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Tuning SMS Eigensolver Performance

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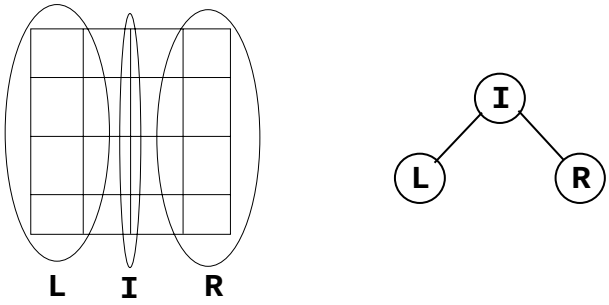
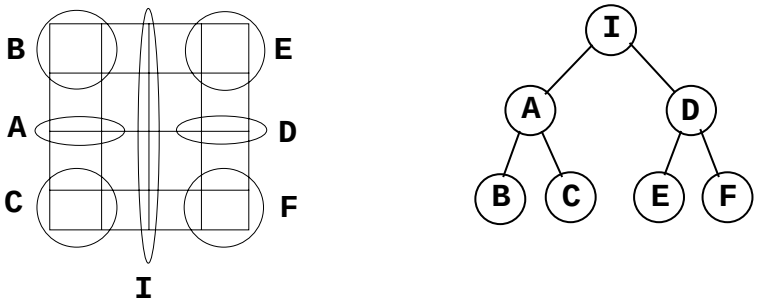
The SMS eigensolver is a high performance method for calculating approximate eigenvalues and eigenvectors for structural vibration problems. As with many approximation methods, there is a tradeoff of performance versus accuracy. The SMS method provides three parameters that a user can adjust to tune the performance/accuracy for a particular problem. To understand how these parameters affect the results, it is important to understand how the SMS method works.

The SMS method works by constructing a **supernode elimination tree** representation of the structural model. The supernode elimination tree is best explained with a simple example. The simple mesh in the figure below is partitioned into sets **L**, **R**, and **I**. The partitions are defined such that set **I** contains the interface between **L** and **R**, while degrees of freedom in **L** and **R** are not connected with each other except through **I**. The supernode elimination tree for this partitioning is represented with the graph on the right:



Each circle in the graph is a supernode that represents several nodes of the FE model. The **L** and **R** partitions can be further divided, and a new elimination tree generated as shown in the following figure:



The SMS method contains powerful heuristics to automatically partition a structure into thousands of supernodes in an elimination tree that has dozens of levels.

The degrees of freedom represented by the supernode are called the supernode's **internal** degrees of freedom. Within a supernode's representation in the stiffness/mass matrix, there are terms that couple the internal degrees of freedom with those of possibly all of the supernodes in the chain that connects that supernode to the top of the elimination tree. Any degrees of freedom of other supernodes with coupling terms in the matrix are called the supernode's **external** degrees of freedom.

The main idea behind SMS is to construct a reduced global eigenvalue problem by approximating the internal degrees of freedom of every supernode in terms of external degrees of freedom and some of the supernode's **fixed interface modes**. Fixed interface modes are calculated using the mass and stiffness of the supernode with the external degrees of freedom constrained to zero. In matrix terms, the internal degrees of freedom can be expressed as:

$$u_i = \begin{bmatrix} \Phi_i & G_i \end{bmatrix} \begin{Bmatrix} \xi_i \\ u_e \end{Bmatrix}$$

where G_i is the static condensation matrix, Φ_i is the supernode eigenvector matrix, and ξ_i are supernode generalized modal degrees of freedom.

The first tunable parameter for SMS relates to how many ξ_i degrees of freedom to keep for each supernode. If no ξ_i degrees of freedom are kept, then the method reduces to a pure static condensation, which will ordinarily give very poor approximations of eigenvectors. If all of the ξ_i degrees of freedom are kept, then the method is a simple change of basis and no approximation is involved. Research in component mode synthesis has shown that very high component modes are not needed to accurately calculate low global modes. Using this principle, the high supernode fixed interface modes are discarded. To use SMS, a desired frequency calculation range must be provided. The upper bound on the frequency range, V2, is multiplied by SMS_PARAM1 to obtain a cutoff frequency for the ξ_i degrees of freedom. The default value of SMS_PARAM1 results in a high fidelity reduced model.

While the reduced eigenproblem is much smaller that the full system model (tens of thousands of degrees of freedom compared to millions of degrees of freedom), it is still too large to efficiently solve directly. Eigenvalues and eigenvectors of the reduced problem are therefore approximated using a two phase approach. The first phase condenses the reduced eigenproblem to an even smaller problem. SMS_PARAM2 is multiplied by V2 to obtain a secondary cutoff frequency. Only ξ_i degrees of freedom corresponding to fixed interface modes less than this secondary

cutoff are retained in the condensed problem. The condensed problem is solved directly, and these eigenvectors are used to construct a starting subspace for the second phase. SMS_PARAM3 is multiplied by V2 to obtain a third cutoff which controls the number of vectors retained in the starting subspace. One step of subspace iteration is performed on the reduced eigenproblem to obtain the final approximate global eigenvalues.

The three SMS tuning parameters must obey the following relationship:

$$\text{SMS\_PARAM1} \geq \text{SMS\_PARAM2} \geq \text{SMS\_PARAM3} \geq 1.0$$

Typically, SMS_PARAM1 has the biggest influence on both the performance and the accuracy. A test model of 1.7 million degrees of freedom was created to demonstrate the effect of this parameter. A frequency range that contained about 770 modes was chosen. The chart below shows both the performance and accuracy of the results for various values of SMS_PARAM1. The chart shows that a performance gain of about 10% can be obtained at the expense of about 3% error in the highest calculated frequency.