Topometry Optimization: A New Capability to Perform Element by Element Sizing Optimization of Structures

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A new capability that allows finding the optimal distribution of sizing dimensions of elements in a given designable region is presented. The new capability allows performing sizing optimization on each element individually, whereas in traditional sizing it is performed by groups. Like topology optimization this capability allows finding the optimal location of void material but is not limited to that, as it can produce continuous variation of the geometric properties. Sizing optimization at the element level has been previously studied for special cases such as thickness distribution of plate elements and spring rates of elastics elements, what is new about this work is that the implementation is done for every type of element that is currently possible to size optimize in the GENESIS program. The implementation generalizes the sizing capabilities already available in the program and inherits most of its key efficiencies. The implementation has been done so that this new capability can be used simultaneously with shape, sizing and/or topography optimization.

Nomenclature

F	=	objective function
g_j	=	jth constraint
m	=	number of constraints
n	=	number of design variables
x_j	=	ith design variable
xl_j	=	lower bound of design variable i
xu_j	=	upper bound of design variable i

I. Introduction

INTENSE competition in the market place today requires engineers to continually search for better and more economic designs. Sometimes models have been refined so much that traditional sizing optimization cannot find enough improvements. In the last few years, GENESIS[1] users have occasionally been forced to substantially modify their models so each element can be designed independently with "element-by-element" sizing optimization. Since this task is tedious, external ad-hoc programs have been put together to break models of shell and/or elastic spring elements so that each element references an independently designed property. However, these programs have not been able to take fully advantage of all designable features of GENESIS. That lead to start thinking: Why not make this task automatic? Why not allow this task be available for every designable element type? The automation of this task and the generalization of sizing optimization from the property level to the element level are discussed in this paper. The resulting capability has been termed topometry optimization.

Before describing in detail what topometry optimization is, it is relevant to mention here that in GENESIS the traditional sizing and shape optimization problem is solved using the approximation concepts approach. In this approach, an approximate analysis model is created and optimized at each design cycle. The design solution of the approximate optimization is then used to update the full model, and a full system analysis is performed to create the next approximate analysis model. The sequence of design cycles continues until the approximate optimum design converges to the actual optimum design. In the mid-seventies Schmit et al. introduced approximation concepts for traditional structural optimization[2,3]. These concepts, in the eighties and early nineties, were refined to improve the quality of approximations[4,5,6]. In GENESIS, in the late nineties, these concepts were used to solve the topology

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optimization problem[7]. Now we use them to solve the topometry optimization problem as they apply naturally as an extension of sizing optimization.

II. The Optimization Problem

The optimization problem can be stated as:

$$\begin{aligned} &\min_{x_i} F(x_1, x_2, \ldots, x_n) \\ &\text{such that:} \\ &g_j(x_1, x_2, \ldots, x_n) \le 0; \quad j = 1, m \\ &xl_i \le x_i \le xu_i; \quad i = 1, n \end{aligned} \tag{1}$$

where F is the objective function, gj are the constraints, xi are the design variables and xl_i and xu_i the side constraints.

A. Design Variables

Design variables are parameters that can change directly or indirectly dimension of elements, grid locations and/or material properties.

B. Objective Function

Any of the considered responses can be used as the objective function for minimization or maximization. Often mass, strain energy or frequencies are used as objective functions.

C. Constraints

Any of the considered responses can be constrained to user-specified limits. Typical constraints are stress, displacements and mass.

III. Optimization Capabilities

Before the topometry optimization capability was implemented in GENESIS there were four types of optimization capabilities available: Sizing, shape, topography and topology. Simultaneous sizing, shape and topography optimization could be handled, while topology optimization was (and still is) performed separately. Topometry was added to work together with the sizing, shape and topography optimization. These types of optimization capabilities are associated with a certain design variable types and they are discussed next to help understanding topometry optimization.

A. Sizing Optimization

In sizing optimization, the element cross-sectional dimensions are typically used as design variables. However, in finite element models, data is provided for properties (i.e., areas, inertias, etc.). To overcome this difference, in GENESIS and other commercial programs, such as MSC.Nastran, the user needs to create equations that relate design variables to the properties. For example, if a bar element has a rectangular section, described by B (width) and H (height), then the following equations relate B and H with the A (area) and I_{zz} (inertia) properties:

$$A = A(B,H) = B*H \tag{2}$$

$$I_{zz} = I_{zz}(B,H) = B*H^3/12 \tag{3}$$

If the cross section of the bar is square, with B being the width and height, the equations become:

$$A = A(B) = B^2 \tag{4}$$

$$I_{zz} = I_{zz}(B) = B^4/12 \tag{5}$$

With these type of equations, GENESIS can update the properties in the finite element model any time the design variables change. Typically, many elements reference a given set of properties. If the set is updated then all elements associated to the set are update simultaneously. In GENESIS, for a bar element the properties are given in entries named PBAR. The properties of quadrilateral and triangular shell elements are given in entries named PSHELL. The reason finite element works with properties and not with dimensions is to allow for any arbitrary section to be used. GENESIS uses equations to update the properties to work with sizing variables instead of properties. Working with properties alone might be much simpler in term of creating data, but that would produce optimal properties that might not correspond to any physical section.

B. Shape Optimization

In shape optimization, scale factors of perturbation vectors are the design variables. The perturbation vectors are input directly or by providing basis vectors. Basis vectors contain alternative grid locations that represent candidate designs. When the user provides basis vectors, GENESIS internally creates perturbation vector as vectorial difference between the provided basis vector and the original grid locations. Currently, GENESIS contains three methods to automate creation of basis or perturbation vectors: the GRID basis vector method, the natural basis vector method and the DOMAIN method[8].

C. Topography Optimization

Topography optimization is a special case of shape optimization. In this type of optimization, the program automatically generates perturbation vectors that are either perpendicular to the designable region or in a given direction[9].

D. Topology Optimization

In topology optimization, the design variables correspond to the element volume fractions. The volume fraction designs simultaneously the material properties E (modulus of elasticity) and ρ (density) with the purpose of getting a 0-1 answer to identify the key elements to keep and the rest to discard.

E. Topometry Optimization

Topometry optimization is a generalization of sizing optimization. Unlike size optimization, where all elements associated to a property data entry are designed with the same values, in topometry optimization each element is designed independently. Topometry data uses sizing optimization data. With sizing optimization data the user provides the information to the program of what relationship their designable properties (like areas and moment of inertia in bars in PBAR entry) and his design variables have to follow. A new data in GENESIS, named DSPLIT, allows the user to select which sizing data will be used for topometry. Internally, the code splits the properties and creates all associated design data to create independent relationships between automatically created design variables and element properties.

Following simple rules, GENESIS will also split all data that references those properties and design variables. Examples of split data are: DLINK, DRESP1, DRESP2 and DRESP3. A DLINK split allows maintaining section properties symmetric or following a predefined linear relationship. For example, the location of the bottom of a composite element, Z0, can be set to be one-half the sum of all the layer thickness of the composite. If the composite is split, then Z0 is calculated consistently. A DRESP2 split allows for responses that are nonlinear functions of split design variable and split responses. Like other ordinary design variables in GENESIS, topometry design variables could be independent, dependent, discrete or continuous.

IV. Responses

Responses are quantities that are calculated by the program and are functions of the design variables. They can be used as the objective function or as constraints of the optimization problem. Understanding what responses are available allows understanding of the capabilities of the program in general and of topometry in particular. A summary of the responses available in GENESIS is presented next.

A. Responses for Shape, Sizing, Topography and Topometry Optimization

Finite Element Responses

Almost every finite element response calculated for analysis can be used in optimization. These responses are displacement, velocities, acceleration; stress; strain; force; strain energy; the temperature obtained by heat transfer analysis, the buckling load factors from stability analysis and the frequencies and mode shapes obtained on a free vibration analysis.

It should be mentioned here that in large-scale topometry optimization problems (large number of variables), element responses cannot be used in practice, as the size of the sensitivity matrix would be too big. In this case, like in topology optimization, global responses should be used (strain energy, frequencies, mass, moment of inertia, etc).

Geometric Responses
Responses that are functions of grid locations, such as volume, area, length, angles, distances, moment of inertias and center of gravity.

Equation Responses
The user can specify nonlinear equations mixing finite element responses with design variables, grid locations and geometric responses to create their own responses.

Subroutine Responses
User-written subroutines can be linked with GENESIS to mix finite element responses with design variables, grid locations and geometric responses to create special responses.

External Responses
An external program can be used to generate responses from other analysis programs for complete multidisciplinary optimization.

B. Responses for Topology Optimization
Available static responses are displacement, strain energy, natural frequencies and mass.

V. Solving the Optimization Problem

In the approximation concepts approach, responses are modeled using approximation functions. Rather than approximating the responses directly, intermediate responses and intermediate design variables are used. This allows the approximation to capture more of the nonlinearities of the responses, which can then be used over a greater range of design variables. In addition, a constraint screening process is used to limit the amount of work required in the sensitivity module. Because approximation functions are used, GENESIS uses move limits to limit how much the design variables and properties (in sizing optimization) move in each design cycle. GENESIS has two optimizers: DOT[10] and BIGDOT[11]. For topometry, topography and topology optimization BIGDOT is the default optimizer, otherwise DOT is the default. BIGDOT is an exterior penalty base optimizer developed by Vanderplaats[12]. This optimizer is capable to solve very large problems[13], and so is key for topometry as well as topology optimization where several hundred thousands of design variables could be involved in the problem.

VI. Comparing Topometry with Sizing Optimization

Every element that can be size optimized in GENESIS now it can be topometrically designed. The same responses and same design data apply to both sizing and topometry. The DSPLIT entry converts sizing information to topometry information. Internally, sizing and topometry variables are treated differently. For example, by default for approximating shell responses we do not use intermediate design variables with topometry (this reduces the size of sensitivity matrix by half). In topometry optimization we do not use move limits on properties, as they add too many constraints to the optimizer. Move limits in topometry are reduced compared to move limits in sizing.

A. Mixing Sizing and Topometry on Same File on Different Elements

In the same problem, it is possible to size optimize certain elements while topometry optimizing others.

B. Mixing Sizing and Topometry on Same problem on Same Elements

It is possible to simultaneously design the same elements with topometry data and with sizing data. For example, a frame structure built with bar element can be designed using sizing variables to get a uniform height and topometry variables to have a variable width in each individual element. Another example for which this type of mixing would be desirable is for designing composites; one layer could be size optimized while others could be topometrically designed.

VII. Comparing Topometry with Topology Optimization

In general, both topology and topometry optimization improve structures by changing individually each element in the designable region. In other words, they are element-by-element optimization methods. However, their aims are fundamentally different. With topology optimizations the user aims to obtain 0-1 answers. Topology optimization can help the user decide which element should be kept and which elements should be discarded from the designable domain. On the other hand, with topometry optimization the user cat get continuous variations of the dimensions of the elements in the design space. In topology optimization the 0-1 answers are achieved by designing the material properties in each element (Young's modulus and density). The topometry optimization is achieved by designing the element properties (beam areas, plate thickness) via the user sizing variables (height, width of bars, thickness of plates, etc). It is interesting to mention here that by using appropriate relationships with appropriate design variables bounds, some topology optimization problems can be also formulated with topometry optimization. This means that elements that can now be topometrically designed in GENESIS can be also "topologically" designed using topometry data. This capability is particularly useful to design elements such as the CBUSH (that references PBUSH) or the CMASS (that references PMASS) for which topology optimization is not available. Figure 1 shows the elements that can be designed with topometry and with topology. The elements are grouped by the property they reference.

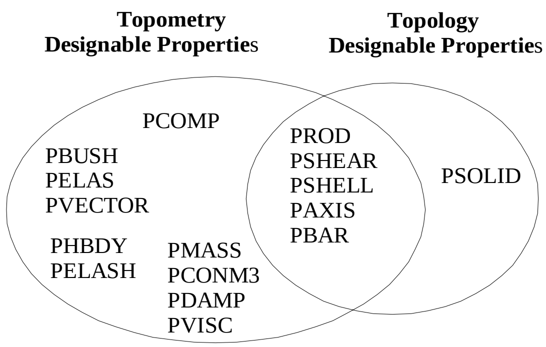

Figure 1. Topometry and Topology Designable Elements Listed by Their Property Data Entries Name

Also, the topometry capability allows to "topologically" design elements for responses that are not currently available to topology optimization such as direct frequency responses, buckling responses or heat transfer responses. Figure 2 shows the responses that are currently available in GENESIS for topometry and for topology.

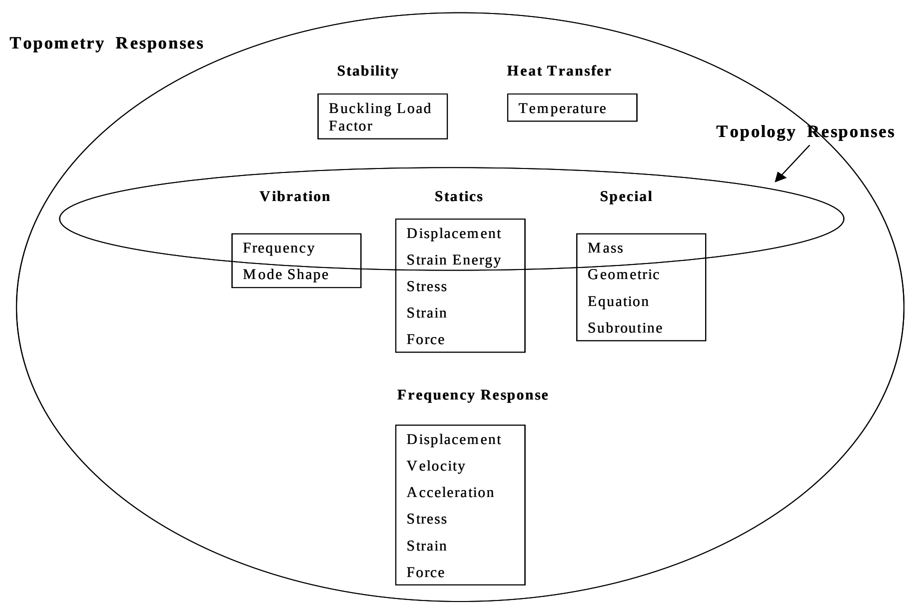

Figure 2. Responses Available for Topology and Topometry Optimization

VIII. Numerical Examples

A. Stiffness Optimization of a Plate

Description of Problem

The purpose of this example is to demonstrate the use of topometry optimization with a very simple structure. Six optimization cases are studied. The first three optimization cases are used to compare results from sizing, topology and topometry. The next two cases are used to demonstrate the use of topometry optimization combined with topography and shape. The sixth case is to study the effect of simultaneously combining topometry, topography and shape. All cases use the same initial mesh, shown in Fig. 3. The structure is a simply supported 18x40 mm plate. The material properties of the plate are E= 207,000 MPa and ν= 0.3. The initial thickness is 0.6 mm. The plate is modeled using a mesh that contains 779 grids and 720 quadrilateral elements. The number of degrees of freedom is 4650. The plate is loaded with a concentrated force of 100 N applied at the center of the plate. In all problems, the objective function is to find the stiffest structure possible using up to 600 mm^3 of material. The stiffness is measured by the reciprocal of the displacement under the load. To compare solutions, the optimal stiffness of each case is normalized by the optimal stiffness obtained by sizing optimization.

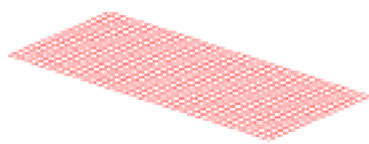

Figure 3. Initial Design Finite Element Mesh

Results

For the first case we use sizing optimization to solve the problem. This problem uses only one design variable. In this case the optimal dimension is trivial and can be calculated by hand by dividing the available volume (600 mm3) and area of the plate (18mm x 40mm). In other words, the theoretical optimal thickness for the first case is 0.83mm. Using GENESIS we obtain the solution shown in Fig. 4a. The calculated displacement is 0.4324. So the optimal stiffness is 2.313. The normalized sizing solution is 1.00. The second case is solved with topology optimization. The optimal displacement is 0.0788 and the normalized stiffness is 5.49. In other words, this solution is approximatly 5.5 times stiffer than the sizing solution. The topology optimization problem was solved using the maximum allowed thickness and mass fraction constraint of 0.46 (600/(1.8x18x40)). The third case corresponds to a pure topometry optimization case. In this case each of the elements was designed with its own design variable. Each of them was allowed to move between 0.1 and 1.8 mm. Figure 4c shows that the solution is almost identical to the topology result. In all figures (except the initial mesh) the color red represent a thickness of 1.8 mm, the color blue 0.1 mm and the rest are continuous variation of the thickness between those two values. The normalized stiffness in this case is 5.76, which is very similar to the one obtained with topology optimization. In this example, if we neglect the elements with minimum thickness we can see that topometry optimization also gave the topology answer.

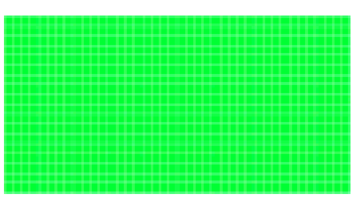

Figure 4a. Sizing Results

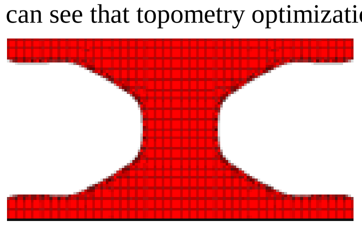

Figure 4b. Topology Results

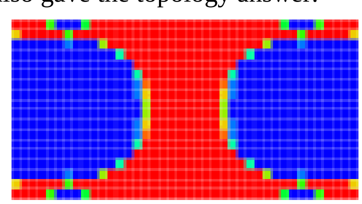

Figure 4c. Topometry Results

In the fourth case, topography optimization data was added to case 3. The aim of topography optimization in this case is to find the optimal bead pattern. In this case, we only use 6 topography design variables to obtain a simple solution. Figure 5 shows different views of the case 4 results. In this case, the normalized stiffness is 7.34. In the fifth case, shape optimization data was added to case 3. The aim of shape optimization in this case is to find the optimal global curvature of the structure along the short side. In this case, we only used 1 perturbation vector that resembles the final answer. The design variable associated to it just scaled the perturbation vector. Figure 6 shows different views of the case 5 results. In this case the stiffness increased to 12.89. In case number 6, topometry, topography and shape optimization data were used together. Figure 7 shows different views of the results. In this case, the stiffness increased to 16.31. This case gives the stiffest design proposal of the 6. This should not be surprising as it is the case with the most design freedom.

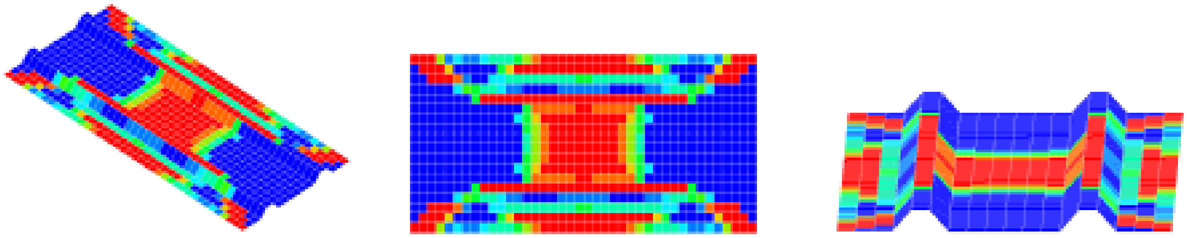

Figure 5. Different Views of Combined Topometry and Topography Optimization Results

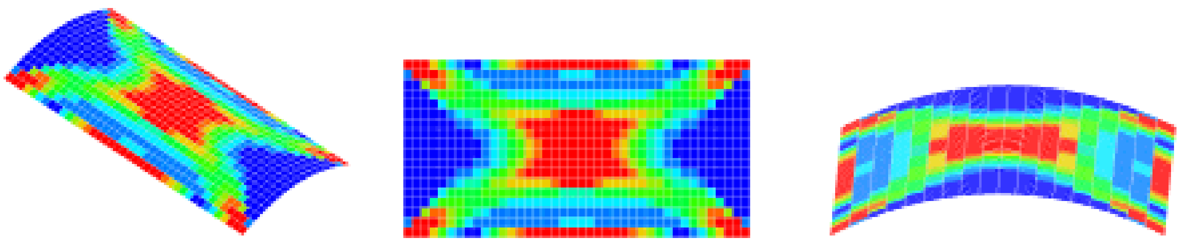

Figure 6. Different Views of Combined Topometry and Shape Optimization Results

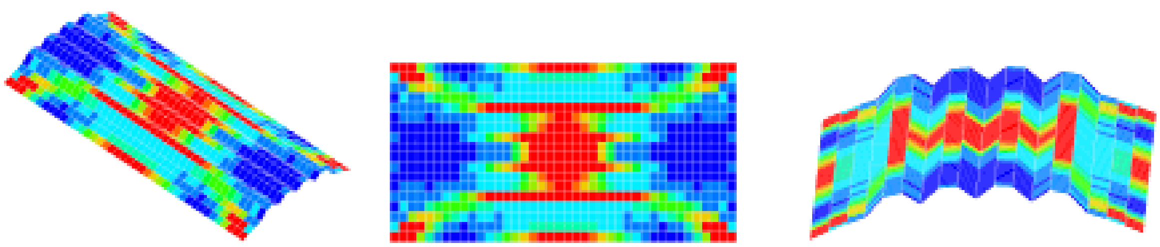

Figure 7. Different Views of Combined Topometry, Topography and Shape Results

Analysis of Results

The design histories of the stiffness optimization of all six cases are presented in Figure 8 and Table 1.

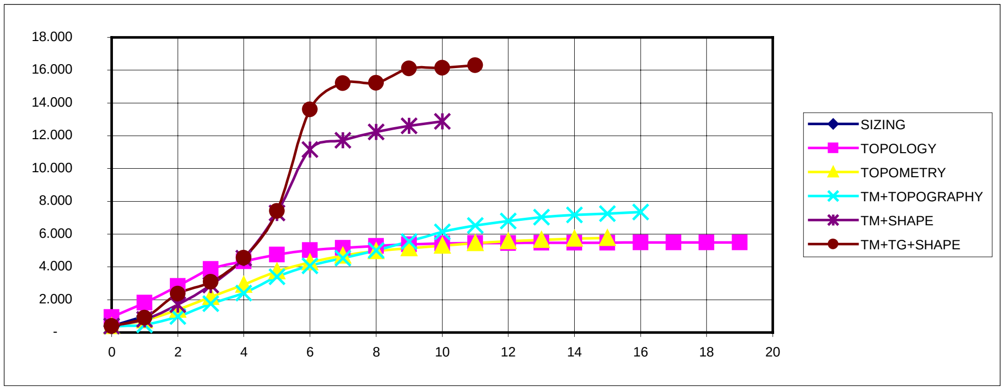

Figure 8. Optimization Histories

Table 1. Normalized Stiffness Optimization Histories

	Normalized Stiffness Optimization History					
	1	2	3	4	5	6
Design Cycles	SIZING	TOPOLOGY	TOPOMETRY	TM+TOPOGRAPHY	TM+SHAPE	TM+TG+SHAPE
0	0.382	0.931	0.382	0.382	0.383	0.382
1	1.000	1.810	0.739	0.470	0.808	0.893
2	-	2.831	1.395	0.969	1.693	2.367
3	-	3.870	2.172	1.767	2.899	3.095
4	-	4.346	2.906	2.409	4.536	4.555
5	-	4.750	3.740	3.400	7.287	7.409
6	-	5.019	4.288	4.059	11.160	13.599
7	-	5.169	4.688	4.543	11.726	15.201
8	-	5.280	4.958	5.002	12.231	15.223
9	-	5.375	5.148	5.549	12.605	16.099
10	-	5.429	5.304	6.136	12.888	16.145
11	-	5.448	5.449	6.522	-	16.309
12	-	5.455	5.562	6.810	-	-
13	-	5.463	5.647	7.030	-	-
14	-	5.472	5.720	7.169	-	-
15	-	5.481	5.763	7.252	-	-
16	-	5.488	-	7.341	-	-
17	-	5.489	-	-	-	-
18	-	5.489	-	-	-	-
19	-	5.489	-	-	-	-
Number of Variables	1	720	720	726	721	727

Figure 9 shows the normalized optimized stiffness of each of the six-optimization cases

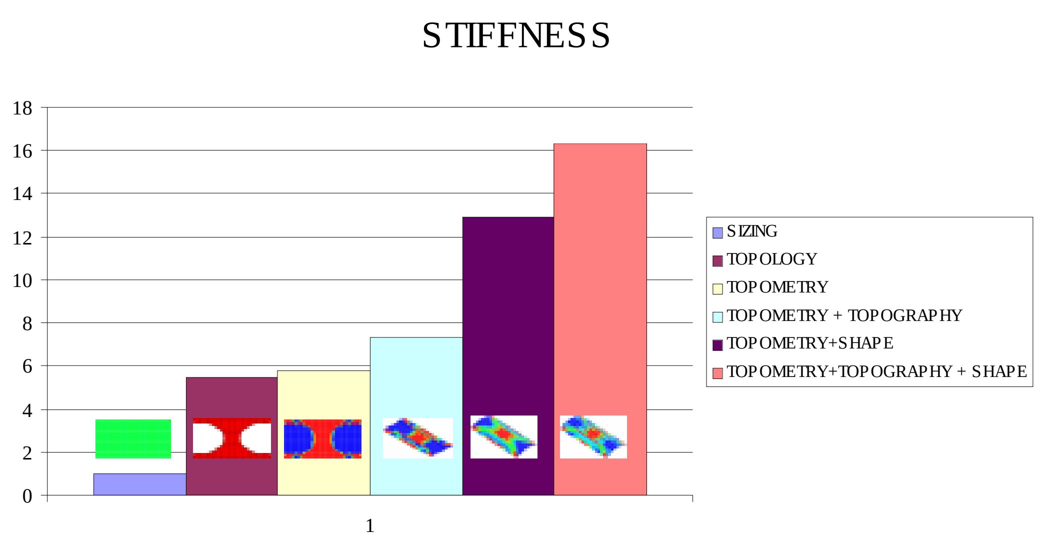

Figure 9. Normalized Optimal Stiffness

B. Stiffness optimization of a car body using topometry optimization

Description of Problem

In this example, topometry optimization was used to find the optimal thickness distribution to reinforce a car body. The car model has 34,560 shell elements referencing 63 PSHELL data entries. The objective function of the problem consists of maximizing the stiffness of the car. The stiffness is measured as the average of the first 12 natural frequencies. Every thickness of every shell element is taken as a design variable so there are 34,560 designs variables. The initial value of the thickness is 1.0 mm. The study allows the thickness to grow to up to 2.0mm. Multiple different cases were solved to get trade-off table to study the relationship between added mass and gained frequencies. Different mass constraints were used in each case. Case 1 allowed adding 5Kg. Case 2 allowed for 10 kg, and so on as shown in Table 3. A case with no mass constraint was also included to obtain the best possible answer and to obtain the limit of the design. To compare results, sizing optimization was run for same cases as topometry.

Results

The optimal topometry for the average frequencies with 15kg constraint is shown in Fig. 10. The blue elements represent the elements that retain their original size which correspond to the lower bound of 1mm. The rest represent the elements needed to be reinforced. The red element represent the element that are more important and reached their upper bound of 2 mm.

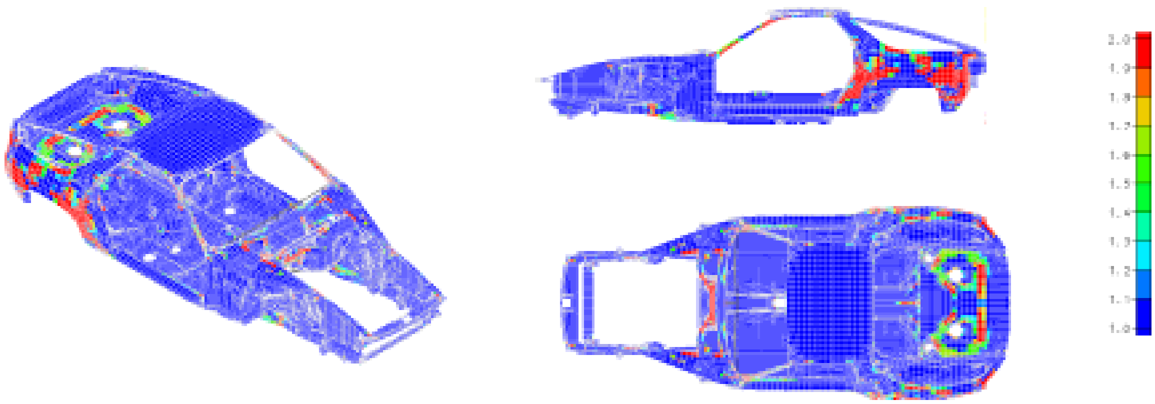

Figure 10. Thickness Distribution, Topometry Optimization with 15 kg

In the sizing optimization problem there are 63-indendendent design variables associated to the 63 PSHELL entries. This 63 design variables design all 34,560 elements in the model. Figure 11 shows the sizing optimization results. Like in the previous case, the red elements represent the elements that has the highest thickness (2.0 mm) the elements in blue did not change size. The rest of the elements have intermediate thickness values.

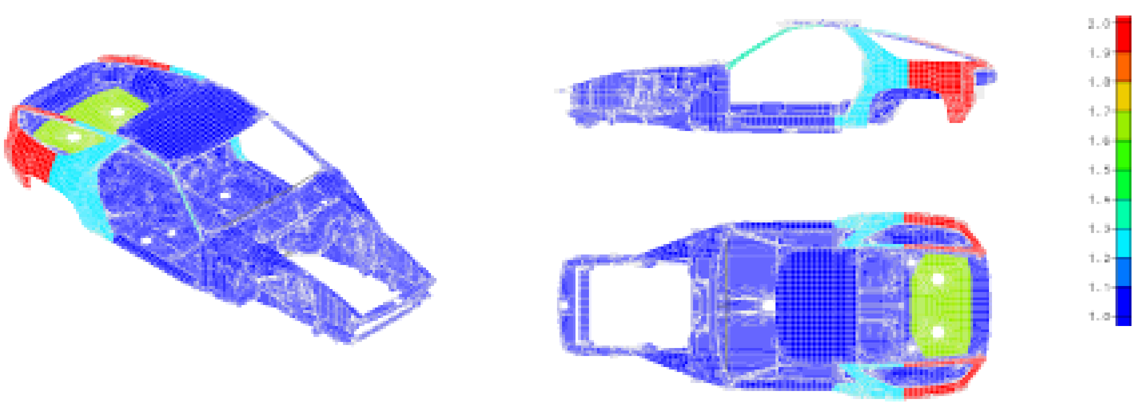

Figure 11. Thickness Distribution, Sizing Optimization with 15 kg

Table 2 and Fig. 12 show the optimization history of the case of adding up to 15 kg. At the end of this optimization run, topometry increased the average frequencies to 56.26 Hz (a 46 % increase). Sizing increased the average frequencies to 48.86 Hz (27%).

Table 2. Topometry and Sizing Optimization Design Histories using up to 15 kg

	Average of 12 Lowest Freqs (Hz)	
Design Cycle	SIZING	TOPOMETRY
0	38.61	38.61
1	42.88	41.14
2	45.43	43.61
3	46.55	45.85
4	47.04	47.97
5	47.62	49.92
6	47.70	51.69
7	48.02	53.28
8	47.91	54.70
9	48.35	55.88
10	48.46	56.01
11	48.38	55.90
12	48.67	56.13
13	48.82	56.19
14	48.85	56.17
15	48.86	56.24
16		56.24
17		56.26

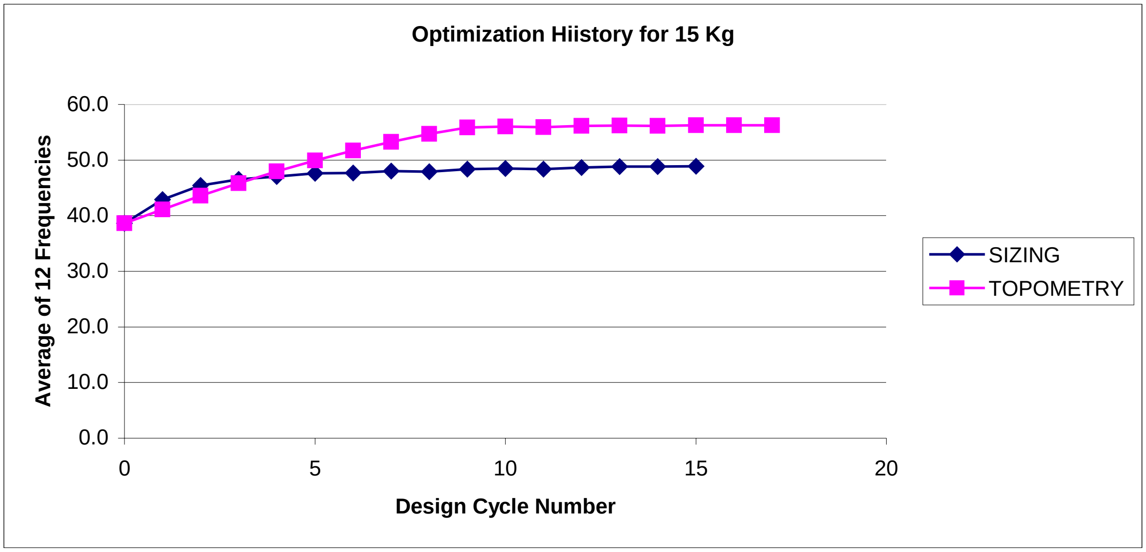

Figure 12. Topometry and Sizing Optimization Design Histories using up to 15 kg

Table 3 shows a trade-off study between added mass and the optimal average of the 12 lowest natural frequencies. The first row in Table 3 shows the analysis results of the original design. The next seven rows show seven mass constrained optimization minimization results. The last two rows of the table show the optimal solution with no mass constraint. The mass reported in the two last runs are the optimal added mass, not the allowed masses like on rows 2 to 8. Table 3 also shows that with sizing optimization the stiffness can be increase to up to 55.71 hz (44%). Topometry optimization can get similar gain with about 15 Kg. Topometry optimization with not mass constraint increased the stiffness to 60.20 Hz (56%).

Table 3. Added Mass vs Optimal Average of 12 Lowest Natural Frequencies

Added Mass [Kg]	Optimal Results (Hz) SIZING	Optimal Results (Hz) TOPOMETRY
0	38.61	38.64
5	44.17	51.85
10	47.10	54.70
15	48.86	56.26
20	50.25	57.47
25	51.21	58.28
30	52.13	58.80
35	52.73	59.20
70	-	60.20
117	55.71	-

Figure 13 shows that adding mass on optimal locations can significantly increase stiffness, but after a certain point no more gains can be archived. The plot lets understand what are the natural limits on the design. The unconstraint minimization gives the best improvements. Obviously the sizing optimization limit is lower than the topometry optimization limit. In this case, for sizing the limit is 55.71Hz and for topometry is 60.20 Hz. If we were to increase the stiffness to say 50 Hz, then either sizing or topometry optimization could be used but sizing would require more mass for same improvement. If we were to increase the stiffness to 60 Hz, sizing optimization could not be used. If we were to increase the stiffness to 70 Hz, then we would need to change methodology (e.g. shape optimization) or add new structural members (e.g., add stiffeners). Although is not shown in this graph, it should be mentioned here that adding mass in non-optimal location could easily reduce the stiffness.

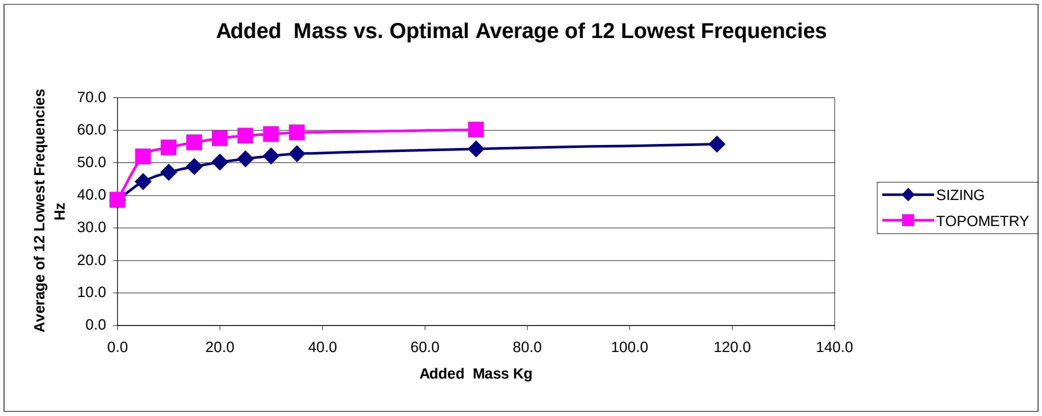

Figure 13. Added Mass vs. Optimal Average of 12 Lowest Natural Frequencies

C. Spot Weld Optimization of Car Body Using Topometry Optimization

The purpose of this example is to show one of the problems that motivated adding topometry to GENESIS. This problem is presented in Ref. 14 so here we just give the key features of the problem. The requirements of the problem were to find a trade-off table for the optimal locations of spot welds. The objective function of each case was to maximize the sum of the first bending and first torsional frequencies. The constraint of the problem was the number of welds to keep. The problem was solved using sizing optimization with 4316 design variables. The variables designed 4316 CVECTOR elements that modeled spot-welds. This problem was optimized six times to study the effect of taking out different numbers of welds. Table 4 shows the results for all optimization cases and the case where all weld were used (100%). This table gives the designer trade-off information that helps to study the influence of the number of welds on the rigidity of the car body.

Table 4. Relation Between Rigidity and Number of Welds

Quantity of kept welds (%)	First torsional frequency (Hz)	First bending frequency (Hz)	Sum of two frequencies (Hz)
30	24.983	35.100	60.083
40	26.662	37.330	63.992
50	29.831	40.755	70.586
60	30.499	42.100	72.599
70	31.312	44.947	76.259
80	31.762	45.718	77.480
100	31.962	46.185	78.147

Figure 14. Optimal Weld Location, 70% Kept Welds and 30% Discarded

The problem was solved without complications in Ref. 14 but the required data was tedious to create. Table 5 shows a comparison of numbers of data entries needed to perform the same problem with regular sizing optimization and with topometry optimization. From Table 5 we can see that topometry optimization can reduce substantially the amount of data needed to solve the problem.

Table 5. Data Necessary For Solving Same Problem with Sizing and Topometry Optimization

	SIZING	TOPOMETRY
DVAR	4316	1
DVPROP2	25896	6
DSPLIT	-	1
DRESP3-Arguments	8632	2
PVECTOR	4316	1

C. Eighteen Rod Truss

Description of Problem

The purpose of this problem is to find the optimal areas of the rod elements and the optimal location of the grids on the lower elements of the truss structure shown in Fig. 15. The analysis model has 18-rod elements (CROD) referencing 4 sets of properties (PROD) and 11 grids. The modulus of elasticity is E= $1.0*10^7$ psi. The problem has one load case that consist in 5 point loads of 20,000 lb each applied on the top part of the truss.

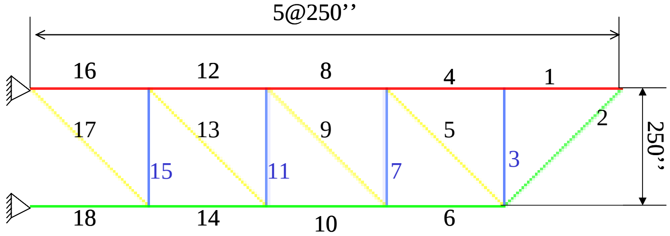

Figure 15. Initial Configuration

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The optimization problem consists of minimizing the mass while satisfying 18 stress and 18 Euler buckling constraints. The problem was solved first using 4 sizing and 8 shape design variables using GENESIS. Afterwards, the problem was solved using topometry and shape optimization. In the second case 26 design variable were used: 18 design variable to design individually each rod and the rest 8 corresponds to the same shape design variables used in the first case.

To avoid Euler buckling of the rod elements, the force in each element must be constrained to be less that the Euler buckling load. This constraint can be written as:

$$F >= Fe = - P*EI/L2 \quad (6)$$

For a circular cross section the equation reduces to

$$B=B(VOL,A,F) = VOL**2/A**4*F >=-78.854E6 \quad (7)$$

Where VOL is the volume of the element, A is the area of the element and F is the axial force on the element.

Results

Using Eq. 7 the problem was solved in GENESIS. Table 6 shows the optimized areas for the two cases studied

Table 6. Optimal Areas

Element ID	Initial	Shape & Sizing	Shape & Topometry		Initial	Shape & Sizing	Shape & Topometry
1	18.00	11.76	10		18.00	39.98	39.98
2	18.00	39.98	11		18.00	11.57	17.78
3	18.00	11.57	12		18.00	11.76	10.62
4	18.00	11.76	13		18.00	5.00	0.89
5	18.00	5.00	14		18.00	39.98	39.99
6	18.00	39.98	15		18.00	11.57	15.64
7	18.00	11.57	16		18.00	11.76	11.58
8	18.00	11.76	17		18.00	5.00	5.31
9	18.00	5.00	18		18.00	39.98	40.00

Figure 16 shows the location of the grids on the final design for the shape and sizing optimization case.

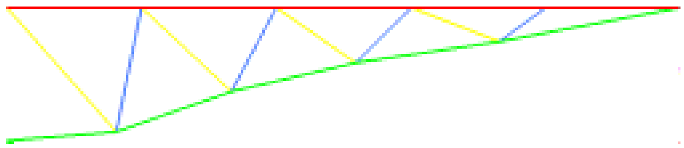

Figure 16. Final Grid Locations for Shape and Sizing Optimization case

Figure 17 shows the location of the grids on the final design for the shape and topometry optimization case.

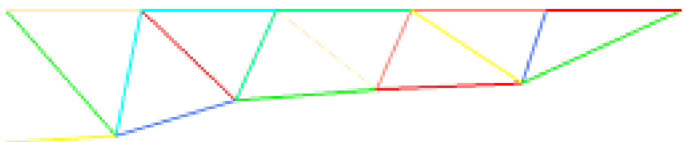

Figure 17. Final Grid Locations for Shape and Topometry Optimization case

Table 7 shows the design history of the two cases studied. In the sizing optimization problem, the mass decreased form 9,032 to 7,805 (13.2% reduction) and the maximum constraint violation in the structure was reduced from 636.8% to 0.0%. In the Second case, using topometry optimization, GENESIS also satisfied the initial maximum constraint violation while the mass decreased from 9,032 to 6,671 (26.1% reduction)

Table 7. Design Histories

	Mass		Maximum Constraint Violation	
Design Cycle	Shape & Sizing	Shape & Topometry	Shape & Sizing	Shape & Topometry
0	9,032	9,032	636.80%	636.80%
1	8,478	7,059	107.70%	268.40%
2	7,730	7,011	808.30%	153.90%
3	7,996	5,732	21.60%	226.50%
4	7,819	6,564	3.20%	98.70%
5	7,792	6,855	2.80%	99.00%
6	7,805	6,663	0.00%	18.40%
7		6,841		1.30%
8		6,643		6.90%
9		6,692		1.40%
10		6,669		0.40%
11		6,663		0.50%
12		6,671		0.00%

IX. Conclusions

A new capability to perform element-by-element sizing optimization has been presented. The use of this capability allows designers to find innovative designs with less effort. It also allows the designer to explore a larger design space than with traditional sizing optimization. The new capability is very flexible as it can be combined with other existing optimization capabilities in the commercial program GENESIS. Implementation aspects were described and examples that illustrate this capability were presented.

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