are built in to the parameterization of the design variables. The proposed method is implemented in the Genesis program and some examples that show its effectiveness are included.

2. Keywords: *Topology, Optimization, Structures, Manufacturing Constraints*

3. Introduction
The topology optimization field has been rapidly developed since it was first introduced in the late eighties [1-2]. In the last decade it has been adopted by multiple automobile, aerospace, and motor sports companies. However, engineering firms in the civil engineering field rarely use this method. The most likely reason for this is that the majority of structures in this field are built with components that are simple and straight, like columns and beams or are flat, like floors and walls in buildings. Topology optimization naturally gives trusses for answers or curved shapes that are not easy or economical to build. The literature shows attempts to improve this by including some manufacturing constraints like symmetry and pattern gradation [3], but the patterns reported still have trusses on them which although can be suitable for tall buildings they are not general enough for majority of buildings. In the literature we can find methods to incorporate manufacturing constraints for casting [4-8] but these methods are intended and are more suitable for structures that are manufactured with dies and presses and not for civil engineering.
In publications [6-7], we introduced a method that produces castable topology designs which has the following characteristics: a) Discipline independent, b) Works with regular or irregular meshes, c) Efficient: It reduces the design variable without increasing the number of constraints and d) It automatically finds the parting plane. The presented method works by parameterizing the design variables of the design domain. In this paper we use new manufacturing constraints that are constructed by modifying the equations previously presented in [7]. Before discussing such modification, we will summarize first the standard formulation of topology optimization and then the formulation we presented in [7].

4. Topology Optimization – Standard Formulation
The topology optimization problem can be stated as:

$$
\begin{aligned}
&Min\ F(\rho_1, \rho_2, \ldots, \rho_n) \\
&\text{such that :} \\
&g_j(\rho_1, \rho_2, \ldots, \rho_n) \le 0; \quad j = 1, m \\
&0.0 \le \rho_i \le 1.0; \quad i = 1, n
\end{aligned}
\tag{1}
$$

where F is the objective function, g_j are standard constraints and ρ_i is the ith design variable.

The topology optimization above can be solved using different methods, in the Genesis program [9] it is solved using a sequence of approximate problems based on the approximation concept approach. Each approximate problem contains approximate functions and is solved using the general-purpose optimizer BIGDOT [10]. The material properties are updated using predefined known rules (Power, Reuss, Voigt and/or combination of the last two) that help to get 0-1 solutions that represent a void or solid state of each of the designable elements in the finite element model.
In the mid-seventies [11-12], Schmit et al. introduced approximation concepts for traditional structural optimization. In or around the eighties [13-19] efficient ways to approximate functions were developed. The idea to design a Young's modulus with a predefined rule for the purpose of creating voids that represent a topological design was presented by Bendsoe et al. in the late eighties[1-2]. The Reuss and Voigt rules are discussed in [21]. The BIGDOT program is a large-scale, non-linear, exterior-penalty based optimizer developed by Vanderplaats [22]. Details on how these key ideas and others are implemented in the GENESIS program can be found in [23].

5. Topology Optimization with Fabrication Constraints Formulation for Casting

To generate designs that are easier to manufacture, we can parameterize the design space so its parameters can produce possible shapes needed in the final results. In the Genesis program, we use the following parameterization to obtain castable designs:

$$
\begin{aligned}
&Min\ F(\rho_1,\rho_2,\ldots,\rho_n)\\
&\text{such that:}\\
&g_j(\rho_1,\rho_2,\ldots,\rho_n)\le 0;\quad j=1,m\\
&\rho_i=\Psi_i^e(\vec{A},\vec{L},\vec{C})=\rho_i^e=\rho_i^e(\vec{A},\vec{L},\vec{C})=\Psi_i^e(\vec{A})*\Phi_i^e(H_i^e\Psi_i^e(\vec{L}))*(1-\Phi_i^e(B_i^e(\Psi_i^e(\vec{C}))))\ \ i=1,n\\
&0.0\le A_k^p,L_k^p,C_k^p\le 1.0;\quad \mathrm{k}=1,NP
\end{aligned}
\tag{2}
$$

where B^e_i and H^e_i correspond to the distance to the top and bottom of the structures for element *i* measured in a given reference direction, A_k^p, L_k^p, C_k^p are parameters that are used as design variables, NP is the number of poles used in the parameterization. Reference [7] discusses the function Ψ_i^e, how to arrive to this formulation and the meaning of all terms in Eq.(2).

6. Parameterization to Obtain Shell-Like Structures

To generate shell-like designs, we use the following parameterization:

$$\rho_i^e=\rho_i^e(\vec{A},\vec{L})=\Psi_i^e(\vec{A})*\Phi_i^e(H_i^e\Psi_i^e(\vec{L}))*(1-\Phi_i^e(H_i^e-T)) \tag{3}$$

where T is the desired thickness of shell. The other terms of this equation are defined in [7]. This parameterization comes from modifying the following equation in [7]:

$$\rho_i^e=\rho_i^e(\vec{A},\vec{L},\vec{C})=\Psi_i^e(\vec{A})*\Phi_i^e(H_i^e\Psi_i^e(\vec{L}))*(1-\Phi_i^e(B_i^e(\Psi_i^e(\vec{C})))) \tag{4}$$

For casting designs as defined in [7] the variable that controls how much to carve from the bottom is:

$$B_i^e(C_i^e)=(H_i^e\max-H_i^e\min)*C_i^e+H_i^e\min \tag{5}$$

But in stamping the variables that control the bottom of the surface of the design are not independent; the variables are offset by the desirable thickness of the shell structure. In other words, we can replace Eq.(5) by the following equation:

$$B_i^e=(H_i^e-T) \tag{6}$$

Using Eq.(6) instead of Eq.(5) and replacing it in Eq.(4) it yields to Eq.(3).

7. Topology Optimization with Fabrication Constraints to Obtain Shell-Like Structures

The topology optimization problem to obtain shell-like structures is as follows:

$$
\begin{aligned}
&Min\ F(\rho_1,\rho_2,\ldots,\rho_n)\\
&\text{such that:}\\
&g_j(\rho_1,\rho_2,\ldots,\rho_n)\le 0;\quad j=1,m\\
&\rho_i=\Psi_i^e(\vec{A},\vec{L})=\rho_i^e=\rho_i^e(\vec{A},\vec{L})=\Psi_i^e(\vec{A})*\Phi_i^e(H_i^e\Psi_i^e(\vec{L}))*(1-\Phi_i^e(H_i^e-T)\ \ i=1,n\\
&0.0\le A_k^p,L_k^p\le 1.0;\quad \mathrm{k}=1,NP
\end{aligned}
\tag{7}
$$

In Eq.(7) the number of parameters (design variables) has been reduced to 2*NP when compared with the formulation in Eq.(2) where the number of parameters is 3*NP. Solving the problem in Eq.(7) allows to generate one shell structure that has a uniform thickness T. To get two or more shells structures we can simply add an extra set of parameters that superimpose the ones already created. If in the above formulation the parameters A_k^p are replaced by a value of 1, then the final shell answers would not have voids on their surface. In this case, the number of parameters is further reduced to NP. The example presented in section 11, shows an answer where two shell layers are required. In the same example, possible voids on the surface of the final shell design are prevented.

8. Topology Optimization with Fabrication Constraints to Obtain Curved Beam-Like Structures

To obtain curved beam-like structures, the same formulation in Eq.(7) can be used. In this case, however, the user needs to use a design domain where only two dimensions are dominant. If the domain has one dimension, W, that is small compared with the other two, then the beams generated by this parameterization will be W by T, where T corresponds to the height of the curved beam. In this case using Eq.(7) alone will yield to only one curved beam per design domain. To obtain two or more curved beams per design domain, superposition of the additional parametric equations should be done.

9. Topology Optimization with Fabrication Constraints to Obtain Plate-Like Structures

To obtain plate-like structures we also parameterize the problem. In this case, the parameterization enforces that designable nodal points which have same coordinates in one specified direction (e.g. Z) must also have the same densities. The specified direction will then correspond to the normal of the generated planes (e.g. Z).

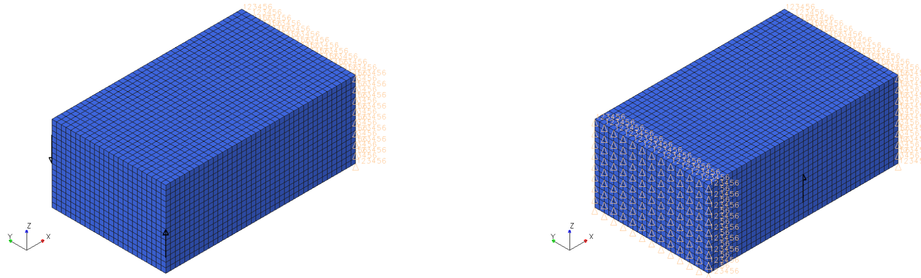
10. Topology Optimization with Fabrication Constraints to Obtain Straight Beams or Columns

To obtain straight beam or straight columns a parameterization that enforces designable nodal points in the design space that have the same two (e.g. X and Y for a specified direction of member Z) of three parametric coordinates must also have the same densities.

11. Example 1. Optimal Shell-Like Topology Optimization of a Solid Block Subject to Torsional Loads

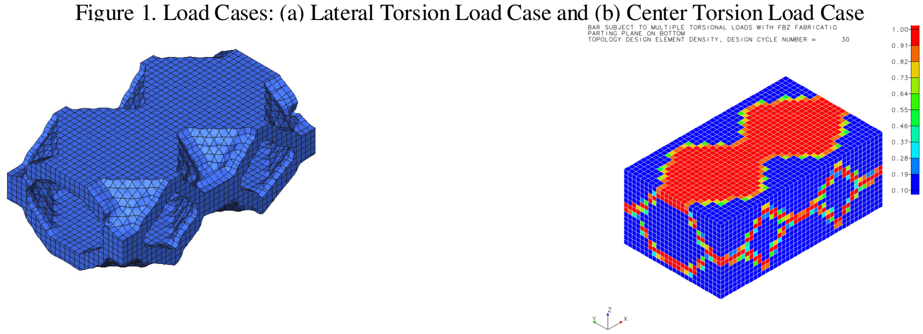
11.1 Description of the Problem

This example demonstrates the use of topology optimization to find an optimal shell-like structure that can be manufactured by joining two stampable sheet metals. The overall dimensions of the structure are 10 m by 6 m by 3 m. The design domain of the structure consists in 15,360 hexahedral elements and 17,425 grids. The Young's modulus is 200 Gpa, the density is 7860 kg/m^3 and the Poisson's ratio is 0.3. There are two torsion load cases. In the first load case, one end is fixed on one end while the other end is subject to a pair of loads that produce torsion. In the second load case, the two ends of the structure are fixed while the center section of the structure is loaded in torsion with a pair of loads in opposite directions. The two load cases can be seen in Fig. 1(a-b). A manufacturing constraint is used to force topology to produce two shell-like structures with no voids on their surface. In addition, symmetry constraints are defined to enforce mirror symmetry with respect to the XZ and YZ planes. The symmetry planes are located at the center of the domain. The stamping direction for this case is the Z direction. The design objective is to minimize the sum of the strain energies of the two load cases subject to mass fraction constraint of 0.25. A prescribed shell thickness of 0.2 m is used.



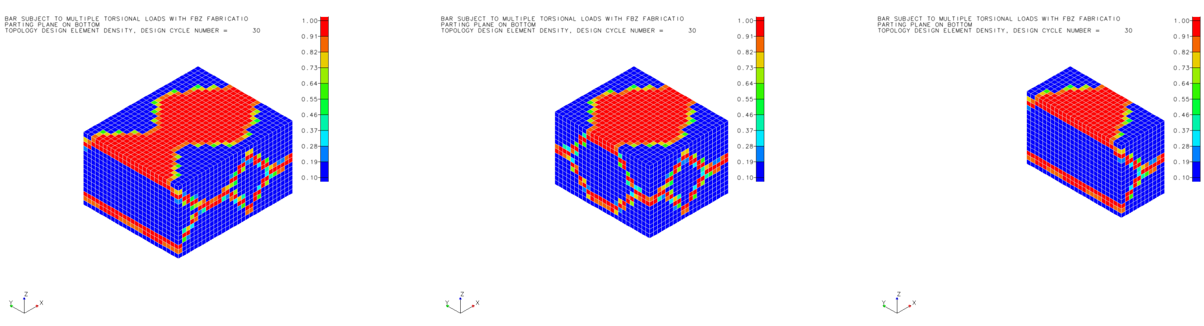
(a) (b)

Figure 1. Load Cases: (a) Lateral Torsion Load Case and (b) Center Torsion Load Case



(a) (b)

Figure 2. Topology Optimization Final Results: (a) Isosurface of Final Design and (b) Density Distribution of Final Design



(a) (b) (c)

Figure 3. Density Distribution of Final Design: (a) Cut Showing Three Quarters of the Structure, (b) Cut Showing one Half of the Structure, and (c) Cut Showing one Quarter of the Structure

11.2 Results

Figures 2(a-b) show that the final structure can be assembled by joining two stamped shells. Figures 3(a-c) show three cross sections in which no interior trusses are formed. Figures 3(a) and 3(c) show that the shells are horizontal at those sections while Fig. 3(b) shows that the shells curve in the center section to be able to carry the point loads of load case 2. Figure 3(b) also shows that the center section is similar to that of the end sections. Figures 2 and 3 show that there are no voids in the surface of the final shells. The final design is double symmetric as prescribed.

12. Example 2. Optimal Support Structure of a Seating Section of a Bleacher Stadium

12.1 Description of the Problem

This example demonstrates the use of topology optimization to find optimal support structures of a seat section of a bleacher stadium. The overall dimensions of the structure are 550 in by 920 in by 450 in. The structure is assembled with 113,436 hexahedral elements and 125,550 grids. There are four load cases. In the first two load cases the structure is subject to lateral loads. In the third load case, the structure is subject to vertical load representing the weight of seats and attendees. The last load case corresponds to frontal loads. Figure 4 shows the four load cases. The design objective is to minimize the strain energy of the four load cases. Three alternative designs are studied and are explained below.

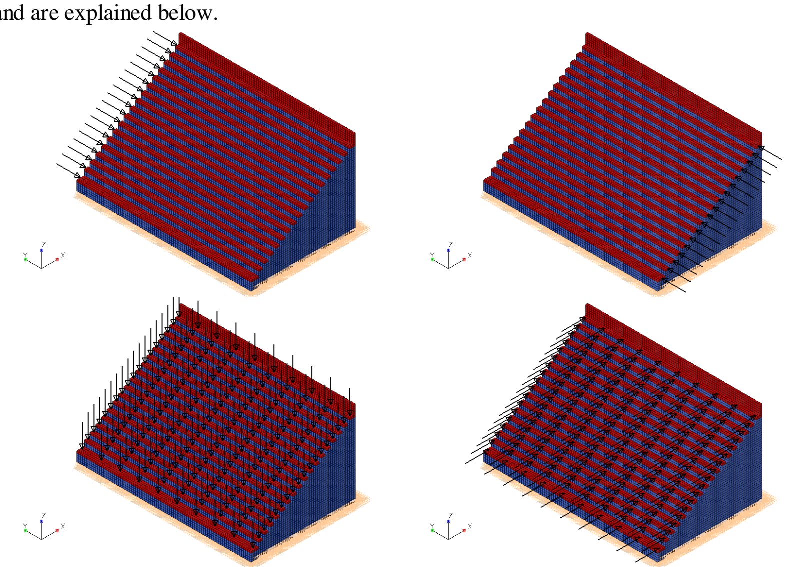


Figure 4. Load Cases

12.2 Alternative Design 1. Wall-Like (Plate-Like) Design

In this case a fabrication constraint is used to enforce that the final solution can be built using straight walls. In addition, a symmetry constraint with respect to the x-z plane that passes through the center of the structure is used. A mass fraction constraint of 0.05 is used to limit the amount of material to be used. The initial design for this case consists on the entire blue region shown in the Fig. 4.

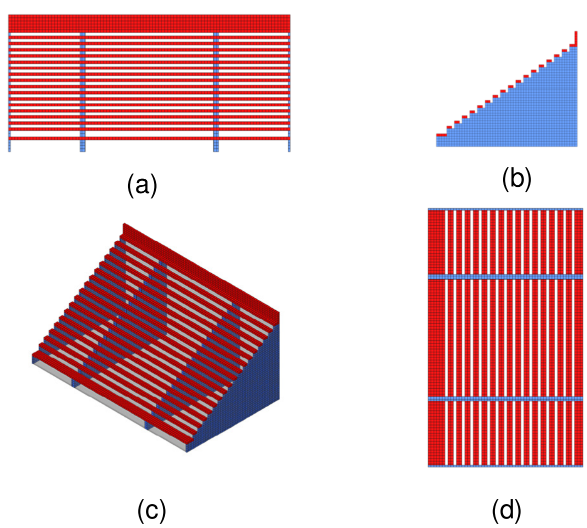


Figure 5. Topology Optimization Results using Uniform Fabrication Constraint to get Straight Walls. Isosurfaces that enclose element With High Density are Shown in Blue: (a) Front View, (b) Side View, (c) Isometric view, and (d) Bottom view

12.3 Results for Alternative Design 1

The blue isosurfaces in Fig 5 show that the final design has 4 straight walls. As desired, in this case, no truss structures were generated. The final design is symmetric as prescribed.

12.4 Alternative Design 2. Free Topology applied to the Walled-Design

A second topology optimization case is studied. In this case, topology optimization without fabrication constraints is used on one wall while forcing the other three to be identical to the first one. In this case, the initial design uses a reduced model obtained from eliminating the element discarded by the previous topology run. This reduced model has 15,032 elements and 27,540 grids. The load conditions are the same as described in section 12.1. A mass fraction constraint of 0.22 is used in this case.

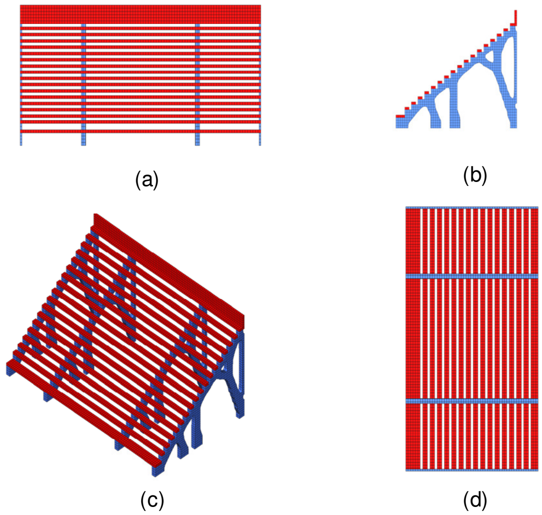


Figure 6. Topology Optimization without new Fabrication Constraints:
(a) Front View, (b) Side View, (c) Isometric view, and (d) Bottom view

12.5 Results for Alternative Design 2

The blue isosurfaces in Fig. 6 show the final design. The isosurfaces enclose the elements with high density and represent the element that needs to be kept. In this case, truss structures were produced as no special fabrication constraints were used to avoid them. The final design is symmetric.

12.6 Alternative Design 3. Fabrication Constraints in Topology applied to the Walled-Design to obtain a Design with Columns

A third topology optimization case is studied. In this case topology optimization is forced to produce straight columns on one wall while forcing the other three to be identical to the first one. In this case, as in the previous case described in section 12.4, the initial design is the walled design in section 12.2. However, in this case the elements shown in green in Fig. 7 are not designed to allow getting a connected structure. A mass fraction constraint of 0.22 is used in this case.

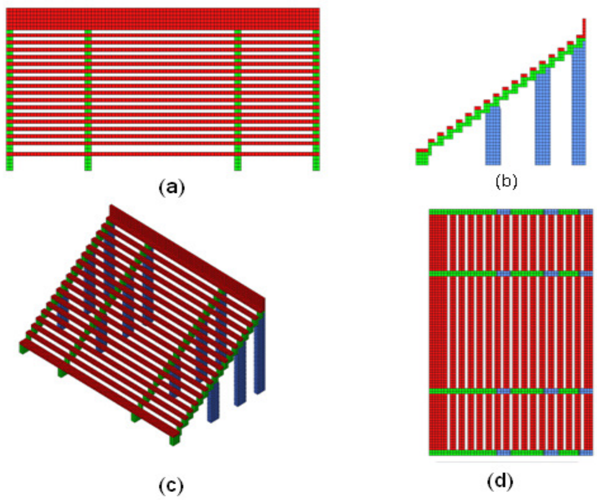


Figure 7. Topology Optimization Results using Fabrication Constraints
(a) Front View, (b) Side View, (c) Isometric view, and (d) Bottom view

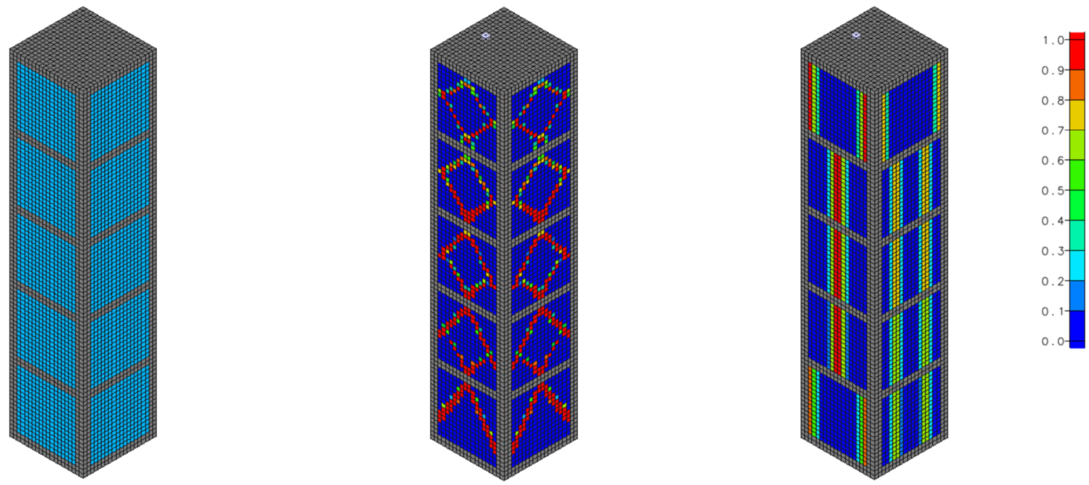
12.7 Results for Alternative Design 3

The blue isosurfaces in Fig. 7 show that the final design has 16 straight columns. As desired, in this case, no truss structures were generated. The final design is symmetric.

13 Example 3. Five-Story Building Design

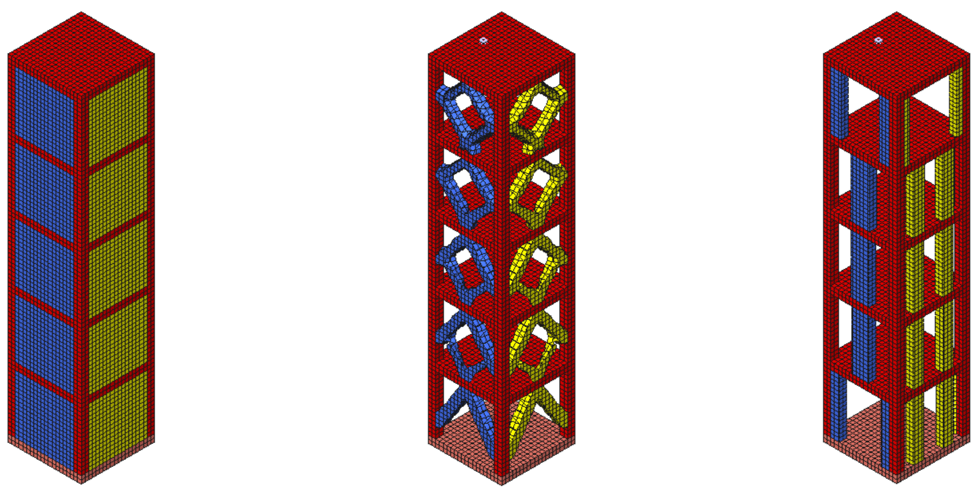
13.1 Description of the Problem

This example demonstrates the use of fabrication constraints to obtain an optimal design proposal with column patterns. This example is also solved with topology optimization without fabrication constraints. The example uses a finite element model of a five-story building shown in Fig. 8(a). There are three load cases in the problem. Two load cases contain lateral loads, while the third load case contains vertical loads. The objective function of the problem is minimizing the sum of the global strain energies of the three load cases. The designable areas in the problem correspond to 20 panels, ten designable panels are shown in light blue in Fig. 8(a), the other ten designable panels are similar to the ones shown and are located behind the structure. There is a mass fraction constraint of 0.2 applied to each of the 20 independent designable panels. The areas in grey (corner columns, floors, and ceiling) shown in Fig. 8 are not designable.



(a) (b) (c)

Figure 8. Density Distributions: (a) Initial Design, (b) Final Design using Topology without Fabrication Constraints, and (c) Final Design using Topology Optimization with Fabrication Constraints



(a) (b) (c)

Figure 9. Comparison of Optimal Isosurfaces: (a) Initial Design, (b) Final Design using Topology without Fabrication Constraints, and (c) Final Design using Topology Optimization with Fabrication Constraints

13.2 Results

Figures 8(b) and 9(b) show the results for topology without fabrication constraints. Figures 8(c) and 9(c) show the results for topology optimization with fabrication constraints. The results of Figures 8(b) and 8(c) show the density distributions. Red densities correspond to the most important elements to be kept while blue densities correspond to elements that should be discarded. The results in Figures 9(b) and 9(c) show the isosurfaces (shown in blue and yellow) that enclose the most important element that should be kept. Free topology (topology optimization without fabrication constraints) produced trusses as expected. On the other hand, the topology results with fabrication constraints produced columns as desired.

14. Summary and Conclusions

A topology optimization method that avoids generating truss-like structures was discussed. Truss-like structures are avoided by parameterizing the design domain with functions that are built to satisfy certain fabrication requirements. The new method can be used to generate design proposals that contain straight members or curved shells making the final results easier to fabricate. An example that shows the generation of a stampable structure was presented. In addition, examples that show the use of the methodology on civil engineering structures were discussed. The presented topology method is currently implemented in the Genesis program and it does not replace existing methods; it complements them.

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