

Multidisciplinary Optimization of a Transport Aircraft Wing using VisualDOC

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Multidisciplinary Design Optimization (MDO) has become popular as engineering systems have become more complex. One of the most challenging applications of MDO is in the field of simultaneous aerodynamic and structural optimization because of the design tradeoffs between lift, drag, weight and strength. In addition, there is an inherent coupling between structural deformations and aerodynamic shape for long range and high-speed transport aircraft. These issues create organizational complexity and potential for high computational expense. This paper demonstrates a technique for aeroelastic MDO that couples aerodynamic optimization with structural optimization as a sub-optimization problem. The organization complexity of MDO is reduced by the sub-optimization problem. Further, by dividing the MDO problem into a multi-level optimization problem, transferring data from one discipline to another is simplified. It is shown that gradient based optimization can be computationally efficient. Finally, it is demonstrated how commercially available software can be effectively used for solving MDO problems.

Introduction

Multidisciplinary Design Optimization (MDO) is generally described as a methodology for the design of systems where interaction between several disciplines is considered. The increasing complexity and performance demands on engineering systems are the driving force behind MDO. Increased performance requirements and economic pressure to reduce the operational costs can not always be met by traditional design processes. However, MDO typically creates a higher level of organization complexity than single discipline design optimization, and the computational demands of MDO can be significantly higher.

This paper describes a solution process for the design of a transport aircraft wing. The process considers the simultaneous optimization of aerodynamic drag and structural strength using a multilevel formulation. Specifically, this paper describes the organizational complexity and how the MDO formulation chosen and

software tools that are employed reduce the organizational burden. Finally, this paper will present the optimization results and describe the computational effort required to solve the problem.

MDO problems are more complex than single discipline optimization problems for two primary reasons. First and foremost is that analysis and design optimization codes for each discipline generally need to interact. The interaction is complex with each discipline possibly having a unique set of design variables and responses. The nature of MDO oftentimes requires that all or at least a large portion of the variables and responses from each discipline be communicated between software modules. Typically, this data transfer is not a simple direct transfer of data, but a more complex abstraction of the data. For example, structural deformations change the aerodynamic shape. Using the finite element method for structural analysis, the deformations are given as nodal coordinate displacements; however, the

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aerodynamic shape may be described as the wing's aspect ratio, sweep, and airfoil section characteristics, particularly, depth to chord ratio.

The complexity of MDO problems is also due to the complexity of the individual disciplines. Most design organizations are divided into disciplinary groups, and these groups must interact. Their interaction is a prelude to the interaction between disciplinary software tools. The choice of disciplinary software tools may dictate the choice and number of design variables, the level of abstraction of the problem, and even the MDO formulation employed. The disciplinary groups must work together to develop system objective functions, design variables, and responses. Decisions on the choice of design variables and on what kind of optimization to employ can have profound effects on how to transfer data between software modules and between the design groups.

For example, Integrated Product Development approach¹ was used in the process of creating the F/A-18E/F aircraft. This method required a shift from cereal to concurrent process strategies, which in turn required closer work of disciplinary teams. One of the important features outlined by this work is development of the database that can be used by many disciplines. Another conclusion drawn from this work was that when a complicated system is designed in industry, often there is no sense in performing optimization based on low-fidelity tools.

Similar to the aircraft companies, the automotive companies tend to develop their own MDO tools. An example of such a tool is Integrated Vehicle Design Analysis system² developed at GM R&D Center over the past few years. It incorporated several disciplinary analysis tools and its own database. Like in the aerospace industry, aerodynamic-structural interaction plays an important role here.

The computational demands of MDO are generally higher than those of single discipline design optimization. Single discipline design optimization has challenged software developers to create robust and computationally efficient algorithms. MDO presents many additional challenges to development teams. First, MDO problems usually have more design variables, and solution times dramatically increase as the number of variables increases. The number of variables may also affect the choice of optimization algorithm. For example, response surface techniques do not scale well to large number of design variables. Second, without careful consideration, MDO problems can produce

discontinuities in the design space that can cause some optimization algorithms to fail. Third, although each discipline may employ linear analyses, the relationship between the disciplines may be nonlinear and require additional computational resources. Finally, MDO oftentimes requires multiple objective functions and thus increases the computational cost.

MDO Formulation

A great variety of approaches is used in industry and academia to implement multidisciplinary optimization concepts. Sobieski and Haftka³ surveyed recent developments in MDO for aerospace design. In their survey, they identified several categories of problem formulations. When only two or three disciplines interact, it is possible to reduce the organizational complexity by focusing on the interaction of the disciplines. Most of the references they cited in this category worked on simultaneous optimization of structures and aerodynamics or structures and control systems. Below are some of the examples of MDO applications.

The interaction of the computational fluid dynamic (CFD) and Finite Element (FE) analysis systems is one of the most interesting and complex MDO problems. The works by Bhardwaj et al⁴, and by Onishi et al⁵, are typical examples of this branch of MDO in application to aircraft. The aerospace industry is a natural first candidate for MDO applications because of economic pressure to reduce operational costs. However, currently MDO is applied to the great variety of areas, other than aerospace. Some of the many other MDO applications include the multidisciplinary optimization of the turbine engine, that was considered in the work of Rohl et al⁶. The automotive industry has started using MDO and developing corresponding tools². Another interesting MDO problem was considered in application to the magnetic levitation trains⁷.

All the MDO application considered above rely upon integration of various disciplinary codes together. Another category of MDO formulations relies on simple analysis tools. Vanderplaats⁸ used this type of formulation in the ACSYNT program. Because of the simplicity of the analysis tools, he integrated the different disciplinary analyses in a single modular computer program. This reduced the organizational complexity and reduced the computational effort. It should also be noted that ACSYNT was useful at the conceptual stage of design rather than at a more detailed level.

Typically, when researches in universities and industry implement MDO, they rely on “in-house” software to tie the disciplines together. The individual disciplines could be represented via response surfaces and the like, or by complex analysis and optimization codes. The common drawback of MDO systems developed in industry is the fact that these systems are proprietary. These systems as a rule are narrowly specialized, that is, created for the particular product with certain requirements. Such systems are hardly expandable for the purpose of creating other products. Considerable effort is required for other organizations to use the same MDO code and adjust it to their needs. On the other hands the MDO systems created in academia in most cases lack fidelity of the analysis and correct requirements for the products.

One of the ways to try to overcome these contradictions is to use flexible commercially available software. Currently there are not many commercially available MDO software packages. Two of the several well-known packages are iSIGHT⁹ and LMS Optimus¹⁰. Salas and Townsend¹¹ of the NASA Langley Research Center formulated a set of requirements for general MDO frameworks. Current commercially available software that is intended for MDO use could hardly meet all of these requirements. All of the packages have their own advantages and disadvantages.

In the present paper we use the commercially available program, *VisualDOC*¹², to couple the different disciplines. Using a relatively simple MDO example, we will demonstrate that when using this particular program and the technique described here, one may replace and/or add disciplines to an MDO problem with minimal effort, as well as considerably increase the complexity of the disciplinary codes involved. One of the main differences of VisualDOC from the other available MDO frameworks is that VisualDOC was developed based upon the well-known general purpose optimization package, *DOT*.¹³ Thus, the optimization procedures used in VisualDOC are well tested and robust.

This paper presents a multilevel formulation for simultaneous optimization of aerodynamic drag and structural weight of a wing. The optimization occurs at two levels. At the system level, the aerodynamic optimization of the airfoil shape is done, where the design variables are the airfoil configuration variables: the aspect ratio and depth-chord ratio for a swept wing. The objective function at the system level is to maximize the range under a constant takeoff gross weight (TOGW) constraint. Here the aerodynamic analysis is

simplified to specifying a “reasonable” distribution of aerodynamic pressure in the chord-wise and span-wise directions.

The structural optimization is considered a sub-optimization problem. The design variables are the membrane thickness of the wing box. The objective of the structural optimization is to minimize the structural weight under stress constraints.

By considering a multilevel MDO formulation, we can significantly reduce the organizational complexity and the computational effort. First, the multilevel approach allows for the isolating the interactions between the two disciplines. This reduces the complexity issue. Second, the multilevel approach lets us simplify the aerodynamic analysis, which may be substituted by a more detailed analysis at a later time. This reduces both the complexity and computational effort. Third, we can employ a state of the art design optimization and analysis program (GENESIS¹⁴) for the structural optimization independent of the aerodynamic analysis and optimization. Therefore, we can take advantage of the computational efficiency of the structural optimization, which makes use of state of the art approximation methods. The next section provides more details on the proposed formulations for each level.

Problem Statement

This section describes each part of the optimization problem and the interaction between them. First the structural problem is described.

Structural Optimization Problem

Figure 1 shows the structural model. It is a simple finite element mesh defined by 72 nodes. The chord lines are streamline oriented, which is not the usual practice, but it simplifies the mesh. The structural box rises above the bottom plane. The nodes at the leading and trailing edges are non-structural and serve to transfer aerodynamic loads acting on the leading and trailing edges. These nodes are connected to the box by rigid links such that they do not add stiffness. The wing box is made up of membrane elements only. For simplicity spar and rib caps are not modeled. Aluminum is considered as the material with a tensile allowable of 50 ksi and compression allowable of 25 ksi. The compression allowable is reduced to allow for buckling of the panels.

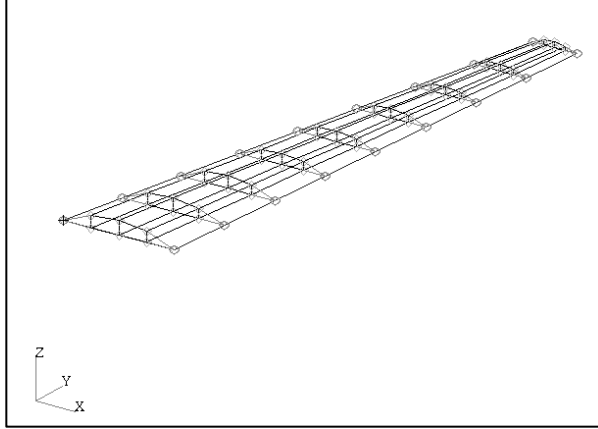


Figure 1: Wing Box Structural Model.

The wing size corresponds to a long-range transport aircraft in the Boeing 767 class. Four design regions are considered in the span-wise direction (Figure 2). Each region consists of two bays. The structural design variables consist of the thickness of 7 membranes in each region, which are two top panels, two bottom panels, and three spar webs. The rib web thicknesses are all treated as constant. There are a total of 28 structural design variables and 148 stress constraints. The objective is to minimize the structural weight.

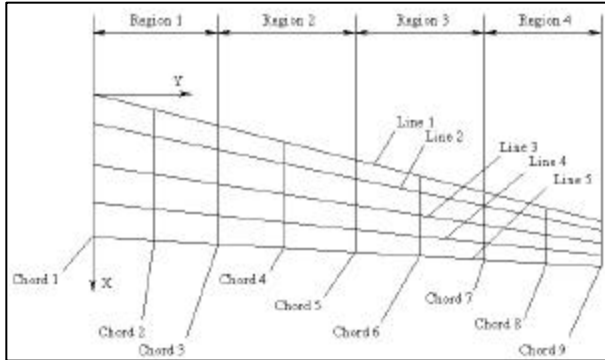


Figure 2: Design Regions.

The loads are applied as nodal forces representing aerodynamic pressure. Figure 3 shows the normalized chord-wise distribution of the loads. The span-wise normalized distribution is shown in Figure 4. These are "reasonable" distributions for a transport wing¹⁵. The aerodynamic pressures are applied only to the bottom surface nodes.

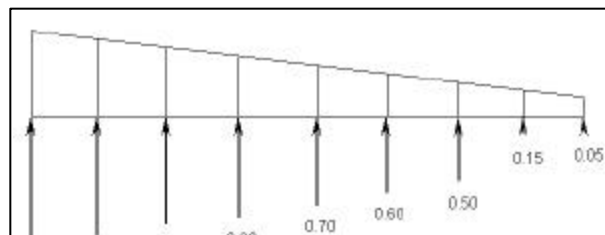


Figure 3: Span-wise Pressure Distribution.

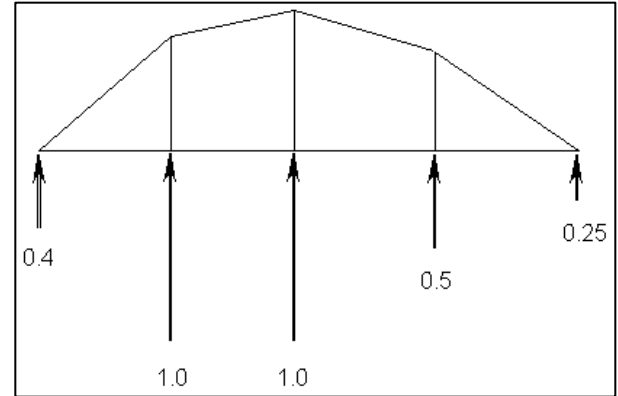


Figure 4: Chord-wise Pressure Distribution.

The wing area is 2100 square feet and the wing loading is approximately 143 lb/sf. The pressure distribution can be converted to concentrated forces at the bottom surface nodes, accounting for the areas associated with each node.

$$F_{i,j} = 143 \cdot LP_i \cdot CP_j \cdot A_{i,j} \cdot (\alpha_{root} + \beta_j) / \alpha_{root_ref} \quad (1)$$

where, i is the line number of the node and j is the chord number of the node; LP_i is the span-wise pressure distribution value for the node (from Figure 3); CP_j is the chord-wise pressure distribution value for the node (from Figure 2); $A_{i,j}$ is the area associated with the node; β_j is the angle of rotation of chord j relative to the root chord, calculated using equation (2); α_{root} is the angle of attack at the root of the wing; α_{root_ref} is the angle of attack at the root of the reference wing. The $(\alpha_{root} + \beta_j) / \alpha_{root_ref}$ term represents the influence of the local chord rotation relative to the root chord. For the initial case, all β_j terms are considered zero, and $\alpha_{root} = \alpha_{root_ref}$.

Once the displacements are calculated in the structural sub-optimization problem; the β_j terms can be calculated using the vertical displacements.

$$\beta_j = (U_{z1,j} - U_{z5,j}) / (X_{1,j} - X_{5,j}) \quad (2)$$

Here, the U_z terms are the Z components of the displacements at the leading and trailing edges for each chord. The denominator is the chord length for each chord.

Two load cases will be considered. The first is a 3.75-G maneuver and the second is a -1.5-G maneuver.

Aerodynamic Optimization Problem

The objective of the MDO problem is to maximize the range of the aircraft under constant TOGW. The

simplified Breguet formula for range when the propulsion system is not considered is given as

$$R = C \cdot (L/D) \cdot \ln(\text{TOGW} / (W_c + W_{opt})) \quad (3)$$

where, R is the range; L is the lift; D is the drag; W_c is the nonstructural weight, W_{opt} is the structural weight, and C is the constant, which we set to 252.08 so, that the range of the reference configuration will be equal to 5000 mi.

Fixing the payload at $0.13 \cdot \text{TOGW}$ and the fuel load at $0.26 \cdot \text{TOGW}$, we can represent W_c as,

$$W_c = 0.61 \cdot \text{TOGW} \quad (4)$$

Since the lift, L , must equal TOGW , the range formula becomes,

$$R = C \cdot (\text{TOGW} / D) \cdot \ln(\text{TOGW} / (0.61 \cdot \text{TOGW} + W_{opt})) \quad (5)$$

Equation (5) is the objective function for the aerodynamic optimization problem that occurs at the system level. The two designable components are D and W_{opt} . W_{opt} is simply the weight from the structural sub-optimization problem factored by 1.3 to represent structural overhead. The drag, D , is determined by the aerodynamic analysis.

In order to simplify the aerodynamic optimization we assume the area of the wing to be constant. We also do not change the sweep of the wing, $p=25/120$, in the process of the aerodynamic optimization.

The aerodynamic analysis is simplified when computing the total drag, D , which consists of the induced drag, D_I , and wave drag, D_w , components. The induced drag depends on the aspect ratio of the wing and is calculated using the following equation:

$$D_I = D_{ref} \cdot 0.4 \cdot A_{ref} / A \quad (6)$$

where, D_{ref} is the total drag computed for the reference wing ($D_{ref} = 40000$ lbs.); A_{ref} is the aspect ratio of reference wing ($A_{ref} = 6.8571$), and A is the aspect ratio of the current wing. The wave drag depends on the wing volume. If we consider the wing area to remain constant, the following formula can represent the wave drag:

$$D_w = D_{ref} \cdot 0.015 \cdot ((t/c) / (t/c)_{ref}) \quad (7)$$

where, $(t/c)_{ref}$ is the reference wing's depth to chord ratio ($(t/c)_{ref} = 0.12$), and (t/c) is the current depth to chord ratio. Therefore, the total drag, D , is given by,

$$D = D_I + D_w + 0.585 \cdot D_{ref} \quad (8)$$

Although the aerodynamic analysis is simplified, the interaction between the aerodynamic analysis, the system level optimization, and the structural design is complex. First, the structural deformations affect the aerodynamic forces acting on the wing. Second, the aerodynamic shape described by both the aspect ratio and depth to chord ratio have a direct influence on the structural model because they dictate the node coordinates. Finally, the resulting structural optimization computes the optimum weight for the current aerodynamic shape. The next section describes the interface between these three steps.

Interface Issues

As the wing deforms, the loads will change as the angle of each chord changes. The total load must still sum to $1/2 \text{ TOGW}$ (one wing supporting half the weight). This is achieved by trimming the wing to a new α_{root} value. Therefore, the α_{root} quantity needs to be determined prior to each structural subproblem since aerodynamic loads are considered constant in the subproblem:

$$\alpha_{root} = \alpha_{root_ref} - \frac{143 \cdot \left[\sum_{j=1}^9 \left(\sum_{i=1}^5 LP_i \cdot A_{i,j} \right) \cdot CP_j \cdot \beta_j \right]}{\frac{1}{2} \text{TOGW}} \quad (9)$$

The total drag is a function of the wing's depth to chord ratio and the aspect ratio. The depth to chord ratio is used to set the new Z-coordinates of the wing box as given in the following formula:

$$Z_{i,j}^{n+1} = Z_{i,j}^0 \cdot (t/c)^{n+1} / (t/c)_{ref} \quad (10)$$

where, i is the line number of the node, j is the chord number of the node, n is the system level iteration number.

The aspect ratio is used to define the new X and Y-coordinates of the bottom surface nodes. Since the wing sweep angle is preserved, we can calculate the new X and Y coordinates of each node using the following two formulae:

$$Y_{i,j}^{n+1} = Y_{i,j}^0 \cdot b^{n+1} / b_{ref} \quad (11)$$

$$X_{i,j}^{n+1} = (X_{i,j}^0 - Y_{i,j}^0 \cdot p) \cdot \frac{cr^{n+1}}{cr_{ref}} + Y_{i,j}^{n+1} \cdot p \quad (12)$$

where, b is the wing span; b_{ref} is the wing span of the reference wing ($b_{ref} = 120$); cr is the root chord length; cr_{ref} is the root chord of the reference wing ($cr_{ref} = 25$).

Optimization Procedure

The optimization problem is divided into two levels. The structural optimization is carried out as a sub-optimization problem. The aerodynamic optimization is done at the system level. The following steps outline the optimization procedure. In these steps $DVAR$ represents the structural design variables, i.e. the shell thicknesses, n is the iteration number at the system optimization level, all the other variables are described previously.

0. Initialize all the design variables and constraints:
 $(t/c)^0 = 0.12$; $A^0 = 6.8571$; $a_{root}^0 = 1.5$;
 $b_j^0 = 0$; $n = 0$; X_{ij}^0 ; Y_{ij}^0 ; Z_{ij}^0 ; $DVAR_k^0$; D^0
1. Calculate concentrated aerodynamic forces at the bottom surface nodes: F_{ij}^n (equation 1)
2. Calculate the location of the structural nodes:
 X_{ij}^n ; Y_{ij}^n ; Z_{ij}^n (equations 10, 11, and 12)
3. Use GENESIS to solve the following optimization problem:
Given: F_{ij}^n ; X_{ij}^n ; Y_{ij}^n ; Z_{ij}^n ; $DVAR_k^n$
Find: $DVAR_k^{n+1}$; U_{ij}^{n+1}
Minimize: structural weight, W
Subject To: strength constraints and gage limits
4. Calculate rotation of the chords: b_j^{n+1} (equation 2)
5. Calculate angle of attack of the chords:
 a_{root}^{n+1} (equation 9)
6. If a_{root} has converged, proceed to step 6, else go to step 1.
7. Calculate adjusted structural weight: $W_{opt} = 1.3 \cdot W$
8. Use VisualDOC to solve the following problem:
Given: $(t/c)^m$; A^m ; $(t/c)_{ref}$; A_{ref} ; W_{opt} ; $TOGW$; D^m
Find: $(t/c)^{m+1}$; A^{m+1} ; D^{m+1}
Maximize: R (equation 3)
Subject To: move limits on (t/c) and A
9. If $(t/c)^{m+1}$ and A^{m+1} have converged, then stop; else go to step 1.

VisualDOC Implementation

The optimization procedure described above was implemented in the commercially available general-purpose optimization system VisualDOC. When defining the system level optimization problem users work in a spreadsheet-like environment where they specify initial values and bounds for the system level design variables and identify responses, constraints (if any), and objective functions. Users also specify the method of optimization that is going to be employed. If

the user does not like the default method, Modified Method of Feasible Directions, (MMFD), then by a click of a button it is possible to change the method to sequential quadratic or liner programming, or to the response surface method. As it was done in our case.

VisualDOC requires the user to supply a subroutine or function that evaluates the responses. Inside this subroutine two system calls were made, for the problem described here. The first calls GENESIS to perform structural analysis and optimization and provide optimal structural weight for the given geometrical configuration. The second system call performs aerodynamic analysis and to calculate range of the aircraft. In the current version of the paper, we used the very simplistic approach to perform aerodynamic analysis. However, current work is being done to replace the simplified aerodynamic analysis with a CFD analysis.

Results

We used two methods to perform global optimization: direct optimization employing MMFD and optimization using response surface approximations. For the case of direct optimization, it took five global optimization cycles and 29 structural optimizations to get to the optimal wing design. When the optimization was performed using the response surface method it required eight global iterations and eight structural optimizations. As a result of the multidisciplinary optimization, the calculated range was increased by more than 25% with respect to the range of original configuration both for the direct optimization and for the optimization using response surface methodology. This result also confirms good quality of the approximations used in VisualDOC since the results obtained using response surface approximations were within 1% of direct optimization results, though the number of required structural evaluations was significantly lower.

Figure 5 shows the history of the objective function (range) when direct optimization was performed. Figure 6 presents the history of the structural weight (objective function of the structural optimization) for all the structural analyses performed for the direct optimization case.

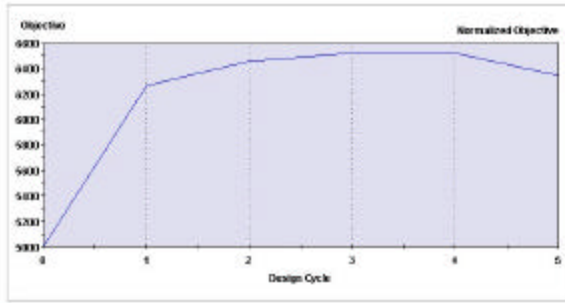


Figure 5: History of the objective function during direct optimization.

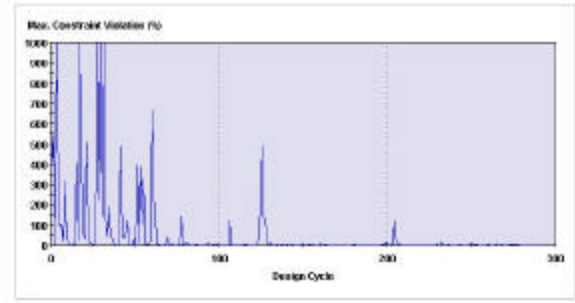


Figure 7: History of the maximum stress constraint violation direct optimization.

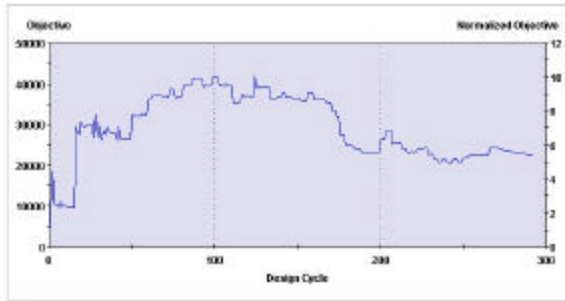


Figure 6: History of the structural weight during direct optimization.

In Figure 6 one can identify start of the new global iterations at the places with the jumps in the values of the structural weight. The reason for these jumps is the change in the aerodynamic shape and aerodynamic loads at the start of the new iteration, while the structure corresponds to the geometry and loads from the previous iteration. One may also notice that as optimization progresses, the jumps become smaller. This is because the changes in the geometry and loads are not that drastic, when the optimization is getting closer to the optimum.

Figure 7 shows the maximum violation of the stress constraints during the structural optimization. Similar to the case of the structural weight, the violation of the constraints becomes less and less severe as the optimization progresses.

Table 1 shows the initial and optimal values of the system level objective function and system level design variables. From these results one can see that both direct optimization and optimization employing response surface methods converged to the same global design. Both design variables are at their bounds. One can also notice that the thickness of the wing (depth to chord ratio) increased, whereas the span of the wing (aspect ratio) decreased in the process of the optimization. The main reason for it could be the low fidelity of the aerodynamic representation. Because of that the optimizer picked up the wing that is more advantageous from the structural point of view. In addition, we did not consider sufficient number of load cases, and optimizer definitely tried to take advantage of it.

Parameter	Initial value	Direct optimization	Response surface optimization
Range (n.mi.)	5,000	6,342	6,403
Depth to chord ratio (t/c)	0.12	0.14	0.14
Aspect ratio	6.8571	5.9165	5.8824

Table 1: Initial and optimal values of the system level objective and design variables.

Conclusions

The multidisciplinary optimization of the transport aircraft wing was performed using the VisualDOC framework. The results obtained were reasonable, taking into account the low fidelity of the aerodynamic analysis: the calculated range was increased by more than 25% with respect to the range of original configuration. The quality of the response surface

approximations proved to be good for this particular problem.

The main difference that distinguishes the current approach from many complicated multidisciplinary works in aeroelasticity is the fact that structural suboptimization was used instead of performing the structural optimization along with the aerodynamic optimization. By creating the structural suboptimization problem, the interaction between the system level and subsystem level is isolated and simplified. Specifically, the only data that is transferred are the aerodynamic pressures and airfoil configuration. This approach proved to be promising, though work with significantly higher fidelity aerodynamic code will be required to confirm the usefulness of this approach.

VisualDOC proved to be a good framework for multidisciplinary optimization. Particularly, it is very convenient to perform prototyping work in the VisualDOC environment, when trying different disciplinary analysis (and optimization) tools is required along with trying different optimization methods and different combinations of input and output parameters.

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