Computational Dynamics in Design Optimization

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The purpose here is to review the state of the art in design optimization relative to dynamics and to identify needs for the future. The paper will begin an overview of optimization. This will include a focus on sensitivity calculations and approximation concepts for efficient optimization. Following this, several examples will be offered to demonstrate industrial applications that are currently possible. Finally, future needs will be discussed.

Nomenclature

A	= cross-sectional area
F	= force in member
F(X)	= objective function
$g_j(X)$	= j-th inequality constraint
K	= stiffness matrix
L	= length of member
m	= number of inequality constraints
M	= mass matrix
n	= number of design variables
P	= structural load vector
S	= search direction
u	= vector of structural displacements
U_k	= numerator of Rayleigh quotient
T_k	= denominator of Rayleigh quotient
$\tilde{U}_k$	= approximate numerator of Rayleigh quotient
$\tilde{T}_k$	= approximate denominator of Rayleigh quotient
X	= vector of design variables
x	= single design varible
δX	= change in design variables
Greek Symbols	
α	= move parameter
σ	= stress
$\bar{\sigma}$	= allowable stress
σ_{ijk}	= stress in element i, component j, load case k
∂	= partial derivative
λ_k	= k-th eigenvalue
Φ_k	= k-th eigenvector
Subscripts	
i	= design variable number
j	= inequality constraint number
x	= derivative with respect to x

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Superscripts

L	= lower bound on design variable
New	= new design
Old	= old design
U	= upper bound on design variable
-1	= inverse
0	= initial value

I. Introduction

Design optimization is by nature very computationally intensive, typically requiring from ten to thirty detailed finite element analyses. Given that models of over a million degrees of freedom are now commonplace, the need for computational efficiency is clear. Furthermore, optimization imposes additional needs beyond basic analysis. These include the calculation of gradients of dynamic responses with respect to the design variables as well as creation of formal approximations for use in the optimization phase.

Although there were various analytic approaches to structural design, numerical optimization began in earnest with the landmark paper of Schmit[1] in 1960. The original work was applied to stress constrained structures. Fox[2] extended this work with the calculation of stress derivatives. Fox and Kapoor[3] expanded this further to include eigenvalue constraints and their derivatives. Later, Nelson offered an improved gradient computation algorithm that is in common use today. Similar methods are available for calculation of direct and modal frequency response. With the ability to calculate eigen responses and their derivatives, the optimization process was greatly improved as compared to finite difference calculations.

During the 1970s and 1980s, approximation techniques were developed for structural optimization. These approximations were primarily developed for stress constraints but were also expanded to frequency constraints. These methods dramatically reduced the number of detailed finite element analyses needed to achieve an optimum design. Also, in recent years eigenvalue analysis methods have been developed to greatly reduce the computational cost of the analysis itself leading to further reductions in optimization time.

In recent years, this technology has been added to several commercial finite element analysis programs and new programs have been created to provide for optimization of real structures.

The purpose here is to review the state of the art in dynamics as related to design optimization and to identify needs for the future. The paper begins with the optimization problem statement. This includes a focus on sensitivity calculations and approximation concepts for efficient optimization. Following this, several examples are offered to demonstrate industrial applications that are currently possible. These include optimization of car bodies to maximize frequencies, ground vibration test correlation and others.

II. The Optimization Problem Statement

Numerical optimization solves the general problem[4] Find the values of the design variables contained in **X** that will;

Minimize
$$F(X) \tag{1}$$

Subject to;
$$g_j(X) \le 0 \qquad j = 1, m \tag{2}$$

$$X_i^L \le X_i \le X_i^U \qquad i = 1, n \tag{3}$$

The function, ***F(X)*** is referred to as the objective or merit function and is dependent on the values of the design variables, **X**, which themselves include member dimensions or shape variables of a structure as examples. The limits on the design variables, given in Equation 3, are referred to as side constraints and are used simply to limit the region of search for the optimum.

The $g_j(\mathbf{X})$ are referred to as constraints, and they provide bounds on various response quantities.

Additionally, we could include equality constraints and these can be included in the original problem definition as two equal and opposite inequality constraints.

A common constraint is the limits imposed on stresses at various points within a structure. Then if $\bar{\sigma}$ is the upper bound allowed on stress, the constraint function would be written, in normalized form, as

$$\frac{\sigma_{ijk}}{\bar{\sigma}} - 1 \leq 0 \tag{4}$$

where i = element, j = stress component and k= load condition.

Objective and constraint functions are interchangeable. For example, we may wish to maximize the fundamental frequency of a structure with limits on mass or minimize mass with limits on frequencies.

Optimization methods closely model what we do in design already. Normally, we begin with a candidate design and ask "How can we change the design to improve it?" Thus, we modify our design as;

$$X^{New} = X^{Old} + \delta X \tag{5}$$

Optimization does much the same thing, but in two steps. First, we ask what direction to move in and then we ask how far to move. That is,

$$X^{New} = X^{Old} + \alpha S \tag{6}$$

where *S* is the search direction and α is the number of steps we move in this direction (partial steps are allowed).

The difference in optimization algorithms is mainly in how we calculate the search direction, *S*, and how we do the "one-dimensional search" to determine α. The key point here is that all variables are considered simultaneously according to their effect on the objective function and all constraints. Also, since this is all automated and today's computers are very fast, we can find an optimum design with much less time and effort than just finding an acceptable design using traditional methods.

This problem statement provides a remarkably general design approach and a multitude of methods are available today for solving this general problem. Much of the theoretical development has been in the operations research community and applications there are widespread today. In engineering, while development has been underway for over forty years, applications have lagged far behind. The time has come for that to change.

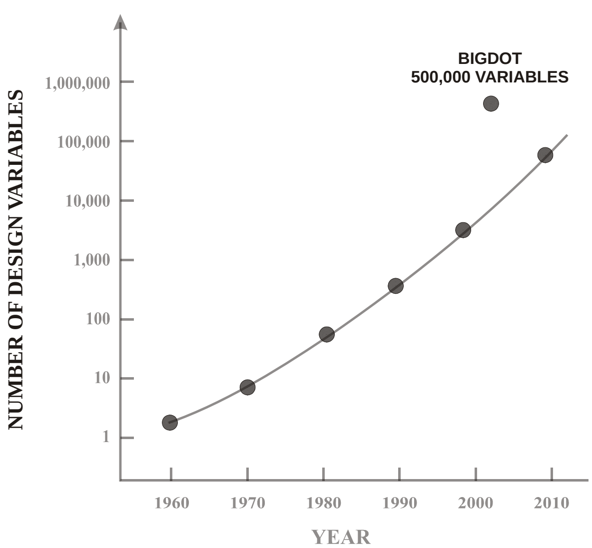

Figure 1. Growth in Optimization Problem Size

III. Optimization Algorithms

There are a multitude of algorithms available for solution of the general optimization task defined above. In structural optimization, gradient based methods are considered to be the most efficient and reliable.

Since its inception in 1960, the size of structural optimization tasks has grown exponentially as shown in Figure 1. Today, for member sizing and shape optimization tasks, several thousand to over a hundred thousand variables are routinely considered. In topology optimization, the number of design variables often exceeds one million. Very large problems with only a few active constraints can be routinely solved as well as small (a few hundred variables) problems with many active constraints. Recently, there has been focus on large numbers of design variables where there are also a large numbers of active constraints[5] so that optimization problem size is no longer a limitation in design optimization.

IV. Gradient Computations

As noted above, the most efficient and reliable optimization algorithms require calculating the gradients (sometimes called sensitivities) of the responses with respect to the design variables. Because most design tasks require static stress limits in addition to dynamic constraints, we will begin with calculation of displacements, from which stress or strain gradients can easily be recovered.

Gradients of displacements are calculated from the basic finite element analyses equations,

$$Ku = P \tag{7}$$

where K is the master stiffness matrix, P is the vector of applied loads and u is the vector of displacements.

Differentiating with respect to design variable X_i and rearranging gives[2]

$$\frac{\partial u}{\partial X_i} = K^{-1}\left[\frac{\partial P}{\partial X_i} - \frac{\partial K}{\partial X_i}u\right] \tag{8}$$

The derivative of the stiffness matrix with respect to the design variable is often calculated by finite difference while the remainder of this computation is carried out analytically. Because the stiffness matrix has already been decomposed, this is a simple and efficient calculation. From this the derivatives of stresses are calculated from the stress recovery equations. Equation 8 is referred to as the direct approach. An adjoint method is also available and the choice of method is normally made automatically, depending on the number of design variables and the number of needed derivatives[6].

Derivatives of eigenvalues and eigenvectors, are calculated in a similar fashion, beginning with the basic eigenvalue equations

$$K\Phi_k - \lambda_k M\Phi_k = 0 \tag{9}$$

The gradient of the eigenvalue with respect to a design variables is now[3]

$$\frac{\partial \lambda_p}{\partial X_i} = \Phi_p^T\left[\frac{\partial K}{\partial X_i} - \lambda_p \frac{\partial M}{\partial X_i}\right]\Phi_p \tag{10}$$

Equation 10 was published by Fox and Kapoor in 1965 and has been superceded by Nelson's method[7]. Nelson's method is mathematically more complicated but also more efficient and is now the method of choice. Nelson also provides eigenvector derivatives, making complex tasks like mode tracking possible. Similar methods are available for direct and modal frequency response as well as flutter speed.

V. Approximation Techniques

By the end of the 1960s it was becoming apparent that numerical optimization was limited to perhaps fifty variables and was computationally too expensive to the a usable design tool. This was particularly emphasized in a paper by Gallatly, Berke and Gibson[8] when they called the 1960s "the period of triumph and tragedy" for structural optimization. Thus, the 1970s began the era of optimality criteria methods. Optimality criteria offered the ability to deal with large numbers of design variables but with a limited number of constraints and without the generality of numerical optimization methods. Numerical optimization methods were given new life in 1974 when Schmit and Farshi[9] published their work on approximation concepts. These methods were based on the concept of creating approximations using the underlying physics to allow for large moves and this reduced the number of detailed finite element analyses from well over 100 to the order of ten. For statically determinate trusses or membrane structures, these approximations were shown to be exact for stress and displacement constraints. Parallel to the development of approximation concepts, the adjoint method for gradient computations was developed.[6,10] Finally, in the late 1970s Fleury and Sanders[11] reconciled numerical optimization and optimality criteria methods by showing that optimality criteria are closely related to duality theory in numerical optimization.

Second generation approximations were created using force approximations[12, 13] instead of the earlier stress approximations. Similarly, Rayleigh quotient approximations were created for eigenvalue constraints.[14] These new approximations expanded the element types to shell and frame elements among others. Importantly, for such elements as frames it was now possible to treat the physical dimensions as design variables and section properties as intermediate variables so that the designer could now deal with the actual variables of interest.

To understand the basic concept of formal approximations, consider the simple rod shown in Figure 2. The objective is to minimize the volume subject to a stress limit. That is, letting the design variable be the cross-sectional area, A,

Minimize
$$AL \tag{11}$$

Subject to;
$$\sigma = \frac{F}{A} \le \bar{\sigma} \tag{12}$$

Note that the objective is linear but the constraint is nonlinear. We could linearize both and repeatedly solve the problem using this approximation. Such an approach is just sequential linear programming and is generally not very reliable or efficient.

Now consider a change in variables so x = 1/A. The problem is now

Minimize
$$\frac{L}{x} \tag{13}$$

Subject to;
$$\sigma = Fx \le \bar{\sigma} \tag{14}$$

We've now converted the problem to one with a linear objective and a nonlinear constraint to one with a nonlinear objective with a linear constraint. Such a problem is better conditioned for optimization. Furthermore, we can create a linear approximation to the constraint and keep the original objective, since it is easily calculated, along with its derivatives.

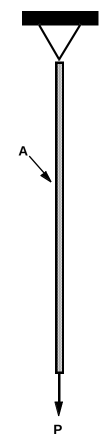

Figure 2. Simple Rod

That is,

$$\sigma \approx \sigma^0 + \nabla \sigma_X \bullet \delta X \tag{15}$$

This approach was offered by Schmit and Farshi[9] in the 1970s and this allowed us to solve structural optimization problems of rods and membranes with an order of magnitude improvement in efficiency.

In the 1980s, Bofang,[12] and Vanderplaats and Selajeghgh[13] proposed approximating the force on the elements instead of approximating the stress.

Thus,

$$\sigma \approx \frac{F^0 + \nabla F_A \bullet \delta A}{A} \tag{16}$$

This is actually a higher order approximation and is also applicable to elements other than rods and membranes.

For eigenvalues, Canfield[14] proposed approximating the numerator and denominator of the Rayleigh quotient,

$$\lambda_k = \frac{U_k}{T_k} = \frac{\Phi_k^T K \Phi_k}{\Phi_k^T M \Phi_k} \tag{17}$$

Letting,

$$\widetilde{U}_k = U_k^0 + \sum_{i=1}^{N} \frac{\partial U_k}{\partial X_i}\left(X_i - X_i^0\right) \text{ and } \widetilde{T}_k = T_k^0 + \sum_{i=1}^{N} \frac{\partial T_k}{\partial X_i}\left(X_i - X_i^0\right) \tag{18}$$

The eigenvalue is now approximated as

$$\widetilde{\lambda} = \frac{\widetilde{U}}{\widetilde{T}} \tag{19}$$

Similar approximations are available for complex eigenvalues and frequency response.

Figure 3 shows the organization of a modern structural optimization program. The general approach is to first perform an analysis and evaluate all constraints. These are then screened to eliminate, temporarily, those that are not critical or near critical. Then, the sensitivity analysis is performed. The approximate problem is then generated and solved. The key points are that the approximations are based on physics and are of very high quality and that the optimizer never actually calls the finite element analysis. The result is that optimization normally requires only 10 or so detailed finite element analyses to achieve an optimum, even when there are very large numbers of design variables and constraints.

In recent years, topology optimization has become popular. Here, given a design volume filled with material, the objective is to find the stiffest structure using a specified fraction of the material. This is a powerful tool for defining an initial structure for later refinement using shape and sizing optimization.

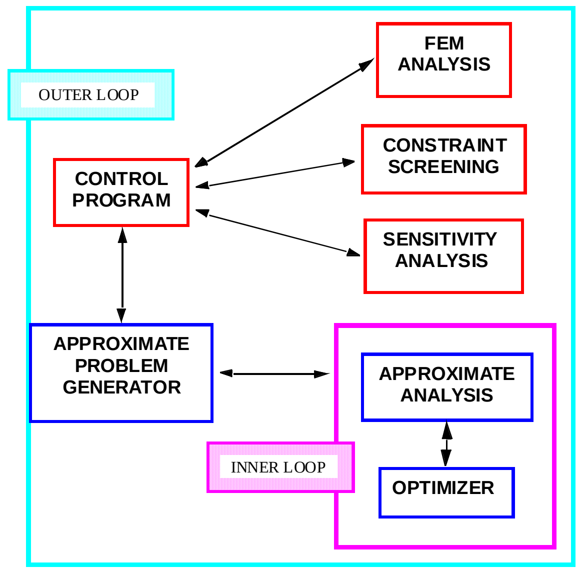

Figure 3. Modern Structural Optimization

VI. Examples

Examples are offered here to demonstrate the power of optimization applied to dynamics related design tasks. These examples are representative of design tasks that are routinely solved, except the real problems often have a much larger number of design variables. Also, finite element models of one million degrees of freedom are becoming commonplace. The examples given here were solved using the GENESIS[15] structural optimization software. This also utilizes the SMS eigensolver[16-18] which is several times faster than the common Lanczos method for large eigenproblems.

A. Car Body Reinforcement

As noted above, structural optimization is more advanced than general purpose optimization because we can calculate gradients of the needed responses and because we have very high quality approximation techniques to provide efficiency and reliability.

Figure 4 shows a car body model which we wish to reinforce to increase the bending and/or torsion frequency. This is a common task in NVH (Noise, Vibration and Harshness) design of automobiles. While this is a test example, many proprietary problems of this nature have now been solved. The largest such problem this author is aware of included 256,000 design variables and was solved on a personal computer.

The approach used here was to allow every element in the model to be optimized for thickness (with a lower bound of the original design) with the constraint that only a specified fraction of the material may be used. Here, 34,560 sizing variables were used. While somewhat difficult to see in Figure 45 (unless viewed in color), reinforcement was added in the areas of the firewall, rocker panels and rear fender areas.

Table 1 gives the increase in bending or torsion frequency for different values of added mass.

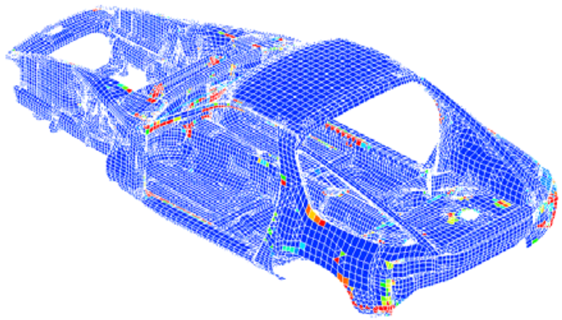

Table 1. Frequency Increases

Added Mass (Kg)	Increase in Frequency (Hz)	
	Maximize First Torsion Frequency	Maximize First Bending Frequency
2.64	4.81	6.42
7.32	7.56	9.89
15.06	9.66	11.22

Figure 4. Car Body Reinforcement

Each optimization required about ten detailed finite element analyses. Without optimization, such tasks often require several months and many more analysis runs without achieving comparable results.

B. Heat Shield Optimization

Figure 5 shows a small (about 30 cm. on a side) heat shield used on an aircraft. The part was modeled with just under 1000 shell elements. The optimization task was created automatically using over 500 shape design variables. It is desired to add a bead pattern in order to increase the first bending frequency without increasing the mass. The initial design has a frequency of 17.4 Hz. The frequency was increased to 59.1 Hz.

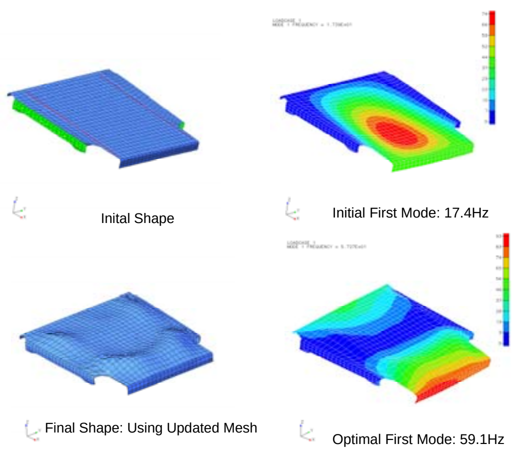

Figure 5. Heat Shield Optimization

C. Ground Vibration Test

A common issue in aeroelastic analysis is to adjust the finite element model to correlate with a vibration test. Figure 6 shows a test article for a business jet and Figure 7 shows the analysis model with the shaker and accelerometer locations.[19]

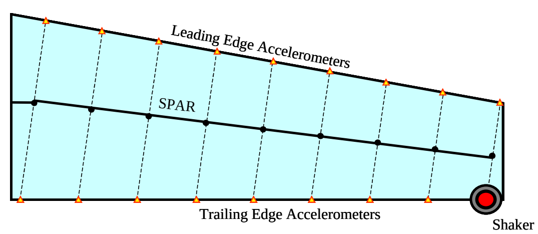

Figure 7. Analysis Model

Figure 6. Test Article

The objective here is to match the first seven eigenvalues and eigenvectors by adjusting the section properties of the beam model. Traditionally, this is done by making many runs, changing the section properties based on experience, to match the results. Here, GENESIS was used to adjust the section properties to match the responses. Table 2 shows the results of the initial model, vibration test, traditional approach and optimization approach. The MAC is the Modal Assurance Criteria and is a measure of how good the mode shapes are captured. Note that, for the initial model, modes 6 and 7 are reversed from the test. Mode tracking was used in optimization so that at the optimum, the calculated modes are in order. The traditional approach was able to achieve this as well. The difference is that the traditional method required about one week while the optimization approach required about one day, achieving comparable results to those achieved by an experienced analyst.

Table 2. Results

Mode	Test Eigenvalue	Initial FEM Eigenvalue	MACii Eigenvector	Traditional Eigenvalue	MACii Eigenvector	Optimization Eigenvalue	MACii Eigenvector
1	21.57	21.03	0.97	21.57	0.96	21.58	0.96
2	31.01	29.23	0.98	31.00	0.98	31.01	0.98
3	55.78	52.63	0.97	55.81	0.97	55.78	0.97
4	60.28	58.72	0.89	60.26	0.88	60.28	0.88
5	63.17	64.10	0.85	63.17	0.85	63.16	0.85
6	86.23	84.50	0.90	86.24	0.93	86.23	0.93
7	90.48	81.82	0.88	90.42	0.87	90.48	0.89

VII. Future Needs

While we can routinely solve eigenvalue/eigenvector related optimization tasks as well as dynamic response problems, other applications are gaining in importance. Acoustics optimization is becoming an important issue for both internal and external vehicle noise. Daily solution of dynamics problems in the nonlinear regime is still mainly a dream. Effective use of distributed and parallel computing requires major effort to take full advantage of coming hardware. Aeroelastic optimization is still uncommon and efficient optimization techniques need development. Significant advances on computational dynamics are still needed. This includes advances in basic analysis capabilities and associated efficient approximation techniques to limit the number of needed full finite element analyses in the optimization process.

VIII. Summary

An overview of the development of optimization emphasizing dynamics problems has been offered. Key technologies that have led to the present state of the art are the ability to calculate gradient information and the use of very high quality approximations. Today, we can routinely solve structural optimization tasks with static and dynamic constraints using thousands of design variables and finite element models with millions of degrees of freedom. Commercial software is available for routine application of this technology. This software is highly refined and can be used with very limited knowledge of optimization theory.

It is noted that research continues in development of even more efficient analysis and optimization methods and application to an even broader range of problems. With increased emphasis on noise considerations, this is expected to be a key area of development.

It is concluded that the state of the art is well refined and is readily available in the commercial environment to improve design quality, reduce design time and increase corporate profits. Indeed, it is argued that no computational technology today is as effective as an advanced design tool as is numerical optimization.

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