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BUCKLING FINITE ELEMENT ANALYSIS AND OPTIMIZATION IN GENESIS

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ABSTRACT

This paper describes the implementation of bifurcation buckling finite element analysis and optimization in the commercial program GENESIS. For optimization, approximation concepts are used to reduce the number of full system analyses. The buckling responses are fully integrated so that in the optimization problem they can be combined with other existing analysis responses resulting from statics, dynamics and/or heat transfer. In addition, these responses can be combined with existing geometric and/or user responses. Example problems using buckling responses are described.

INTRODUCTION

Designers use buckling analysis to insure the stability of their designs. Bifurcation buckling analysis consists in finding the critical load factor that multiplies the applied loading such that for loading greater than that factor, there are multiple solutions (i.e., the point at which the load-displacement path bifurcates). Buckling analysis is a well-established discipline and many papers and books on the theory can be found. See, for example, references 1 and 2. In this paper, the discussion of theory is limited to the basic equations, and more emphasis is placed on implementation issues. This work explains the implementation of linear finite element analysis to solve the buckling problem.

The optimization problem in GENESIS is solved using the approximation concepts approach[3]. In this approach, an approximate analysis model is created and optimized at each design cycle. The design solution of the approximate optimization is then used to update the full model, and a full system analysis is performed to create the next approximate analysis model. The sequence of design cycles continues until the approximate optimum design converges to the actual optimum design. When compared to optimizing using full model structural analyses, the approximation concepts approach typically reduces the number of analyses required to find an optimum design by an order of magnitude.

Approximation concepts for traditional structural optimization (sizing and shape) were introduced by Schmit et al., in the mid-seventies [4,5]. In the eighties and early nineties, these concepts were refined to improve the quality of approximations[6,7]. In the late nineties these refined concepts were used to solve the topology optimization problem[8].

This paper discusses the application of these refined approximations to the buckling responses. This work also discusses the optimization capabilities added to GENESIS related to bucking analysis and other existing optimization capabilities that can be used simultaneously with buckling responses.

BUCKLING ANALYSIS

The type of buckling analysis implemented corresponds to bifurcation analysis. The following governing equation is used:

$$[K]\{\phi\}=\lambda[K_g]\{\phi\} \qquad (1)$$

where [K] is the system stiffness matrix, $[K_g]$ is the system geometric stiffness matrix, $\{\phi\}$ the buckling mode shape and λ is the critical load factor.

The stiffness and geometric stiffness matrices, [K] and $[K_g]$, are generated internally by GENESIS. The eigenvalues and eigenvectors, λ and $\{\phi\}$, are solved for by GENESIS using either the subspace iteration or the Lanczos eigen solvers.

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FINITE ELEMENTS

In this work the geometric stiffness matrices for the rods, bars, uniform and non-uniform beams, shear panels, membranes, shells, composites, and 3-D solid elastic elements were implemented. The geometric stiffness matrices for most of these elements were derived from the following general equation:

$$\begin{aligned} U_g^e &= \frac{1}{2}\int_V \omega_x^2(\sigma_y+\sigma_z)dV \\ &+\frac{1}{2}\int_V \omega_y^2(\sigma_x+\sigma_z)dV \\ &+\frac{1}{2}\int_V \omega_z^2(\sigma_x+\sigma_y)dV \\ &-\frac{1}{2}\int_V 2\omega_x\omega_y\,\tau_{xy}\,dV \\ &-\frac{1}{2}\int_V 2\omega_y\omega_z\,\tau_{yz}\,dV \\ &-\frac{1}{2}\int_V 2\omega_z\omega_x\,\tau_{zx}\,dV \end{aligned} \tag{2}$$

where, U_g^e is the element nonlinear strain energy of the buckling state, ω_x, ω_y and ω_z are the rotations, σ_x, σ_y, σ_z, τ_{xy}, τ_{yz}, and τ_{zx} are the element stresses.

The element stresses and rotations are calculated from nodal displacements using the following equations:

$$\{\sigma\}=[D]\,[B]\{u\} \tag{3}$$

$$\{\omega\}=[R]\,\{\overline{U}\} \tag{4}$$

where [D] is the stress-strain constitutive matrix, [B] is the strain-displacement matrix, {u} is the displacement vector resulting from the applied static loading of interest, [R] is the rotation-displacement matrix, and $\{\overline{U}\}$ is the displacement vector for the potential buckled configuration.

The nodal displacements are calculated from the governing equation of the static loadcase for which the stability analysis is being performed.

$$[K]\,\{u\}=\{P\} \tag{5}$$

where {P} is the consistent load vector.

In this work the contributions of the rigid elements, RROD, RBAR and RBE2, were calculated by recovering the force in each rigid link, and then treating each rigid link as a rod element for the purpose of geometric stiffness calculation.

The system geometric stiffness matrix is obtained by assembling the element geometric stiffness matrices.

All other elements available in GENESIS such as the user supplied elements (GENEL) or interpolation elements (RBE3) can be used, but they do not contribute to the system geometric stiffness matrix.

THE OPTIMIZATION PROBLEM

The optimization problem can be stated as:

$$\begin{aligned} &\min_{x_i} F(x_1,x_2,\ldots,x_n) \\ &\text{such that:} \\ &g_j(x_1,x_2,\ldots,x_n)\le 0;\quad j=1,m \\ &x_{il}\le x_i\le x_{iu};\quad i=1,n \end{aligned}$$

where F is the objective function, g_j are the constraints, x_i are the design variables and x_{il} and x_{iu} the side constraints.

DESIGN VARIABLES

Three types of optimization are currently implemented in the GENESIS program: Sizing, shape and topology. Simultaneous sizing and shape optimization can be handled, while topology optimization is performed separately.

For this work, buckling responses were implemented for sizing and shape optimization problems.

An important feature of GENESIS is that there are no built-in restrictions on the number of design variables that can be used. For shape and sizing optimization, several hundred variables are commonly used, while for topology, there are typically tens of thousands of variables.

Sizing Optimization

In sizing optimization, the element cross-sectional dimensions are typically used as design variables. To link the design variables to the properties of the finite elements, the user creates equations that relate design variables to properties. For example:

$$I_{yy} = 1/12\, B\, H^3 \tag{6}$$

$$A = B\, H \tag{7}$$

Shape Optimization

In shape optimization, scale factors of perturbation vectors are the design variables[9]. The perturbation vectors are input directly or by providing basis vectors. A perturbation vector is the vectorial difference between a basis vector and the original grid locations (see Figure 1). Basis or perturbation vectors can be automatically created in GENESIS [10].

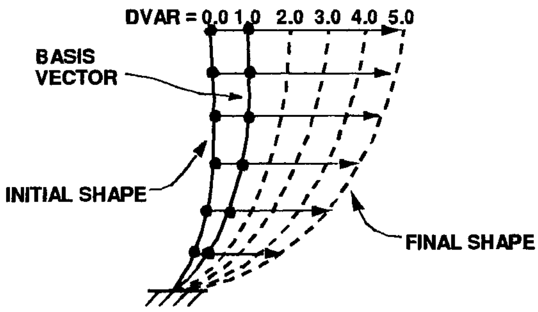

Figure 1. A shape optimization basis vector.

RESPONSES

Responses are quantities that are calculated by the program and are functions of the design variables. They can be used as the objective function or as constraints of the optimization problem. In bucking optimization, the key response is the buckling load factor.

Other existing responses that can be selected simultaneously with the bucking responses are:

Finite Element Responses

Almost every finite element response calculated for analysis can be used in optimization. These responses are displacement; velocity; acceleration; stress; strain; force; strain energy; natural vibration frequency; natural mode shape component; and temperature.

Geometric Responses

Responses that are functions of grid locations, such as volume, area, length, angles, distances, moment of inertias and center of gravity.

Equation Responses

The user can specify nonlinear equations mixing finite element responses with design variables, grid locations and geometric responses to create their own responses.

Subroutine Responses

User-written subroutines can be linked with GENESIS to mix finite element responses with design variables, grid locations and geometric responses to create special responses.

External Responses

An external program can be used to generate responses from other analysis programs for complete multidisciplinary optimization.

OPTIMIZATION

Objective Function

Any of the considered responses can be used as the objective function for minimization or maximization. Because the cost of a structural component is often proportional to its mass, the typical objective in structural optimization is to minimize the mass.

Constraints

Any of the considered responses can be constrained to user-specified limits. Typically, constraints are applied on stresses or deflections. In buckling optimization, the critical load factor is usually constrained to be greater than a safety factor.

Optimizer

The user can select the well-established DOT optimizer[11] or a new optimizer, BIGDOT, which is being developed by Gary N. Vanderplaats. BIGDOT is designed for very large scale optimization problems, and should be selected when there are large numbers of design variables.

APPROXIMATION CONCEPTS

In the approximation concepts approach, responses are modeled using approximation functions. Rather than approximating the responses directly, intermediate responses and intermediate design variables are used. This allows the approximation to capture more of the nonlinearities of the responses, which can then be used over a greater range of design variables. In addition, a constraint screening process is used to limit the amount of work required in the sensitivity module.

Intermediate Design Variables

Sizing variables:

For most elements such as rods, bars, shear panels, and shell GENESIS uses the element properties as intermediate design variable. For laminated composite elements, two options are available: (a) the thicknesses and angles are used directly; (b) the terms of the constitutive matrix are used as intermediate design variables[12,13].

Shape variables:

In shape optimization, the shape design variables are used directly.

Intermediate Responses

Intermediate response are used in GENESIS whenever is possible to improve the quality of the approximations. In this work, the Rayleigh quotient approximation (RQA) method is used to approximate the buckling load factor. This approximation was presented by Canfield[7] and consists of using the following expression:

$$\lambda = \frac{U}{U_g} \tag{8}$$

where U represents the linear modal strain energy and U_g the nonlinear strain energy of the buckled state.

Canfield proposed to approximate U and U_g separately and calculate the approximate load factor from these values.

In this work the RQA method was chosen because its generality (it can be used for any type of element) and because with it, multimodal problems (repeated eigenvalues) can be solved[7]. The ability to solve for multimodal problem could be fundamental because as Olhoff and Rasmussen showed, the correct optimal design of columns is bimodal[14].

Constraint Screening

Constraint screening is a technique to reduce the computational time. The idea is to disregard, in a given design cycle, all constraints that are far from being violated. In GENESIS this technique is used extensively. In this work, this technique has been applied to reduce the number of buckling load factors constraints.

SENSITIVITY ANALYSIS

The sensitivity of the required intermediate responses with respect to the intermediate design variables are calculated using the following relationships:

$$U = \frac{1}{2}\phi^T K \phi \tag{9}$$

$$U_g = \frac{1}{2}\phi^T K_g \phi \tag{10}$$

$$\frac{\partial U}{\partial x_i} = \frac{1}{2}\phi^T \frac{\partial K}{\partial x_i}\phi \tag{11}$$

$$\frac{\partial U_g}{\partial x_i} = \frac{1}{2}\phi^T \frac{dK_g}{dx_i}\phi \tag{12}$$

In equations (11-12), the sensitivities of the mode shape are ignored. This approximation is made because it greatly reduces the computational time without significant lose in accuracy.

The geometric stiffness matrix, K_g, is in general a function of both the design variables, x, and the displacements, u. Therefore, the derivative of $[K_g]$ is calculated using the chain rule as follows:

$$\frac{d[K_g]}{dx_i} = \frac{\partial[K_g]}{\partial x_i} + \sum_{j=1}^{N} \frac{\partial[K_g]}{\partial u_j}\frac{\partial u_j}{\partial x_i} \tag{13}$$

In addition, the sensitivities of the displacements are calculated solving the following equations:.

$$[K]\frac{\partial\{u\}}{\partial x_i} = \left\{\frac{\partial\{P\}}{\partial x_i} - \frac{\partial[K]}{\partial x_i}\{u\}\right\} \tag{14}$$

APPROXIMATE PROBLEM

Response approximations

In GENESIS, most response approximations use the conservative approximation approach first developed by Starnes and Haftka[15] and later refined by Fleury and Braibant[16]:

$$G(X) = G(X_0) + \sum h_i(x_i) \tag{15}$$

Where,

$$h_i(x_i) = \begin{cases} \left.\frac{\partial G}{\partial x_i}\right|_{X=X_0} (x_i - x_{0i}) & \text{if } x_i \left.\frac{\partial G}{\partial x_i}\right|_{X=X_0} > 0 \\ -\left.\frac{\partial G}{\partial x_i}\right|_{X=X_0} \left(\frac{1}{x_i} - \frac{1}{x_{0i}}\right) x_{0i}^2 & \text{if } x_i \left.\frac{\partial G}{\partial x_i}\right|_{X=X_0} \le 0 \end{cases}$$

G(X) is the function being approximated.

X_0 is the vector of intermediate design variables where the approximation is based.

x_i is the i^{th} intermediate design variable

x_{0i} is the base value of the i^{th} intermediate design variable

The modal strain energy and modal nonlinear strain energy are calculated using this expression as intermediate responses to calculate the buckling load factor.

Sensitivities of response approximation

The optimizers require the calculation of the derivatives of the actual responses with respect to the actual design variables. That calculation is divided into four parts: a) the partial derivatives of the actual responses with respect to the intermediate responses; b) the partial derivatives of the intermediate responses with respect to the intermediate design variables; c) the partial derivative of the actual response with respect to the intermediate design variable; and d) the derivative of the intermediate design variable with respect to the actual design variables.

Using the RQA method, the derivatives of the actual response with respect to the intermediate responses are given by:

$$\frac{\partial \lambda}{\partial U} = \frac{1}{U_g} \tag{16}$$

$$\frac{\partial \lambda}{\partial U_g} = -\frac{U}{U_g^2} \tag{17}$$

These derivatives are calculated analytically using the above equation and are updated each iteration, of the approximate problem phase.

The partial derivatives of the intermediate responses with respect to the intermediate design variables are calculated once, per design cycle, in the sensitivity module using Eq. (11-12), and are not changed during the approximate optimization phase.

With the RQA method, the partial derivatives of the actual response with respect to the intermediate design variables are zero because the Rayleigh quotient is not an explicit function of the intermediate design variables.

The partial derivatives of the intermediate design variables with respect to the actual design variables are calculated using the explicit relationships between the intermediate design variables and the actual design variables. For example, for a rectangular beam with actual designable variables H (height) and B (width), the following derivatives are calculated for the intermediate design variable, I_{yy}:

$$\frac{\partial I_{yy}}{\partial B} = \frac{H^3}{12} \tag{18}$$

$$\frac{\partial I_{yy}}{\partial H} = \frac{BH^2}{4} \tag{19}$$

These derivatives are calculated using the finite difference method, and they are updated each iteration during the approximate problem phase.

The chain rule of partial differentiation is used to combine these four parts to calculate the approximate derivatives of the actual responses with respect the actual design variables.

Move Limits

The use of approximation techniques requires limiting how much the design variables can move in each design cycle. Therefore, temporary bounds on the design variables are applied. These temporary bounds are constructed using the following relationships:

$$X_{Li} = X_i - \max(DELX \cdot |X_i|, DXMIN) \tag{20}$$

$$X_{Ui} = X_i + \max(DELX \cdot |X_i|, DXMIN) \tag{21}$$

Where: X_{Li} and X_{Ui} are the temporary bounds for the design variable, X_i, in the current design cycle. DELX is typically 0.5 and DXMIN 0.1 in shape and sizing optimization.

If the temporary bounds lie outside the real bounds, then the real bounds are used.

GENESIS also uses automatic move limits adjustments to improve the performance of the program[17].

CONVERGENCE CRITERIA

The optimization process is terminated when one of the following three criteria is satisfied:

Soft convergence

The optimization process is stopped if the approximate optimization problem did not change the design variables. This type of termination is termed *soft convergence.*

Hard convergence

The optimization process is stopped if the objective function is not changing and there are no violated constraints. This type of termination is termed *hard convergence.*

Maximum number of iterations

In shape and sizing optimization using the approximation concepts described, takes typically 10 design cycles to get close to the final results. So even if the previous criteria are not satisfied the optimization is stopped.

PROGRAM CHART

Figure 2 shows the flowchart of GENESIS.

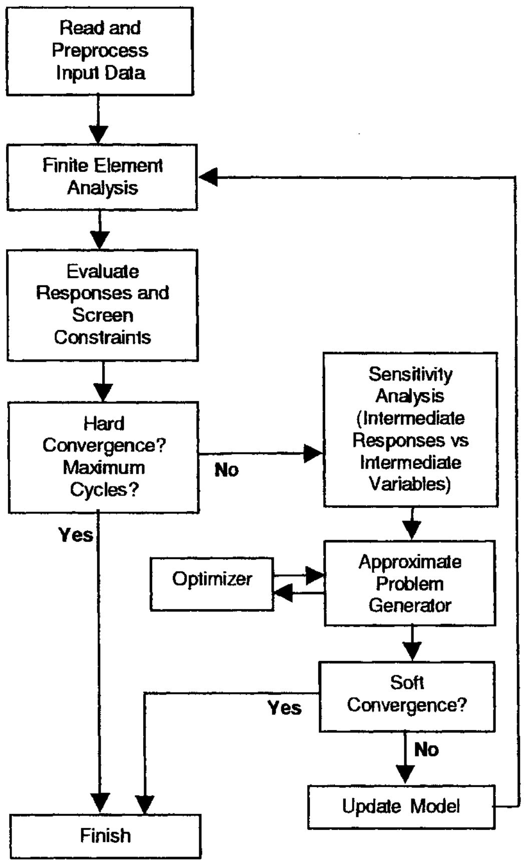

Figure 2. Optimization flowchart.

ADDITIONAL CONSIDERATIONS

For shape optimization in GENESIS, the user may choose to use mesh-smoothing[18]. This option reduces the distortions of the mesh and allows for greater shape changes without re-meshing.

EXAMPLE RESULTS

TOWER OPTIMIZATION

The first example corresponds to a tower structure, which was previously solved by Khot[18]. The initial area of all rod members is 2.03 in^2. The Young's modulus (E) is 1.0E7 psi and the weight density is 0.1 lbs/in^3

Figure 3. shows the finite element model that consists on 41 truss members and 22 grids. A static load consisting on two points loads of 5000 lbs each are applied on the top of the structure.

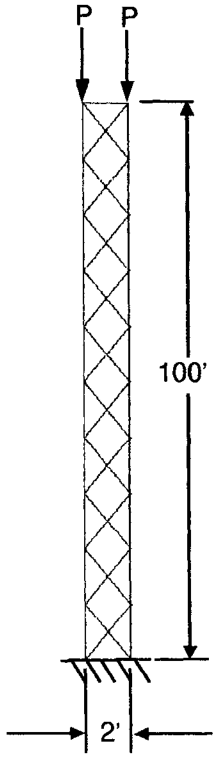

Figure 3. Tower model.

The objective of the problem is to minimize the mass of the tower subject to buckling constraints. The first buckling load factor is constrained to be above 1.0, while the second is constrained to be above 1.1. Twenty one design variables are used to design the areas of the 41 truss members, so that the structure remains symmetric.

Figures 4 show the buckling modes of the original configuration. These modes matches those presented in Ref. 19.

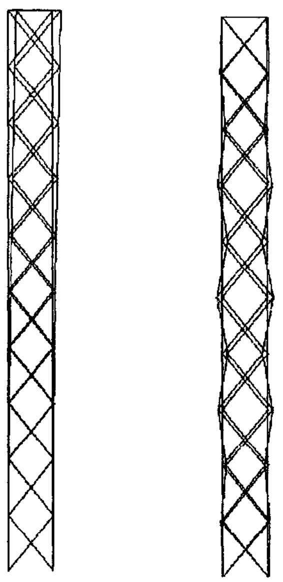

Figure 4. Buckling mode shapes.

GENESIS reduced the mass by 38% and converged with soft convergence after 5 design cycles. That compares well with results in Ref. 19, where the mass reduction achieved was similar. It is interesting to note that in Ref. 19, this problem was solved using optimality criteria. The above results were obtained using only defaults parameters. After changing the GMAX parameter that controls the tolerance for constraint violations to 0.05%, GENESIS converged to a similar answer in 4 design cycles.

Table 1
Weight Design History of Truss Tower

DESIGN CYCLE	REF.19 EQ. 21	REF 19 EQ 24	GENESIS DEFAULT	GENESIS GMAX=0.05
0	989.82	989.82	989.82	989.82
1	689.18	775.22	712.20	712.20
2	616.14	664.36	644.88	628.82
3	608.56	625.00	621.42	610.01
4	608.54	613.89	611.46	608.94
5	608.31	610.12	609.967	
6	608.23	608.72		
7		608.29		
.8		608.23		

OPTIMIZATION OF ELASTIC BEAM CONNECTED TO A RIGID BAR

The second example corresponds to a elastic beam of length 100 in. pinned in one end and built-in at the other to a rigid bar of length 25 in. The rigid bar is connected to a frictionless pin on the right end of the structure. The elastic beam has an initial uniform circular cross section, with a diameter of 1 in. The Young's modulus (E) is 1.0E7 psi and the density is 2.589E-4 (12 slug/in^3). A static load of 50 lbs is applied horizontally on the left end of the structure.

Figure 5 shows the finite element model that consists on 20 bar elements, 1 RBAR element and 22 grids.

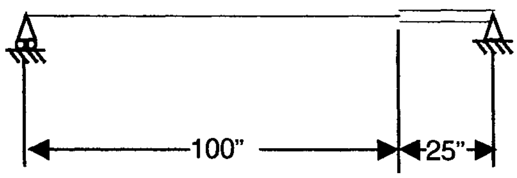

Figure 5. Beam model.

The objective of the problem is to minimize the mass of the elastic beam, while preventing it from buckling. The first buckling load is constrained to be above 1.1 to allow for a 10% factor of safety.

Two sets of design variables are selected. In the first set, one design variable controls the diameter of the 20 bar elements. The second set of design variables contains 20 variables that control the diameter of each bar element.

Figures 6 show the first buckling mode in the original configuration.

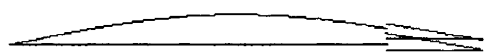

Figure 6. Buckling mode shape.

In is interesting to note that this mode shows that the rigid element also participates in the mode. This is because GENESIS does not ignore the geometric stiffness matrix of the RBAR element.

The solution for case 1 is presented on the following table.

Table 2
Mass Design History of Elastic Beam connected to Rigid bar - Case 1

DESIGN CYCLE	DIAMETER [in.]	MASS 10E-3* (12 slug)	BUCKLING LOAD FACTOR
0	1.0000	20.33	6.49
1	0.7070	10.16	1.62
2	0.6002	8.86	1.23
3	0.6439	8.43	1.12
4	0.6422	8.39	1.10

The theoretical answer for the case of uniform diameter is:

$$D = \left\{ P\lambda_c \frac{L^2}{(kL)^2 E\pi} 64 \right\}^{0.25} \qquad (22)$$

where k is the solution of the following equation:

$$kL = -\frac{L}{R}\tan(kL) \qquad (23)$$

Using Newton's method to solve for equation (23), kL is calculated as 2.5704. Evaluating the equation (22) using $\lambda_c = 1.1$, the analytical diameter is found to be 0.6417. That verifies the GENESIS answer of 0.6422.

The solution for case 2 is presented on the following table.

Table 3
Mass Design History of Elastic Beam connected to Rigid bar - Case 2

DESIGN CYCLE	MASS 10E-3* (12slug/in^3)	BUCKLING LOAD FACTOR
0	20.33	6.49
1	10.16	1.62
2	8.17	1.21
3	7.68	1.11
4	7.66	1.10
5	7.66	1.10
7	7.66	1.10

For the one variable case, *GENESIS* reduced the mass by 59% and converged with soft convergence after 4 design cycles. For the 20 design variable case, *GENESIS* reduced the mass by 62% and converged with soft convergence after 7 design cycles. In both cases the constraint was satisfied and the load factor was reduced by 83%. It is interesting to note that this problem has repeated eigenvalues due to the symmetry of the section and the RQA method did not have problems with them.

CYLINDRICAL TUBE OPTIMIZATION

The final example is a thin shell cylindrical tube with stiffened cutouts subjected to axial compression. Figure 7 shows the finite element model of the tube. The tube was divided into ten bands along its length. The thickness of the shell elements in each band was allowed to vary independently. The objective was to maximize the lowest buckling load factor subject to a constraint on the total mass. The beta method was used to allow the optimizer to consider the lowest 15 modes during the approximate optimization. If this is not done, then mode switching from cycle to cycle creates severe oscillations during the design process. The mass constraint was violated by 15% in the initial configuration. The optimization was able to satisfy the mass constraint, while increasing the minimum buckling load factor by 10%, converging in 16 design cycles.

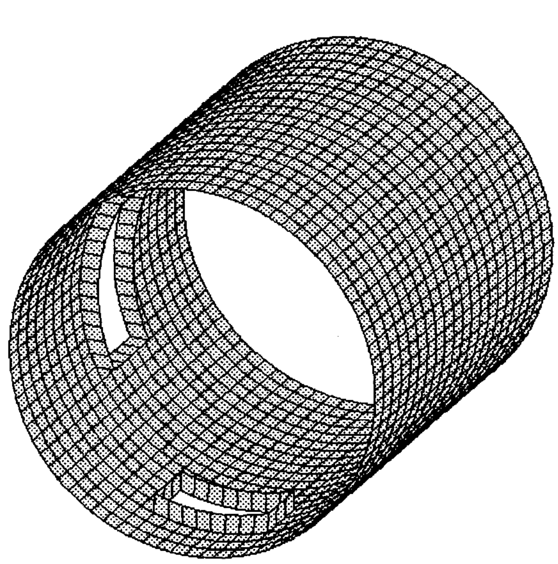

Figure 7. Cylinder model with stiffened cutouts.

Figure 8 shows the buckling mode shape of the lowest buckling load factor of the initial configuration. Figure 9 shows the first buckling mode shape of the optimized configuration.

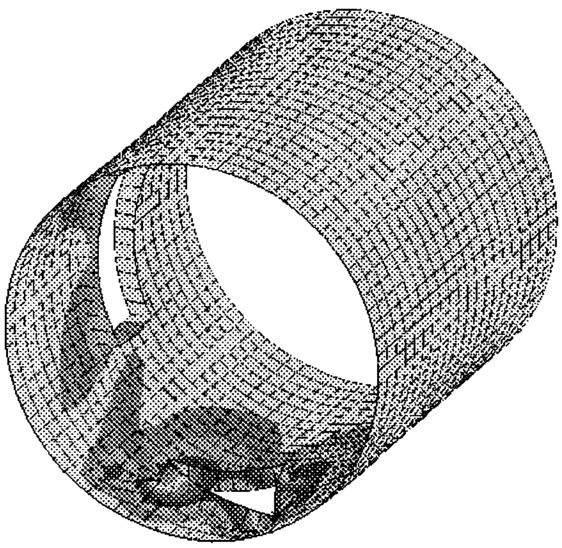

Figure 8. First buckling mode shape of the initial configuration.

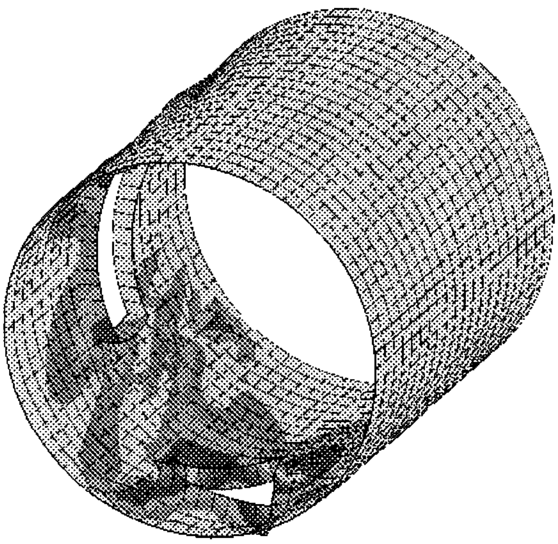

Figure 9. First buckling mode shape of the optimized configuration.

CONCLUSIONS

Buckling finite element analysis and optimization was integrated into GENESIS. The paper discusses the equations used to build the elemental geometric stiffness matrices and the use of approximation concepts to efficiently solve the buckling optimization problem. Examples that illustrate the new buckling capabilities are presented. The implemented Rayleigh quotient approximation method is also discussed. The implementation performs well, and for most problems convergence can be expected to be within ten design cycles. Exceptions could be problems with multiple repeated eigenvalues, where the convergence could be slower, as in the third example.

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