Design Optimization – History and Prospects

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Abstract
Design optimization has been under development for nearly 45 years. Here, the development of optimization in engineering will be reviewed. Examples will be presented to demonstrate the level of sophistication possible in applying this technology. Prospects for widespread use in industry will be discussed. It is concluded that, while much research always remains, optimization technology has matured to the point where it can and should be used routinely for engineering design.

1 Introduction

We have now seen nearly forty five years of intensive development in structural and general purpose optimization research. This has culminated in numerous commercial products that are available today to solve design problems of remarkable size and complexity. These basic developments, together with modern graphical interfaces, makes it possible to use this technology with very little formal training in optimization theory.

Despite the widespread availability of this technology, it is seldom taught as a design tool by universities and remarkably underutilized by industry. Yet the motivation to use optimization is compelling. For automobiles, a ten percent mass reduction will increase fuel economy by about seven percent. Only a one percent economy improvement will save about three billion dollars per year in the U.S. at the pump. Similarly, by reducing the mass of a commercial aircraft by about two hundred pounds adds a paying passenger for the life of the aircraft. A one pound reduction in the mass of a spacecraft will either add a pound of payload or save about $20,000 per flight to space. Even beyond the cost argument, the savings in natural resources through the use of optimization could be immense.

The purpose here is to briefly review the development of optimization leading to the current state of the art, offer examples to demonstrate the power of optimization and discuss the potential of using this technology to benefit society.

2 Optimization Concepts and History

Structural optimization dates to the work of Maxwell [1] in 1869 and Mitchell [2] in 1904. The modern, computer based, era of structural optimization was ushered in by Schmit's classical paper in 1960 [3], though in his 1981 review of Structural Synthesis development[4], he credits a paper by Klein in 1955 [5] for providing some key ideas.

The basic problem solved by numerical optimization (or more formally, mathematical programming) is

Find the set of design variables, ***X***, that will

$$\text{Minimize } F(X) \tag{1}$$

Subject to;

$$g_j(X) \le 0 \quad j = 1, m \tag{2}$$

and

$$X_i^L \le X_i \le X_i^U \quad i = 1, n \tag{3}$$

where $F(X)$ is the objective function, $g_j(X)$ are inequality constraints and Eq. 3 defines lower and upper bounds on the individual design variables. There are n design variables and m constraints. Equality constraints can be included as well and, for purposes of this discussion may be thought of as two equal and opposite inequality constraints.

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Here, we will briefly offer a narrative of the development of general optimization algorithms followed by development of structural optimization. It is assumed that the reader has a basic knowledge of the subject, so a mathematical description of the methods and concepts will be forgone. Most of these details may be found in Ref. 6.

3 Optimization Algorithms

During the 1950s and early 1960s, random search methods were popular, where the components of the X vector were chosen randomly, an analysis was performed and if an improved design was found, it was kept. This was repeated until no progress could be made or computer resources were exhausted (the usual case). The choice of random values could be the actual values of X_i or perturbations of these values. Some researchers observed that, after some time, they could create a vector from the worst to the best design and accelerate the process by moving in this direction. One might observe that this is a (rather poor) gradient search. These methods are easy to program but are very inefficient and are limited to only a few variables.

Focus during the 1960s included Sequential Linear Programming (SLP) [7], Sequential Unconstrained Minimization Techniques (SUMT) [8] and Feasible Directions methods [9]. Though some non-gradient based methods were also developed during this period, these gradient based methods were generally considered to be more efficient and reliable.

The 1970s saw development of the Augmented Lagrange Multiplier [10] and Generalized Reduced Gradient methods [11]. These methods had the advantage that they have a strong theoretical basis in the Kuhn-Tucker conditions for optimality. The idea is that, by creating an algorithm that will drive the design to a Kuhn-Tucker point, improved efficiency and robustness will result. During the late 1970s, development of response surface methods began [12,13] and has continued since.

The 1980s were a period of refinement ending with renewed interest in random methods in the engineering community and Sequential Unconstrained Minimization Techniques by the operations research community. The random (and related) methods include Genetic Search [14], Simulated Annealing [15] and related methods that attempt to mimic natural evolutionary processes. The Sequential Unconstrained Minimization Techniques focused on interior point methods based on the Kuhn-Tucker conditions [16].

Throughout the 1990s, Genetic Search algorithms were the focus of considerable research by the engineering community and a new method called Particle Swarming was added [17]. Meanwhile, the operations research community focused on interior point methods and continued to refine these. For engineering problems, an exterior penalty function method was developed for solution of very large scale continuous and discrete variable problems [18].

As optimization algorithms have improved, the size and complexity of the engineering applications has grown. Figure 1 shows the trend in engineering problem size beginning in 1960. While there is considerable scatter in the data to create this figure, it is seen that there has been an exponential growth in problem size. Recent developments (BIGDOT) [18] allow us to solve both continuous and discrete optimization problems of very large size.

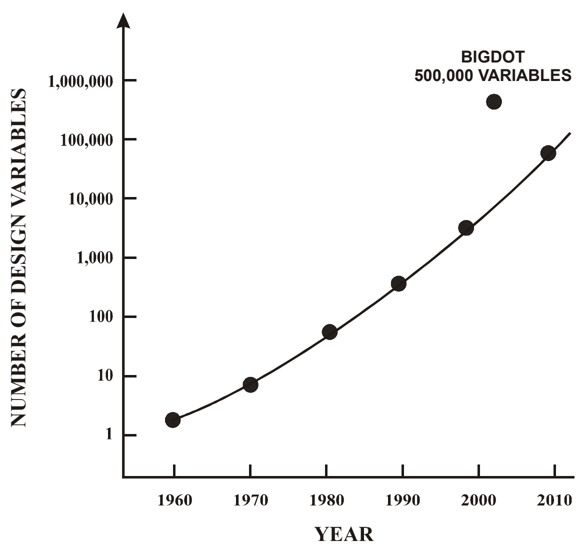

Figure 1. Growth in Optimization Problem Size

3.1 Structural Optimization

Structural optimization began in earnest with Schmit's classical paper in 1960 [3]. This ushered in the era of numerical search methods which were more general than previous analytically based methods such as Shanley's work published in 1952 [19]. The 1960s saw a great deal of research in structural optimization, dealing mainly with member sizing of trusses, frames and shell structures. Initially, gradients were calculated by finite difference methods. It was not until 1965 that gradients were calculated analytically and this happened with such little fanfare that the original published work by Fox [20] on calculating gradients analytically is relatively unknown and seldom referenced.

By the end of the 1960s it was becoming apparent that numerical optimization was limited to perhaps fifty variables and was computationally too expensive to the a usable design tool. This was particularly emphasized in a paper by Galletly, Berke and Gibson in 1971 when they called the 1960s "the period of triumph and tragedy" for structural optimization [21]. Thus, the 1970s began the era of optimality criteria methods. Optimality criteria offered the ability to deal with large numbers of design variables but with a limited number of constraints and without the generality of numerical optimization methods. Numerical optimization methods were given new life in 1974 when Schmit and Farshi published their work on approximation concepts [22]. These methods were based on the concept of creating approximations using the underlying physics to allow for large moves and reduced the number of detailed finite element analyses from several hundred to the order of ten. For statically determinate trusses or membrane structures, these approximations were shown to be exact for stress and displacement constraints. Parallel to the development of approximation concepts, the adjoint method for gradient computations was developed [23,24]. Finally, in the late 1970s Fleury and Sanders [25] reconciled numerical optimization and optimality criteria methods by showing that optimality criteria are closely related to duality theory in numerical optimization.

For a detailed understanding of the development and state of the art at the end of the 1970s, Schmit's AIAA History of Key Technologies [4] paper is an excellent resource.

The 1980s were a period of refinement and the initial steps of creating commercial structural optimization software. Second generation approximations were created using force approximations [26 - 27] instead of the earlier stress approximations. Similarly, Reileigh quotient approximations were created for eigenvalue constraints [29]. These new approximations expanded the element types to shell and frame elements among others. Importantly, for such elements as frames it was not possible to treat the physical dimensions as design variables and section properties as intermediate variables so that the designer could now deal with the actual variables of interest.

4 Examples

Examples are presented here to demonstrate the breadth of design tasks that can be routinely solved with modern commercial optimization software. Most problems solved in a purely research environment are not sophisticated enough to be useful here and most real commercial problems are proprietary and cannot be published. Therefore, these examples fall somewhere between academic and real commercial products. The general application examples presented here are solved by VisualDOC [30] and the structural optimization examples are solved by GENESIS [31]. The capabilities demonstrated here are considered typical of what is available commercially.

4.1 Shape Optimization of a Pin

Figure 3 shows a cutaway of a symmetric structure with a load on the steel pin. The outer structure is ceramic and the intermediate portion is an adhesive. The objective is to change the shape of the outer structure to minimize the maximum stress with deformation limits. This is a nonlinear contact problem solved by the ABAQUS [32] analysis software. Nine shape variables were used and the maximum stress was reduced by eleven percent. This is typical of the improvement optimization provides for an existing design.

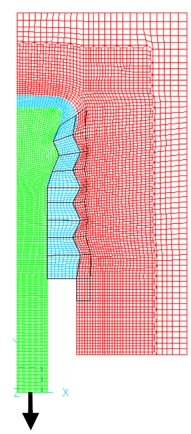

Figure 3. Shape Optimization

4.2 Hybrid Vehicle Control System

Figure 4 shows a parallel hybrid vehicle where the control system is analyzed by a research program called ADVISOR [33]. This is a multiobjective problem where we wish to maximize fuel economy while minimizing hydrocarbon and nitrous oxide emissions with limits on acceleration and grade climbing ability.

Economy was increased by 6.5 percent while hydrocarbon emissions were reduced by 3.6 percent and nitrous oxide emissions were reduced by 11.5 percent.

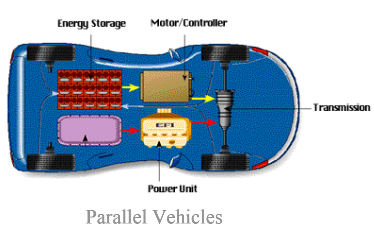

Figure 4. Hybrid Vehicle

4.3 Heat Sink Optimization

Figure 5 shows a heat sink, typical of that used by a computer processor. The analysis program used here is FLUX2D [34]. The objective is to minimize mass with limits on heat dissipation, maximum temperature in the CPU and maximum temperature in the chassis. Starting from an arbitrary initial design, the mass was reduced by 47 percent while satisfying all constraints.

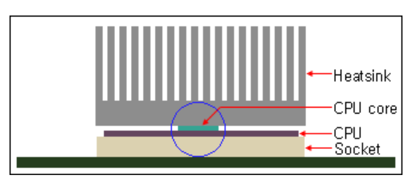

Figure 5. Heat Sink Optimization

4.4 Airfoil Shape Optimization

Figure 6 shows a two dimensional airfoil which is optimized to maximize the lift to drag ratio. The design variables are the mean line camber, chordwise position of the maximum camber, maximum thickness and angle of attack. Beginning with a simple NACA symmetric airfoil, the maximum lift to drag ratio was increased from 4.9 to 66.3. The analysis program used here was the GAMBIT/FLUENT [35] software and the optimum was achieved using 15 analyses.

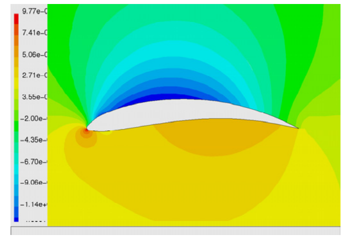

Figure 6. Airfoil Optimization

4.5 Cooling Optimization

Another example using Fluent is shown in Figure 7, which is a two dimensional model of an engine block, gasket and head cooling model. The objective is to maximize the fluid flow velocity averaged at ten locations. The design variables are the diameter and locations of the holes in the gasket, where the holes must be within the larger holes in the head and block. Additionally, constraints were imposed to insure a minimum flow velocity at ten locations. The optimization required 24 calls to Fluent and the constraints were overcome after 15 of those calls.

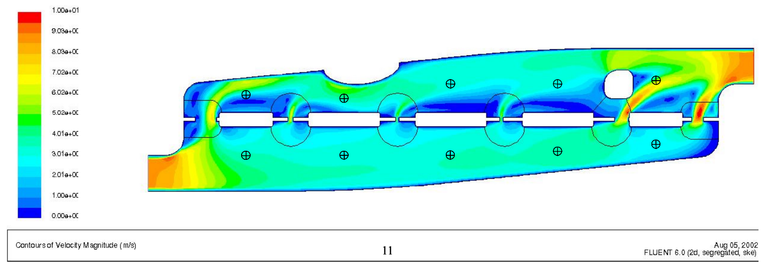

Figure 7. Cooling System Optimization

4.6 Pump and Piping System Optimization

As a final general purpose optimization example, consider the design of a chemical plant with numerous pumps and pipes. The Mercury [36] software has optimization integrated into it, including discrete variables, which are common in this type of system. The analysis is capable of estimating initial and life cycle costs of systems.

Figure 8 show the graphical interface for problem setup. Table 1 shows the results for a typical design. Two cases were considered. The first was to minimize initial cost and the second was to minimize life cycle cost. It is interesting to note the trade-off between initial and life cycle costs. If the initial cost is minimized, life cycle cost is increased, while if life cycle cost is minimized, initial cost is increased. This software was used to design four plants for minimum life cycle costs. An average of fifty percent savings was realized for these four plants.

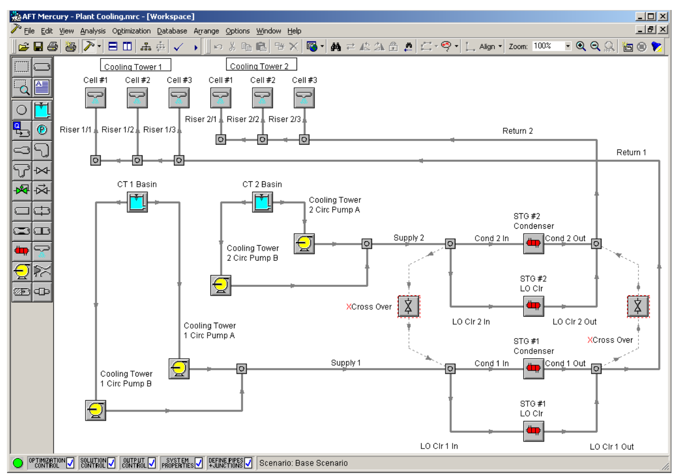

Figure 8. Mercury Graphical Interface

4.7 Car Body Reinforcement

As noted above, structural optimization is more advanced than general purpose optimization because we can calculate gradients of the needed responses and because we have very high quality approximation techniques to provide efficiency and reliability.

Figure 9 shows a car body model which we wish to reinforce to increase the bending and/or torsion frequency. The approach used here was to allow every element in the model was optimized for thickness (with

Table 1. Mercury Optimization Results

Optimized System Cost in U.S. Dollars

Criteria	Initial (Material + Installation)	Operating	Total
Initial Cost	923,000	9,406,000	10,328,000
Life Cycle	1,461,000	4,315,000	5,775,000

a lower bound of the original design) with the constraint that only a specified fraction of the material may be used. Here, 34,560 sizing variables were used. While somewhat difficult to see in Figure 12 (unless viewed in color), reinforcement was added in the areas of the firewall, rocker panels and rear fender areas.

Table 2 gives the increase in bending or torsion frequency for different values of added mass.

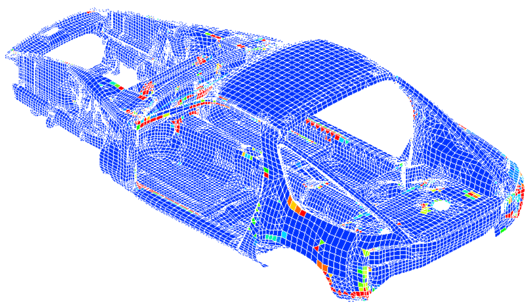

Figure 9. Car Body Reinforcement

Table 2. Frequency Increases

Added Mass (Kg)	Increased Frequency (Hz)	
	Maximize First Torsion Frequency	Maximize First Bending Frequency
2.64	4.81	6.42
7.32	7.56	9.89
15.06	9.66	112.15

4.8 Topology Optimization of a Simple Support

Figure 10 shows topology optimization of a simple support. This was a 100,000 variable example where the density of each element was designed. The key feature here is that manufacturing constraints were imposed to insure that the part could be cast.

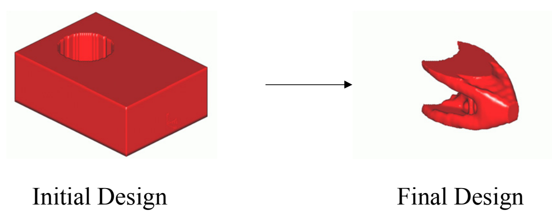

Figure 10. Support

4.9 Topology Optimization Without Manufacturing Constraints

If topology optimization is performed without considering manufacturing issues, very attractive structures are often produced but cannot be easily manufactured. Figure 11 is such an example where just over one million design variables were used. This structure was optimized to minimize strain energy under the applied load.

It is noteworthy that topology optimization seldom produces a final part, even though manufacturing constraints are used. This is because topology optimization normally does not include stress and other constraints. However, it does identify load paths and provides a very good starting point for shape and sizing optimization.

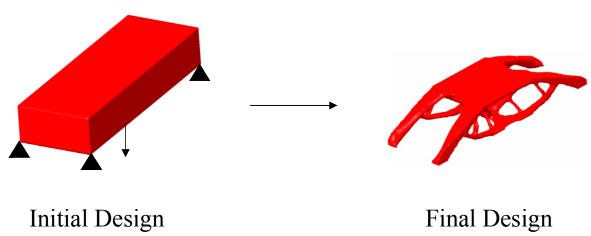

Figure 11. Skeletal Support

4.10 Heat Shield Optimization

Figure 12 shows an actual heat shield where it is desired to increase the first fundamental frequency. The approach used here is to perform shape optimization without changing the thickness of the elements. This is done by automatically generating “beads” to stiffen the structure. Optimization increased the frequency from 9.4 Hz to 40.1 Hz.

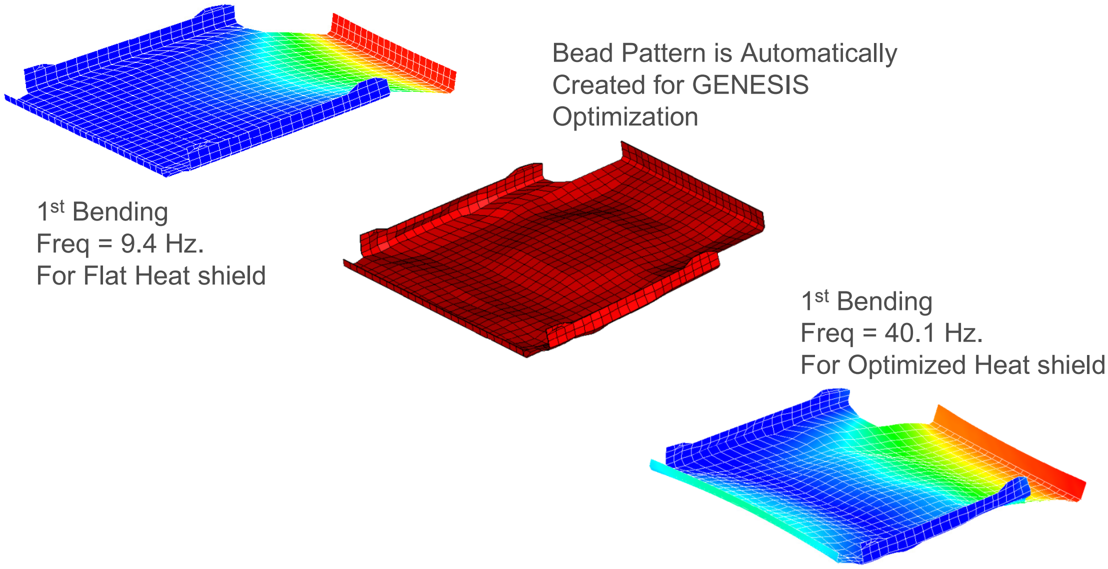

Figure 12. Heat Shield Optimization

5 Prospects

In 1978, this author predicted that optimization would be commonplace by 1985. Clearly, that did not happen and predictions are no longer made. However, we can judge that major progress has been made and that now optimization is becoming recognized as a powerful design tool. Also, numerous companies now provide commercial optimization software. Some of this is specific to structural optimization and other is intended for general purpose applications.

The examples presented here are either actual commercial applications or are examples that are similar to commercial applications which cannot be shown due to proprietary considerations. Experience has shown that if we optimize an existing structural part such as a steering knuckle, we always reduce the mass by five percent or more with no loss in strength. For ride quality improvement of an automobile (NVH – Noise, Vibration, Harshness), we can virtually always increase the first frequency of a car body by ten percent for the cost of under one percent added mass. Thus, we now have compelling examples to demonstrate that optimization can significantly improve designs with very limited cost and effort.

In recent years, we've seen considerable interest in topics such as six sigma design. As a management tool, companies have invested immense amounts of time and money in this concept. Although some claim that product quality has been improved, there is little information to demonstrate that the benefits have exceeded the costs.

Now consider what is possible if a major corporation makes it a corporate policy to use optimization whenever possible. First, unlike six sigma concepts, optimization does not require that everyone use it. Optimization will improve the quality or reduce design time for any system, subsystem or component to which it is applied. Therefore, the benefits can easily be measured. Sufficient experience now exists to be confident that the benefit will exceed the cost. With encouragement from top management, publicizing these successes will generate more and more use of optimization. When one or more major corporations focus on optimization technology this way, its use will surely spread rapidly. Without top level management support, acceptance of optimization will continue to progress slowly as it has for the past many years. In other words, without high level corporate support, as a friend has said, "Progress will be made one retirement at a time." On the other hand, the companies that strongly embrace this technology will have a clear competitive edge for the betterment of all.

6 Summary

A narrative of the development of optimization leading to the current use of this technology in industry has been offered. Development of this technology has followed two distinct tracks. One is optimization algorithms for general applications and the other is special techniques for structural optimization. The distinction is that structural optimization methods create a high quality approximation based on physics (as opposed to simple linearization) to improve efficiency and robustness, and then use a general purpose optimizer to solve this approximate problem.

A variety of applications have been presented to demonstrate the power available today. It is noted that some of these examples are not actual commercial applications because those tend to be proprietary. Indeed, to the best of this author's knowledge, the largest structural sizing optimization problem solved in industry exceeds 250,000 design variables with topology optimization problems exceeding two million variables.

It is concluded that, while much research and development always remains, the state of the art is well refined and is readily available in the commercial environment to improve design quality, reduce design time and increase corporate profits. Indeed, it is argued that no computational technology today is as effective as an advanced design tool as is numerical optimization.

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