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AUTOMATIC GENERATION OF BASIS VECTORS FOR SHAPE OPTIMIZATION IN THE *GENESIS* PROGRAM

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ABSTRACT

*†A procedure to simplify the generation of basis vectors for shape optimization of complex structures is presented. The approach involves the creation of geometric regions (termed domain elements). Control perturbations are applied to corner and/or midside nodes of the domain. These control perturbations are used together with the domain geometry to automatically generate perturbations for all of the nodes in the region using standard linear and/or quadratic isoparametric interpolation functions. Multiple types of domain elements are presented to facilitate the creation of basis vectors of a wide range of structures.

INTRODUCTION

The use of shape optimization in commercial structural optimization programs, such as *GENESIS*[1] and MSC/NASTRAN[2], requires that alternative shapes for the structure be specified. These alternative shapes, known as basis vectors, form the design space for the problem. The structural optimization program finds the optimal shape that is a linear combination of the basis vectors. The creation of basis vectors can be difficult and time consuming. Therefore, it is important to have tools that simplify this task.

There are three basic procedures to automatically (or semi-automatically) create basis vectors. The first procedure uses special load cases to generate displacements. These displacements are added to the original grid locations to create the basis vectors. Examples of this procedure can be found in references 1 and 3. Basis vectors created with this method have been termed *natural basis vectors*.[1-2] The second method is based on the original geometrical features from which the finite element mesh is derived. The geometrical features are perturbed, and a new finite element mesh is generated. The new meshes correspond to the basis vectors.[4] This method requires access to the original CAD data, as well as to an automatic meshing program. This method can be difficult to use if the original mesh and the mesh generated from the perturbed geometry do not have a one-to-one correspondence.

The third method applies perturbations based on pseudo-geometrical features directly to the original finite element mesh. This approach was implemented in the AutoDV program developed by Altair Computing to generate basis vector for *GENESIS* or MSC/NASTRAN.[5] Hitachi has also implemented this approach in their CADAS pre/post-processor. Examples of this third method can be found in reference 6. In this paper, a generalization of the capability of AutoDV, that is implemented in *GENESIS*, is presented. The generalization consists of expanding the number of domain elements available. In AutoDV, 2D and 3D elements are used. In this paper, we expand this to "rigid", 1D, 2D, 3D and axisymmetric domain elements. In subsequent sections, the domain element concept is explained and examples using domains are presented.

MATHEMATICAL FORMULATION

Figure 1 shows a simple structure and a basis vector. The vectorial difference between a basis vector and the initial shape of the structure corresponds to the perturbation vector. The equations used to internally calculate the new locations of the grids are:

$$\left.\begin{aligned} X_i &= X_{io} + \sum_j DV_j (XB_{ij} - X_{io}) \\ Y_i &= Y_{io} + \sum_j DV_j (YB_{ij} - Y_{io}) \\ Z_i &= Z_{io} + \sum_j DV_j (ZB_{ij} - Z_{io}) \end{aligned}\right\} \qquad (1)$$

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where: X_i, Y_i and Z_i are the updated coordinates of the grid *i*.
X_{io}, Y_{io} and Z_{io} are the initial coordinates of the grid *i*.
XB_{ij}, YB_{ij} and ZB_{ij} are the components of the j^{th} basis vector corresponding to grid *i*
DV_j is the value of design variable *j*.

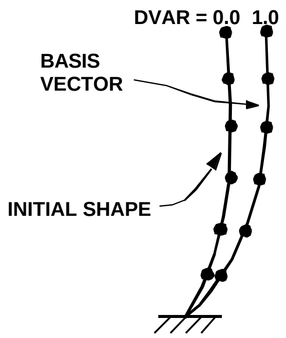


Figure 1. Basis vector.

Figure 2 shows how different values of a design variable affect the final location of the grids. It can be seen that a value of zero cause the grids to remain at the initial locations and a unit value of the design variable causes the grids to move to the basis vector. Values greater than one or less than zero produce a shape that is an extrapolation of the original shape and the basis vector. Values between zero and one produce a shape that corresponds to an interpolation between the original shape and the basis vector.

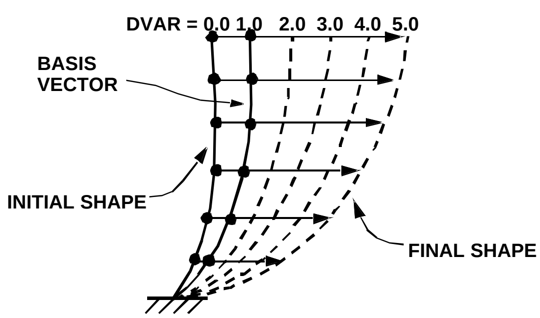


Figure 2. Shape changes possible with basis vector. (Note: Only positive values of the design variable are shown. Negative values are also possible.)

DOMAIN ELEMENTS

Each domain elements is specified by a shape type, a corner nodes list, a set of nodes (usually, but not necessarily, the nodes are physically inside the domain element), and optionally, a list of design variables valid for the domain. Seven types of domain elements implemented in *GENESIS* and two that are currently being implemented are discussed. The basic shapes and characteristics of the domain elements are presented in the Table I. The appendix contains figures of each domain type.

Table I. Domain Types and Descriptions

Domain Type	No. of Shape Functions	Characteristics
RBE2	1	rigid
BAR	2-3	1D elastic
TRIA3	3-6	2D elastic
QUAD4	4-8	2D elastic
TRIAX3	3-6	Axisymmetric elastic
QUADX4	4-8	Axisymmetric elastic
TETRA	4-10	3D elastic
PENTA	6-15	3D elastic
HEXA	8-20	3D elastic

The RBE2 domain element is used to create a rigid translation of all its associated nodes. This domain simply perturbs all nodes that belong to the domain with the same perturbation that is applied to the control node.

The BAR domain is used to create perturbation vectors that vary along one dimension for 1, 2 or 3 dimensional structures. If a node in the list of domain nodes is not physically along the line of the bar, then the interpolation is calculated for the projection of the node onto the BAR domain element. The results are then applied to the original node of the BAR domain.

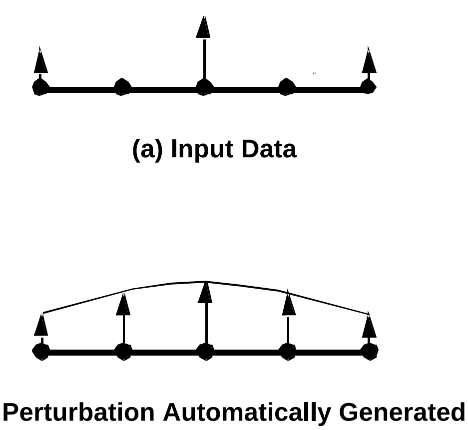


Figure3. BAR Domain.

In Figure 3a, a BAR domain that has 5 grids is used to create a basis vector. Two perturbations (which need not be equal in magnitude) are applied to the “corner” grids, and one perturbation is applied to a midside grid. Figure 3b shows the perturbations that are generated by this domain with these perturbations. In this example, a quadratic shape is obtained because of the presence of a midside perturbation.

The TRIA3 and QUAD4 domains are used to create perturbations that vary in two dimensions for 2 or 3 dimensional structures. If an associated node is not actually contained in the plane of the TRIA3 or QUAD4, then the node is projected to the plane for the purpose of calculating the perturbation. This projection allows these elements to be used for 3 dimensional structures.

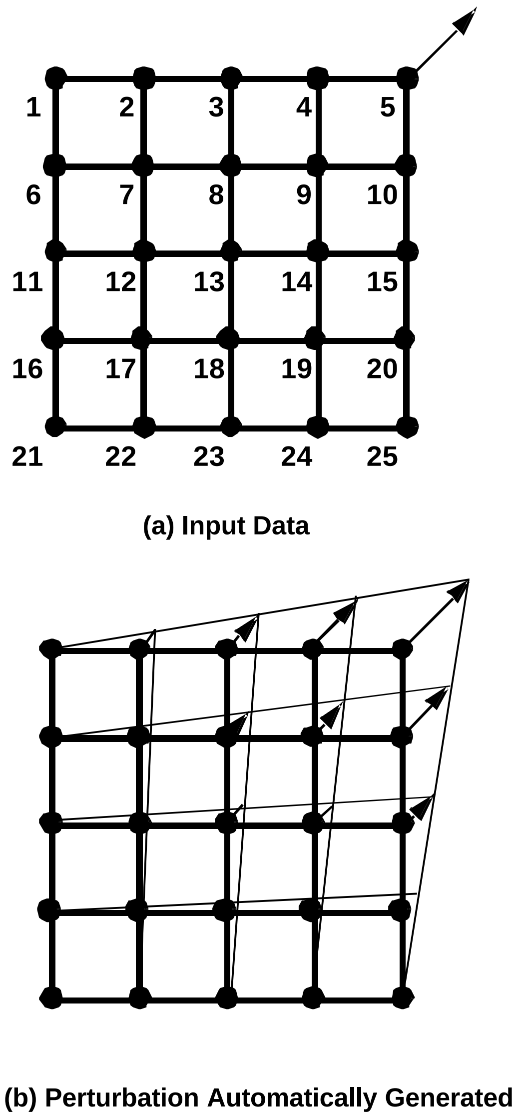


Figure 4. QUAD4 domain.

In Figure 4a, a QUAD4 domain which has 4 corner grids, (1,5,21,25), and 21 interior grids is used to create a basis vector shown in Figure 4b. This example shows that with one control perturbation, it is possible to automatically generate a complete basis shape. Note that the edges that have no perturbation are not moved in the basis vector. In this example, a bilinear shape is obtained because there are no midside perturbations.

The axisymmetric domains, TRIAX3 and QUADX4, are used to create axisymmetric perturbations varying radially and axially for 3 dimensional structures.

The TETRA, PENTA and HEXA domains are most useful to create perturbations that vary in 3 dimensions for 3 dimensional structures.

PROCEDURE TO CONSTRUCT BASIS VECTORS

The procedure to construct basis vectors assumes that a domain type, its corner nodes, its list of domain nodes, and the corner and/or midside perturbations have been selected. The procedure is applied to every grid belonging to the domain. There are 3 steps involved:

Inverse Mapping

This step consists of finding the parametric coordinates associated with every grid i belonging to the domain. In other words, find (ξ_i, η_i, ζ_i) such that:

$$\left.\begin{aligned} X_i &= \sum_j X_j N_j(\xi_i, \eta_i, \zeta_i) \\ Y_i &= \sum_j Y_j N_j(\xi_i, \eta_i, \zeta_i) \\ Z_i &= \sum_j Z_j N_j(\xi_i, \eta_i, \zeta_i) \end{aligned}\right\} \qquad (2)$$

where: X_i, Y_i and Z_i are the coordinates of grid i.

X_j, Y_j and Z_j are the coordinates of the j^{th} corner grid.

N_j are the shape functions corresponding to the domain type.

ξ_i, η_i, ζ_i are the parametric coordinates of grid i.

In *GENESIS*, the Newton-Raphson method is used by rewriting the problem as:

$$\left.\begin{aligned} F_{xi} &= X_i - \sum_j X_j N_j(\xi_i, \eta_i, \zeta_i) \\ F_{yi} &= Y_i - \sum_j Y_j N_j(\xi_i, \eta_i, \zeta_i) \\ F_{zi} &= Z_i - \sum_j Z_j N_j(\xi_i, \eta_i, \zeta_i) \end{aligned}\right\} \qquad (3)$$

Then the problem is reduced to finding the parametric coordinates, (ξ_i, η_i, ζ_i), such that:

$$\left.\begin{aligned} F_{xi} &= 0 \\ F_{yi} &= 0 \\ F_{zi} &= 0 \end{aligned}\right\} \qquad (4)$$

Perturbation Calculation

This step consists of calculating the perturbation of each grid associated with the domain using the shape functions corresponding to the domain type. In this case the equations are:

$$\left.\begin{aligned} P_{xi} &= \sum_j P_{xj} N_j(\xi_i, \eta_i, \zeta_i) \\ P_{yi} &= \sum_j P_{yj} N_j(\xi_i, \eta_i, \zeta_i) \\ P_{zi} &= \sum_j P_{zj} N_j(\xi_i, \eta_i, \zeta_i) \end{aligned}\right\} \quad (5)$$

where: P_{xi}, P_{yi} and P_{zi} are the perturbation of grid *i*.

P_{xj}, P_{yj} and P_{zj} are the control perturbations applied to the corner nodes (and possibly midside nodes) of the domain.

Basis Vector Generation

This step consists of adding the calculated perturbation to the grid locations to get the coordinates of grid *i* for the basis vector.

$$\left.\begin{aligned} XB_i &= X_i + P_{xi} \\ YB_i &= Y_i + P_{yi} \\ ZB_i &= Z_i + P_{zi} \end{aligned}\right\} \quad (6)$$

This third step is not always performed in *GENESIS* because *GENESIS* accepts either perturbation or basis vectors as input. The reason for accepting both is that *GENESIS* can automatically mix generated basis or perturbation vectors with manually created basis or perturbation vectors.

When a grid is referenced by several domains, the perturbations for that grid are calculated by averaging the results of equation (5) for each domain before applying equation (6).

The same domain can be used to generate several basis vectors by using different sets of control perturbations, each associated to a different design variable. The domain element itself can also be restricted to certain design variables, such that control perturbation of other design variables will not use that domain.

USE OF DOMAIN ELEMENTS

DATA CREATION

The data required to define a domain is the domain element corner grids and a set of internal grids. Any preprocessor that can create elements and grid sets can be used to create domains. Domain creation has been made easier with the introduction of commercially available interfaces specifically designed for *GENESIS* shape optimization support. The examples created for this paper were done using Femb/Genesis.[7] Femb/Genesis is a special version of the FEMB program developed by Engineering Technology Associates, Inc.[8] A shape optimization interface for MSC/PATRAN based on the *GENESIS* domain element is also available.

DOMAIN USAGE

The domain elements can be used by covering part of the structure or the whole structure. Different domain element types can be mixed in one model. The domain elements can be larger than the structure. The basic idea is that the domain elements (except the RBE2) can be seen as rubber bands that create basis vectors throughout the domains by stretching their borders. The amount of stretching corresponds to the corner or midside perturbation.

TRANSITION DOMAINS

Domain elements are used to create smooth basis vectors, which will minimize distortion of the finite element mesh. A key use of domains is to smooth the transition from the portion of the model that is not designed to the part of the model that is designed. For this purpose, additional *transition domains* are added, such that perturbations are applied only to one side of the domain. The domain will then interpolate the perturbations down to zero on the side with no perturbations (See, for example, Figure 4b).

SPECIAL FEATURES

GENESIS accepts domains that overlap each others. For example, this allows one large domain to stretch the whole structure, while smaller domains inside the big domain design interior parts of the structure.

GENESIS prints the perturbation vectors created by the DOMAINS using a standard eigenvector post-processing format. In that way, most commercially available post-processing programs can be used to visualize the basis vectors. This feature allows the domains and perturbations to be verified and, if necessary, corrected.

DOMAINS VERSUS NATURAL BASIS VECTORS

The natural basis vector method creates basis vectors by scaling displacements obtained from special loadcases. Basis vectors created using domain elements have an advantage over this method in that there is no Poisson's ratio effect. The domain represents a pseudo-geometrical entity, that the user has absolute control over. The data input required for the domain method is typically much less than that required by the natural basis vector method to create similar basis shapes. Also, basis vectors created with domain elements do not require the solution of a large system of equations.

However, the two methods are not mutually exclusive, and may be combined if desired. In *GENESIS* both methods are available. In a single problem, one set of basis vectors can be generated using domains and another set of basis vectors can be generated using the natural basis vector method.

EXAMPLES

CANTILEVER PLATE WITH 3 HOLES

The first example consists of a cantilever plate with three holes. In this example, 27 QUAD4 domains are used to define 9 basis vectors. Figure 5 shows the initial configuration of the plate, along with 11 of the domains. Nine rectangular domains are positioned on and around each hole, as shown in the figure for the left hole. Note that the corner nodes of the domain element do not necessarily correspond to nodes of the structure. Control perturbations are applied to the corner nodes and midside nodes of the domain to change the shape of the hole in different ways. Three different possible hole shapes for each hole are considered, as shown in Figure 6. The first uses a uniform compression of the central domain to change the radius of the circular hole. The second shape is generated by control perturbations only in the vertical direction, which results in the circular hole being transformed into an ellipse. The third uses quadratic domain variations to perturb the circular hole to a square shape.

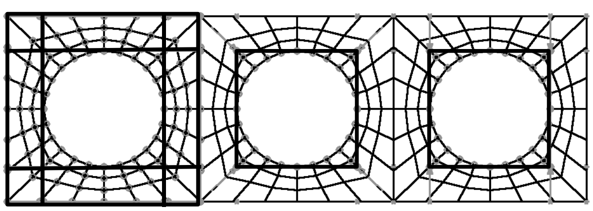


Figure 5. Domains for cantilever plate with 3 holes.

Figure 7 shows the optimized shape of the plate, when subjected to both stress and tip-displacement constraints. Note that the plate thickness was allowed to vary in the optimization. The thickness tapers from left to right as expected (compare reference 2).

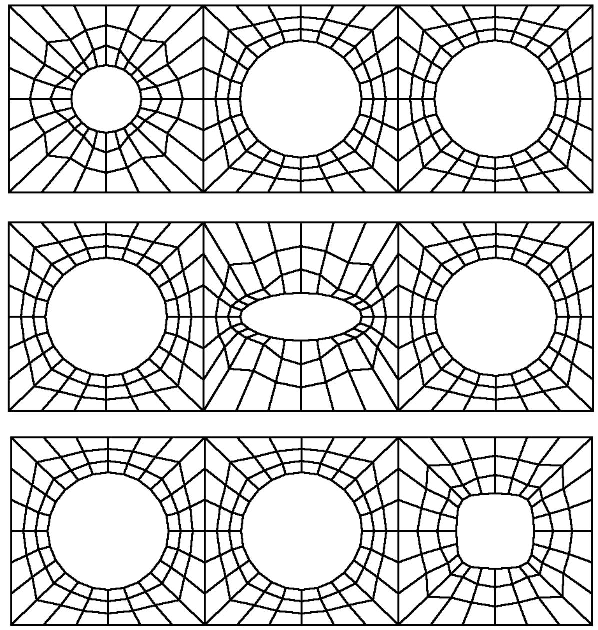


Figure. 6. Example basis vectors for cantilever plate model.

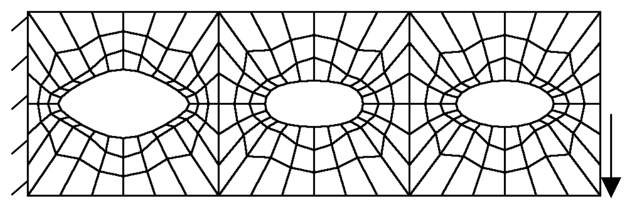


Figure. 7. Optimized shape of cantilever plate model.

CANTILEVER BEAM

The second example is a solid cantilever beam, patterned after an example given in reference 3. The finite element model consists of 60 solid elements and 132 grids. Three loadcases consisting of tip shear, tip moment, and end torque are considered. Three HEXA domains are used to define 9 basis vectors. The first domain covers the entire beam. Four basis vectors are generated with this domain (See Figure 8a-d). The first two are linear tapering in the y- and z- directions, respectively. The next two allow quadratic variations in shape along the entire length of the beam.

The other two domains each cover half of the beam (fixed end to midpoint, midpoint to free end). The final 5 basis vectors are generated using these two domains (See Figure 8e-i). The final basis vector demonstrates how two domains can be used together to generate a smooth, piecewise quadratic variation.

Figure 9 shows the initial and optimized configurations of the beam.

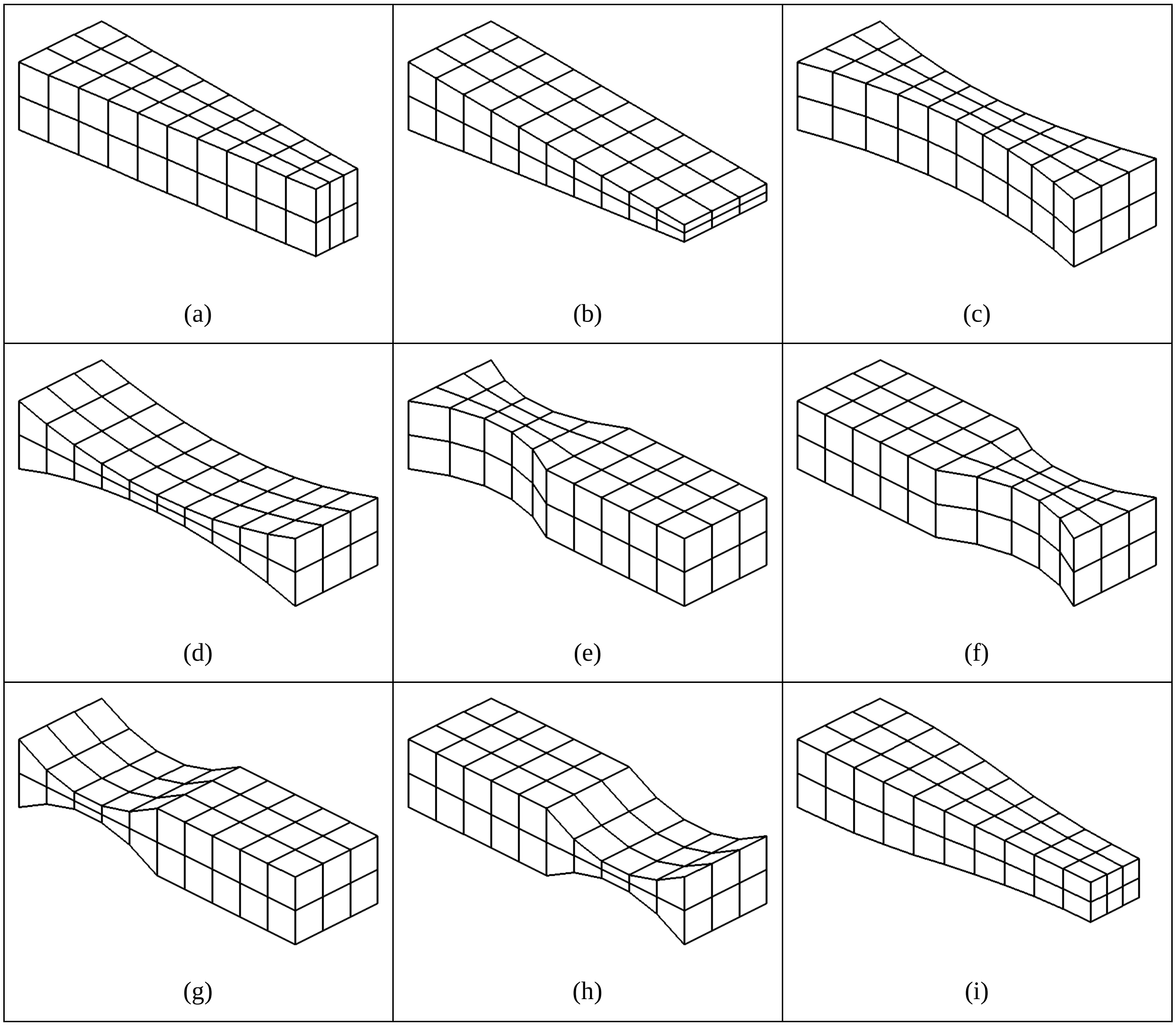


Figure 8. Basis vectors for cantilever beam model.

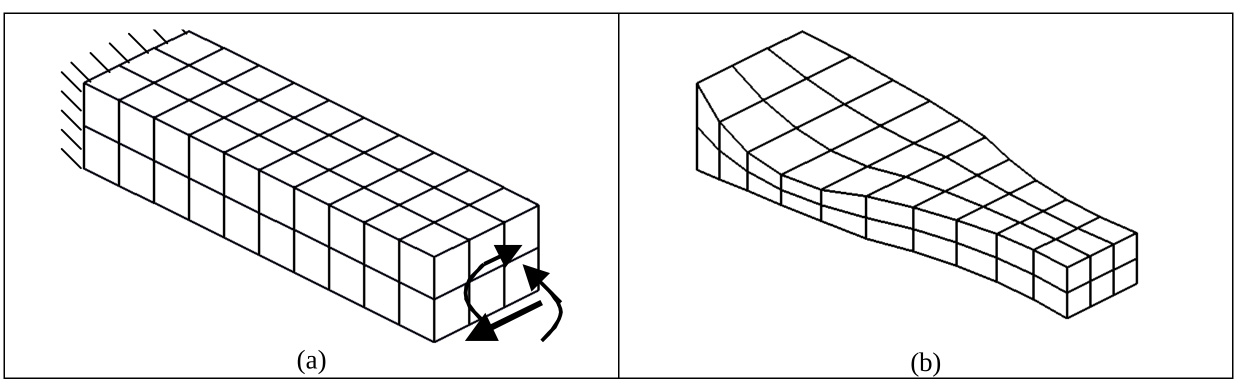


Figure 9. Initial and optimized configurations for cantilever beam model.

CONNECTING ROD

The third example is a connecting rod modeled with solid elements. One quarter of the rod is modeled, and appropriate symmetry boundary conditions are used. Figure 10 shows the original configuration of the connecting rod model.

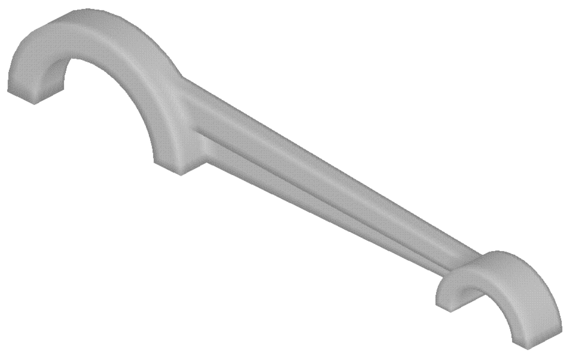
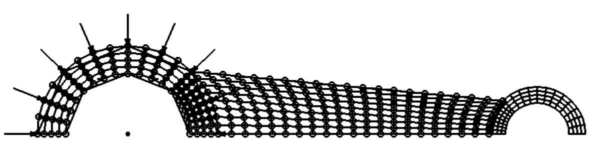
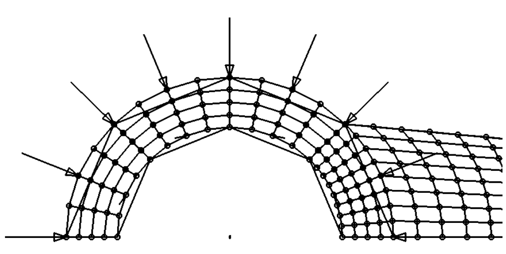


Figure 10. Connecting rod model.

In this example, 10 domains are used to create various basis vectors. To design the thickness of the large end of the rod, 4 QUAD4 domains were created (See Figure 11). Nine perturbations associated with a single design variable were applied to the corners and midpoints of these domains. Although the domain edges are draw as straight lines in Figure 11b, the domain edge is actually forced to follow the outer edge of the large end by the presence of the midside perturbations. Also, zero perturbations are explicitly applied to the midside grids of the inner edge to force the domain geometry to follow the inner curve.



(a) Entire model



(b) Enlargement

Figure 11. Domains and perturbations to change large end thickness.

Note that QUAD4 domains are used, even though the model is 3 dimensional. This is because the perturbations desired for this design variable vary only in the two dimensions of Figure 11. Also note that there is a fifth domain shown in the figure covering the arm of the connecting rod. This domain serves as a transition domain to smoothly vary the model from the large end (that is design by this design variable) to the small end (that is not designed by this design variable).

Figure 12 shows the basis vector that is generated by the domains and perturbations shown in Figure 11. Note that the quadratic variation along the 4 domains very closely preserves the circular shape of the end.

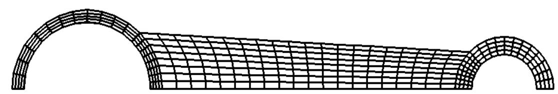


Figure 12. Basis vector generated by domains and perturbations from Figure 11.

Figure 13 shows a HEXA domain used to design the thickness and width of the flange along the arm of the connecting rod. Note that this domain overlaps the domain covering the arm, as shown in Figure 11. Domain overlapping is allowed by restricting domains to only interpolate the perturbations of a given set of design variables.

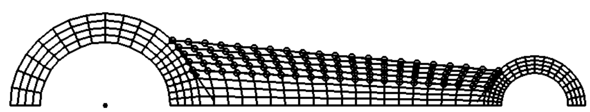


Figure 13. Domain used to design flange height and width.

Two additional basis vectors (associated to different design variables) were created. One was generated by applying a single midside perturbation to the domain covering the arm of the connecting rod. Four QUAD4 domains were used to design the thickness of the small end of the rod in a manner similar to that used for the large end. Figure 14 shows the converged shape of the structure.

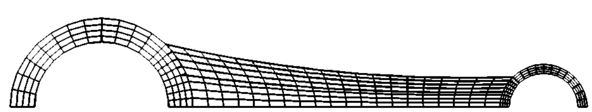


Figure 14. Optimized shape of connecting rod model.

CONCLUSIONS

A general procedure to create basis vectors for shape optimization of structures is presented. The domain represents a pseudo-geometrical entity which can be used to smoothly interpolate perturbations throughout a finite element mesh to create basis vectors for shape optimization. This method has proven to be easy to use, and produces final designs that are generally smoother than those produced by other methods, such as the natural basis vector method.

ACKNOWLEDGEMENTS

Dr. Dhiren Marjadi originally suggested to the authors to add the 2D and 3D domain elements to *GENESIS*.

REFERENCES

1) *GENESIS* Structural Optimization Software, Version 4.0 User's Manual. VMA Engineering, Colorado Springs, CO, USA, August 1997.
2) Kodiyalam, S., Vanderplaats G.N., Miura, H., "Structural Shape Optimization with MSC/NASTRAN", AIAA/ASME/ASCE/AHS/ASC 31St Structures, Structural Dynamics, and Materials Conference, Long Beach, CA, pp. 150-160, April 2-4,1990.
3) Zhang, Jane., "Modified Thermal Load Approach for Automatic Generation of Basis Vectors in Shape Optimization", 5th AIAA/NASA/USAF/ISSMO Symposium On Multidisciplinary Analysis and Optimization, pp. 384-390, Sept. 7-9, 1994.
4) GENESIS/I-DEAS Interface User Manual, VMA Engineering, pp. 1-9, July 1992.
5) AutoDV, A Fortran77 Code for Generating Basis Vectors for Shape Optimization, Altair Engineering, Inc. pp. 1-17, 1994.
6) Thanedar, P.B., Sankaranarayanan, S. and Chirehdast, M., "Topology & Shape Optimization for Product Design", AMD-Vol. 227. Design Optimization with Applications in Industry. ASME, pp. 163-175, 1997.
7) Femb/Genesis, Version 2.0. User's Manual. VMA Engineering, Colorado Springs, CO, USA, August, 1997.
8) FEMB, Version 26.5 User's Manual. Engineering Technology Associates, Inc., Madison Heights, MI, USA, 1996.

APPENDIX – DOMAIN ELEMENTS