

Test-Analysis Correlation with Design Optimization

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ABSTRACT

The correlation of a finite element model to vibration test data is a complex problem. This paper offers an approach to enhance this process by means of optimizing the analytic model to a set of mode shapes and frequencies. Design optimization provided by the GENESIS¹ program is used to assist FEM correlation with the objective to automatically match both frequency and shape for all target modes simultaneously. Multiple design variables are used to vary structural stiffness quantities. Mode tracking algorithms are employed to track mode shapes regardless of frequency.

INTRODUCTION

The correlation of a FEM with test data is often a laborious task generally performed manually. The task is usually composed of sequential steps including the comparison of analytic to test frequencies and shapes, an assessment of what parameter(s) are needed to be changed and by how much, making these changes by manually editing the bulk deck, and rerunning the FEM analysis. This iterative process continues as the analyst works to tune the FEM, essentially one mode at a time. This paper offers an approach to reduce some of the difficulties associated with tuning a model by utilizing design optimization. To evaluate this application a finite element model of a horizontal stabilizer of a business jet was used. A vibration test was performed to provide a set of frequencies and mode shapes that would be used to correlate the finite element model. The correlation problem was posed as a structural design optimization problem. The structural optimization program GENESIS was used in this study. The test frequencies were assigned as design constraints and element properties were assigned as design variables. The design objective was to minimize the differences between the test and analytical

mode shapes by varying the beam properties subject to the frequency constraints. An essential capability was the desire to correlate the many modes from the test simultaneously. After all, it would be counter productive to fine tune mode 7 at the expense of obliterating the correlation of modes 1-6. GENESIS provides mode tracking algorithms that enabled the tracking of mode shapes regardless of frequency, which facilitates simultaneous correlation of all target modes.

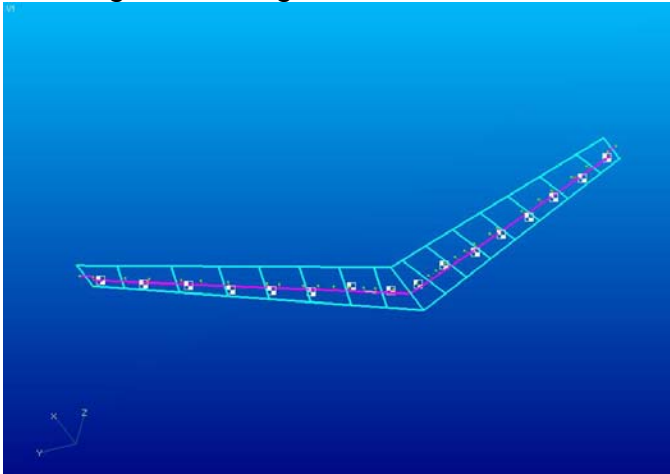
OPTIMIZATION

The optimization approach² used here is based on numerical optimization search methods that iteratively change the key parameters (design variables) in order to minimize or maximize one or more responses (objective functions) with limits on other responses (constraints). Bounds are imposed on the design parameters to limit the region of search. In the GENESIS program, the design variables may include member dimensions and nodal locations and responses may include mass, stress, frequency, and displacement. In the particular application discussed here, the responses of interest are frequencies and mode shape coefficients.

ANALYTIC MODEL

The analytic model was constructed to determine the primary structural bending and torsion properties of the test article. To this end, the FEM consisted of a series of lumped masses attached to massless beams with stiffness. The mass properties of each mass element are defined by the mass value and mass moment of inertia at its center of gravity. The structure was sectionalized into eight mass bays per side, 16 mass bays in total; see Figure 1. The stiffness properties of each beam are defined by bending in two planes and torsion about the shear center. The bending stiffness is determined by the area moment of inertia about the neutral axis at each end of the beam. The torsion stiffness is determined by the torsional stiffness about the shear center at each end of the beam. The spanwise stiffness distribution was discretized with 16 beams per side, 32 beams in total. All beams were changed symmetrically from side to side.

Figure 1 - Diagram of FEM and GVT



VIBRATION TESTING

In keeping with the analysis, the test article was instrumented such that bending in two planes and torsion would be observable. Accelerometers were oriented normal to the datum at several spanwise locations on the front and rear spars of each side of the test article; see Figure 2. These accelerometers allow for the observance of vertical bending and torsion type motions. Additional

accelerometers were oriented within the datum plane and normal to the rear spar line to allow for observance of fore and aft bending. In all, there were 17 spanwise stations of measurement in total. At each station there were three accelerometers, two vertical and one fore and aft, for a total of 51 measurement degrees of freedom.

Figure 2 - Test Setup



Separation of the structural rigid body modes from the desired structural flexible modes was accomplished by suspending the test article using soft supports, thus simulating near free-free boundary conditions. Excitation was provided by electro-magnetic shakers attached to the test article via a stinger. Input force was measured with dynamic load cells. Testing utilized a burst random signal. The sampling parameters were chosen such that the response of the structure had sufficient time to decay within the sampling period. In the usual manner, the measurement consisted of the acquisition of the input and response time series. These data were then transformed into auto- and cross- power spectra from which frequency response functions (FRF) were calculated. Modal analysis of the FRFs was performed to determine a set of frequencies and mode shapes that represent the measured physical system. Analysis of the test data extracted the first seven flexible modes of the structure; four vertical bending, one fore-aft bending, and two torsion.

CORRELATION

The correlation process is, in general, a two step process. The first step is to correlate the geometry of the test with that in the analysis. Careful attention is given to the geometric arrangement of the accelerometers and hence the resulting mode shapes. The location of each accelerometer and its orientation is added to the analytic model. In this way the degrees of freedom of the test are directly relatable to degrees of freedom in the analysis. This allows for direct mode shape comparison. The second step in the correlation process is correlation of the test frequencies and mode shapes to those predicted by the analysis. The traditional correlation environment generally consists of direct comparison of mode shapes. However, comparison of shapes is often accomplished by computing the Modal Assurance Criteria (MAC) between the test and analytic data. Either method employed here would reveal that some modes are out of order, or crossed. This is important to note since an optimization process only focused on frequency could result in a tuned model with the correct set of frequencies but the incorrect set of shapes. A work around might be to manually uncross the modes prior to the employ of optimization. However, the GENESIS mode tracking algorithms allow for crossed modes and make the work around unnecessary.

The optimization goal is the same as the analysts' goal which is to use the set of test modes as the basis for tuning the analytical model. The tuning process consists of adjustment of the beam section properties such that the test and analytical frequencies and mode shapes would begin to correlate. The correlation is achieved by minimizing the least squared error function that monitors how well the test and analytical frequencies and mode shapes match. The test data consisting of frequencies and associated mode shapes are provided to the program as design constraints and the objective respectively. In this example, the first seven flexible modes are chosen. The design variables are likewise

identified. The design variables, with imposed limits, consist of the section properties of the beam elements.

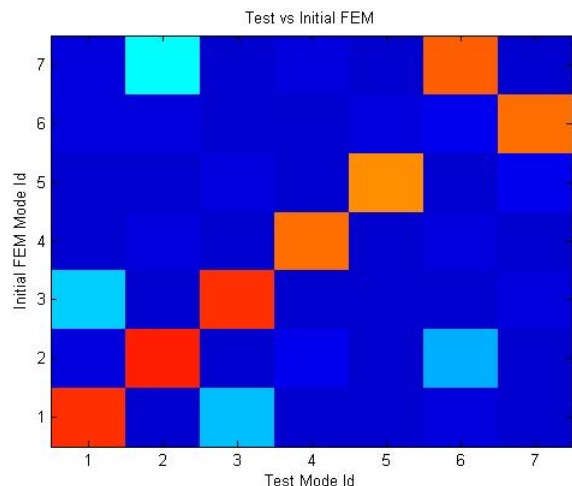
RESULTS

The results consist of comparisons of frequencies and mode shapes of the test data to the initial FEM prediction and the tuned FEM. This is easily performed by computing the MAC between the test data and the initial FEM and the MAC between the test data and the tuned FEM.

Comparison of the test data to the initial FEM predictions is provided in Figure 3. In this example, the initial FEM is a good start to predicting the results of the test. Frequencies are reasonably close to those found, there are no missing modes, and the MAC between the test modes and FEM modes is reasonably good. The frequencies are listed in numerically ascending order except for Initial FEM Modes 6 & 7. These two modes are out of order or switched with respect to the test modes.

Figure 3 - Test vs Initial FEM

Mode	Test	Initial FEM	MACii
1	21.57	21.03	0.97
2	31.01	29.23	0.98
3	55.78	52.63	0.97
4	60.28	58.72	0.89
5	63.17	64.10	0.85
6	86.23	84.50	0.90
7	90.48	81.82	0.88



The MAC graph in Figure 3 graphically depicts the correlation of the Test modes with the Initial FEM modes. Note: blue represents a MAC value approaching zero, while red represents a MAC value approaching one.

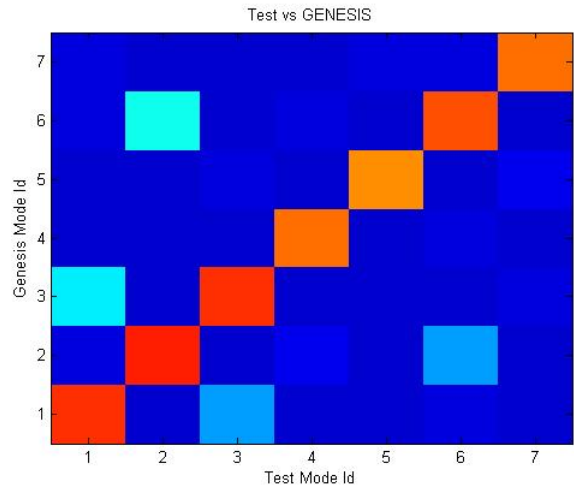
Since there aren't any major problems with the model, the tuning process can be focused on improving the frequency correlation, correcting the mode order, and improving the shapes.

GENESIS was implemented to tune the FEM. The design objectives are to improve the correlation between the test mode shape and the FEM shape. This is performed by minimizing the error between the mode shapes in a least squares sense. Care must be taken to assure the shapes are normalized in a consistent manner. In this example, the mode shapes were normalized by the selection of a specific grid DOF for each target mode. The selection of this grid and DOF was based on its significance to the shape. The design constraints consisted of the target frequencies. The design variables were the beam section properties.

The execution was performed with imposed limits that controlled how much GENESIS was allowed to vary the design variables. The results of this process are provided in Figure 4. This tuning process resulted in the frequency match much improved, as expected via the constraints. Also, the mode shape order has been corrected and the MAC-values maintained.

Figure 4 - Test vs GENESIS Result

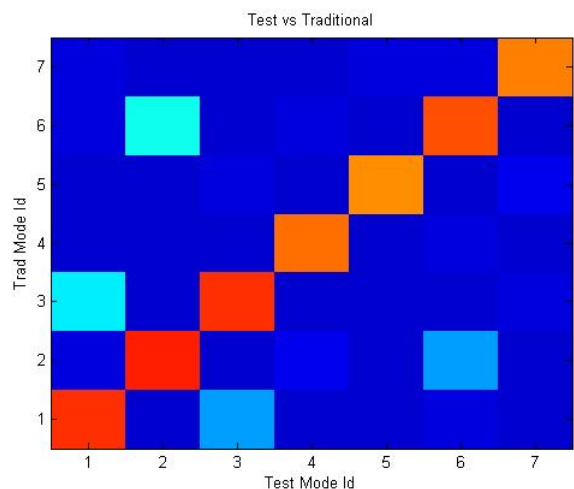
Mode	Test	Genesis	MACii
1	21.57	21.58	0.96
2	31.01	31.01	0.98
3	55.78	55.78	0.97
4	60.28	60.28	0.88
5	63.17	63.16	0.85
6	86.23	86.23	0.93
7	90.48	90.48	0.89



For comparison purposes, the results of a traditionally performed tuning process are shown in Figure 5.

Figure 5 - Test vs Traditional Methods

Mode	Test	Traditional	MACii
1	21.57	21.57	0.96
2	31.01	31.00	0.98
3	55.78	55.81	0.97
4	60.28	60.26	0.88
5	63.17	63.17	0.85
6	86.23	86.24	0.93
7	90.48	90.42	0.87

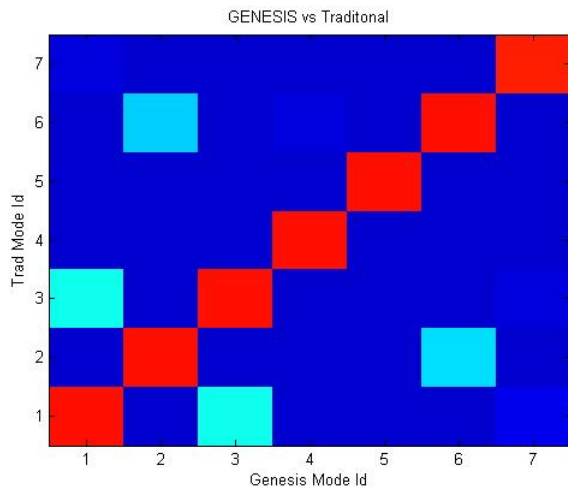


In the traditional method and the GENESIS method the tuning goals have been met. The frequency match has been greatly improved, modes 6 and 7 occur in the correct order, and the MAC values between test and analysis remain high. Furthermore comparison of the

results of the traditional method and the GENESIS method suggest equivalent results, since the frequencies and shapes compare well.

Figure 6 - Traditional Methods vs GENESIS

Mode	Traditional	Genesis	MACii
1	21.57	21.58	1.00
2	31.00	31.01	1.00
3	55.81	55.78	1.00
4	60.26	60.28	1.00
5	63.17	63.16	1.00
6	86.24	86.23	1.00
7	90.42	90.48	0.98



Thus, the model would appear now to be tuned with a comparable outcome. But, what about the beam section properties? Are they reasonable and make sense? Figure 7, shows the beam properties and their variation with span.

Note: The beam property magnitudes and span locations have been normalized.

Figure 7a - Vertical Bending Stiffness

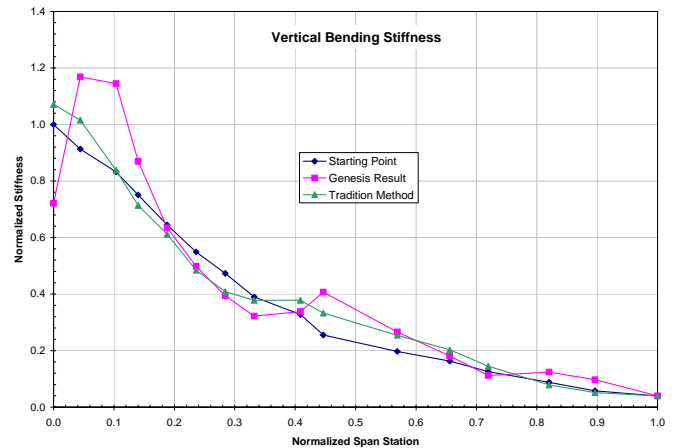


Figure 7b - Fore/Aft Bending Stiffness

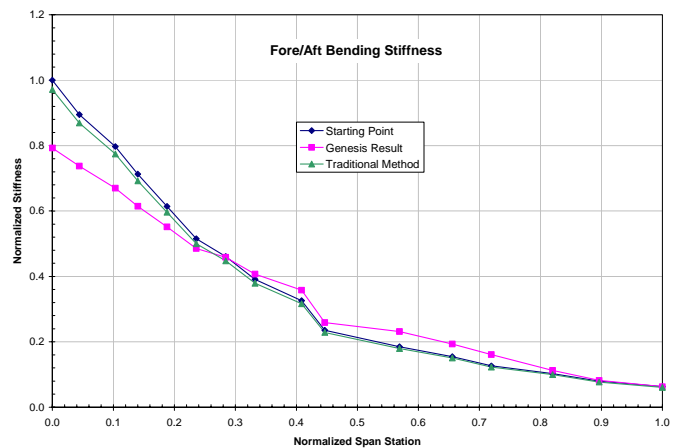
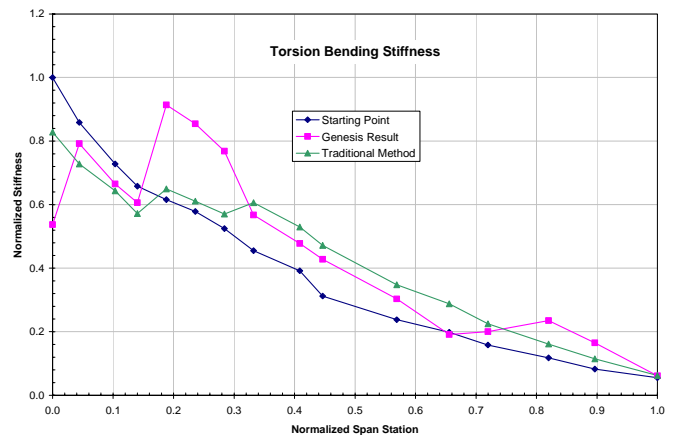


Figure 7c - Torsion Stiffness

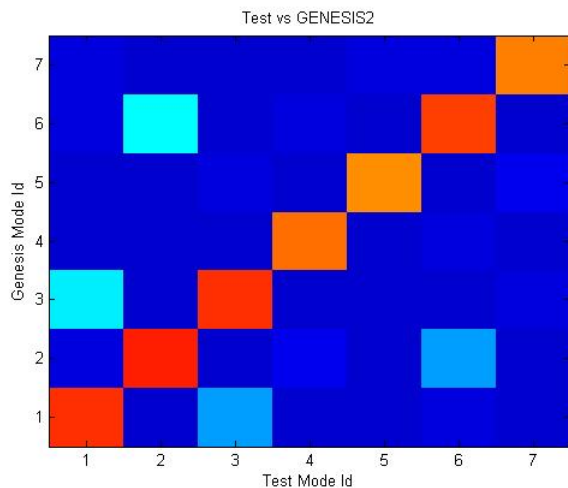


The outboard values, in general, are somewhat comparable, or acceptable. However, the root values show differences that would not yet be acceptable. That is to say, the model is not yet tuned.

The tuning process may need to include additional constraints to provide for some shape manipulation of the stiffness curve, much like that which would be employed in tuning a FEM using traditional methods. In this case, constraints were applied such that each successive outboard stiffness element was less in magnitude than the previous inboard stiffness element. Application of these added constraints resulted in an equivalent solution. Figure 8 shows that the frequencies match well, the mode shapes are in the correct order and the MAC-values are high.

Figure 8 - Test vs Additional Constraints

Mode	Test	Genesis2	MACii
1	21.57	21.58	0.96
2	31.01	31.00	0.98
3	55.78	55.78	0.97
4	60.28	60.31	0.88
5	63.17	63.17	0.86
6	86.23	86.23	0.94
7	90.48	90.45	0.86



The stiffness curves produced as a result of additional curve manipulation strategies are shown below.

Figure 9a - Vertical Bending Stiffness

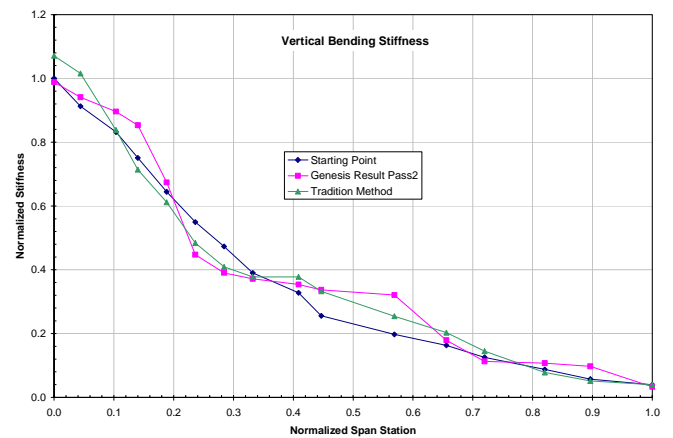


Figure 9b -Fore/Aft Bending Stiffness

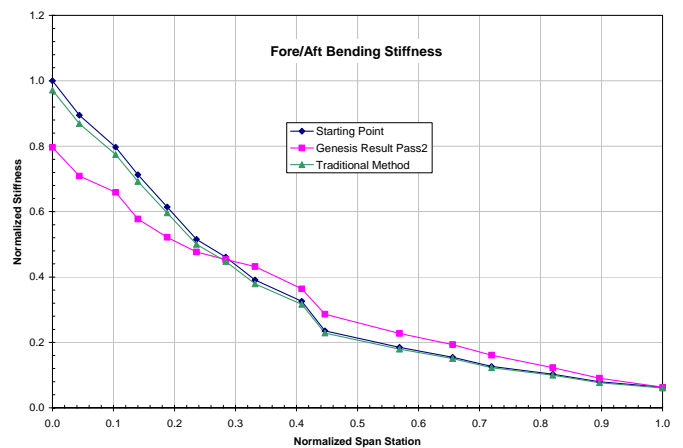
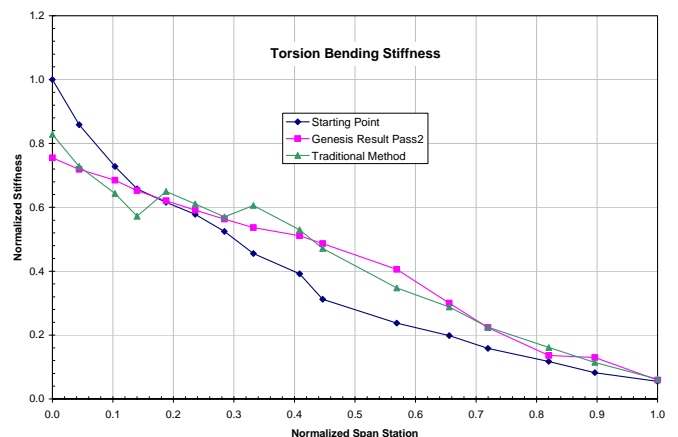


Figure 9c - Torsion Stiffness



CONCLUSION

A Test and Analysis correlation process has been developed using the structural optimization program GENESIS. In general,

this process is capable of producing results that are comparable to those determined using the usual traditional approach. The advantage being that the use of GENESIS allowed for this problem to be worked in little more than a day. As opposed to a weeks worth of adjusting and running repeated NASTRAN jobs as would be necessary with the traditional method. This process requires, as in the traditional method, limits and controls placed on the variables and constraints. The application of engineering expertise is necessary to produce a meaningful and useful result, just as required by the traditional method. The advantage with this process is the time savings to produce results.

This paper presented a brief evaluation of applying a structural optimization program for the purpose of tuning finite element models to vibration test data. The structural optimization program used in this work was GENESIS. GENESIS is an advanced structural optimization program that allows the user to conveniently insert the test data as design criteria, directly uses the typical finite element (NASTRAN) model, accepts property data as design variables, and allows for limits and controls to be imposed. The use of this program provides for opportunities in reducing the time to tune a FEM. The model worked here is relatively simple, it is expected that with the move toward more detailed and large FEMs these approaches will be required to provide satisfactory tuning of the model within a reasonable amount of time.

¹ GENESIS Structural Analysis and Optimization Version 7.5, Vanderplaats Research & Development, Inc., Colorado Springs, CO, 2002.

² Vanderplaats, G. N., "Numerical Optimization Techniques for Engineering Design", VR&D, Colorado Springs, CO, 2001