DYNAMIC FINITE ELEMENT ANALYSIS AND OPTIMIZATION IN GENESIS

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ABSTRACT

This paper describes several aspects in the implementation of dynamic finite element analysis and optimization in the commercial program GENESIS. Dynamic capabilities discussed are: normal mode analysis, frequency response, and Guyan reduction with or without Craig-Bampton modes. Approximation concepts used in optimization to reduce the number of full system analyses are also discussed. Most of the dynamic responses in shape and sizing optimization are fully integrated so that in the optimization problem they can be combined with other existing analysis responses resulting from statics, buckling and/or heat transfer. In addition, these responses can be combined with existing geometric and/or user responses. In topology optimization the normal modes analysis is available and frequency responses can be combined with displacements and strain energies calculated from static analysis. Example problems using dynamic responses are presented.

INTRODUCTION

Engineers use dynamic analysis to insure and to improve the quality of their designs. Dynamic analysis is a well-established discipline and many papers and books on the theory can be found. This work explains the implementation of linear dynamic finite element analysis in the GENESIS program¹ to solve the dynamics problem.

The optimization problem in GENESIS is solved using the approximation concepts approach. In this approach, an approximate analysis model is created and optimized at each design cycle. The design solution of the approximate optimization is then used to update the full model, and a full system analysis is performed to create the next approximate analysis

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model. The sequence of design cycles continues until the approximate optimum design converges to the actual optimum design. When compared to optimizing using full model structural analyses, the approximation concepts approach typically reduces the number of analyses required to find an optimum design by an order of magnitude.

Schmit et al. introduced approximation concepts for traditional structural optimization, in the midseventies ^{2,3}. In the eighties and early nineties, these concepts were refined to improve the quality of approximations^{4,5,6}. In the late nineties these refined concepts were used to solve the topology optimization problem⁷.

This paper discusses the application of these refined approximations to the dynamic responses. This work also discusses the optimization capabilities added to GENESIS related to dynamic analysis and other existing optimization capabilities that can be used simultaneously with dynamic responses.

NORMAL MODES ANALYSIS

The following governing equation is used:

$$[K]\{\phi\} = \omega^2[M]\{\phi\}$$
(1)

where [K] is the system stiffness matrix, [M] is the system mass matrix, $\{\phi\}$ the mode shape and ω is the frequency.

The stiffness and mass matrices, [K] and [M], are generated internally by GENESIS. The eigenvalues and eigenvectors, ω and { ϕ }, are solved in GENESIS using the subspace iteration solver, the Lanczos eigensolver or the newly implemented SMS eigensolver.

The system eigenvalue problem may be reduced to a user specified set of degrees of freedoms using the Guyan reduction method. The quality of the Guyan reduction results can be improved in GENESIS by augmenting the static condensation vectors with Craig-Bampton modes.

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FREQUENCY RESPONSE ANALYSIS

The following governing equation is used:

$$([K]+i[Ks]){u(\omega)}+[B]{v(\omega)}+[M]{a(\omega)}={P(\omega)}(2)$$

where [K] is the system stiffness matrix, [Ks] is the system structural damping matrix, [B] is the system damping matrix, [M] is the system mass matrix, $\{u(\omega)\}$, $\{v(\omega)\}$ and $\{a(\omega)\}$ are the displacements, velocities and accelerations respectively and ω is the applied frequency.

Two methods to solve this equation are available: the direct and the modal method.

STRUCTURAL MATRICES

The system stiffness matrix is obtained by assembling the element stiffness matrices. The elastic elements currently available are: springs (CELAS1, CELAS2, CVECTOR), rods (CROD), uniform and non-uniform beams (CBAR, CBEAM), shear panels (CSHEAR), shells and composites (CTRIA3, CQUAD4), axisymmetric (CTRIAX6) and 3-D solid elements (CTETRA, CPENTA, CHEXA, CHEX20).

GENESIS can generate the elemental mass matrices of all available elastic element using the consistent or lumped mass formulations. In addition, scalar mass elements (CMASS) or user defined mass (CONM2 or CONM3) can be added to the system mass matrix.

Two types of damping can be provided: Structural damping or viscous dumping.

THE OPTIMIZATION PROBLEM

The optimization problem can be stated as:

$$\begin{array}{l} \underset{x_{i}}{\text{Min }} F(x_{1}, x_{2}, \dots, x_{n}) \\ \text{such that:} \\ g_{j}(x_{1}, x_{2}, \dots, x_{n}) \leq 0; \quad j = 1, m \\ x_{il} \leq x_{i} \leq x_{iu}; \quad i = 1, n \end{array}$$
(3)

where F is the objective function, g_j are the constraints, x_i are the design variables and x_{il} and x_{iu} are the side constraints.

Three types of optimization are currently implemented in the GENESIS program: Sizing, shape and topology. Simultaneous sizing and shape optimization can be handled, while topology optimization is performed separately. Each of these types of optimizations is associated with different design variable types and they are discussed next.

DESIGN VARIABLES

Sizing Optimization

In sizing optimization, the element cross-sectional dimensions are typically used as design variables. To link the design variables to the properties of the finite elements, the user creates equations that relate design variables to properties. For example:

$$I_{yy} = 1/12 B H^3$$
 (4)

$$A = B H$$
(5)

Shape Optimization

In shape optimization, scale factors of perturbation vectors are used as the design variables. The perturbation vectors are input either directly or by providing basis vectors. A perturbation vector is the vectorial difference between a basis vector and the original grid locations. Basis or perturbation vectors can be automatically created in GENESIS. Currently, there are 3 methods to do so: The GRID Basis vector method, the natural basis vector method and the DOMAIN method⁸.

Topology Optimization

In topology optimization, the design variables correspond to the element volume fractions. In general, there is one design variable per designable element. The exception is when the user requires a symmetrical design in which case the number of design variables is reduced according to the type of specified symmetry.

Responses

Responses are quantities that are calculated by the program and are functions of the design variables. They can be used as the objective function or as constraints of the optimization problem.

Responses for Shape and Sizing Optimization

In normal mode analysis and in Guyan reduced analysis, the available responses are frequencies and mode shape components. In frequency response analysis, the available responses are: displacement, velocity, acceleration, stress, strain and force. Other existing responses that can be selected simultaneously with the dynamic responses are classified as following:

Finite Element Responses

Almost every finite element response calculated for analysis can be used in optimization. These responses are: static displacement, stress, strain, force, strain energy; the temperature obtained by heat transfer analysis and the buckling load factors from stability analysis.

Geometric Responses

Responses that are functions of grid locations, such as volume, area, length, angles, distances, moment of inertias and center of gravity.

Equation Responses

The user can specify nonlinear equations mixing finite element responses with design variables, grid locations and geometric responses to create their own responses.

Subroutine Responses

User-written subroutines can be linked with GENESIS to mix finite element responses with design variables, grid locations and geometric responses to create special responses.

External Responses

An external program can be used to generate responses from other analysis programs for complete multidisciplinary optimization.

Responses for Topology Optimization

Topology optimization is only available for normal modes and static load cases. In normal modes analysis the key responses are the natural frequencies. Other existing responses that can be selected simultaneously with the frequency responses are displacements and strain energies from a static analysis and the structural mass fractions.

OPTIMIZATION

Objective Function

Any of the considered responses can be used as the objective function for minimization or maximization. Because the cost of a structural component is often proportional to its mass, the typical objective in structural optimization is to minimize the mass.

Constraints

Any of the considered responses can be constrained to user-specified limits. Typically, constraints applied on frequency response load cases are on stresses or deflections. In normal mode load cases typically the natural frequencies are used.

Optimizer

The user can select the well-established DOT optimizer⁹ or a new optimizer, BIGDOT, which has recently been developed by Vanderplaats. BIGDOT is designed for very large-scale optimization problems^{10,11}, and should be selected when there are large numbers of design variables (1000 or more design variables is considered a large number for DOT).

APPROXIMATION CONCEPTS

In the approximation concepts approach, responses are modeled using approximation functions. Rather than approximating the responses directly, intermediate responses and intermediate design variables are used. This allows the approximation to capture more of the nonlinearities of the responses, which can then be used over a greater range of design variables. In addition, a constraint screening process is used to limit the amount of work required in the sensitivity module.

Intermediate Design Variables

Sizing intermediate design variables

For most elements such as rods, bars, shear panels, and shell GENESIS uses the element properties as intermediate design variable. For laminated composite elements, two options are available: (a) the thicknesses and angles are used directly; (b) the terms of the constitutive matrix are used as intermediate design variables^{12,13}.

Shape intermediate design variables

In shape optimization, the shape design variables are used directly.

Topology intermediate variables

In topology optimization, the Young's modulus, E, and the element density, ρ , are used as intermediate design variables⁷.

Intermediate Responses

Intermediate response are used in GENESIS whenever is possible to improve the quality of the approximations. One example of intermediate responses is the modal energies used to calculate the natural frequencies responses. Next the use of these intermediate responses is explained:

In GENESIS the Rayleigh quotient approximation (RQA) method is used to approximate the natural frequencies. This approximation was presented by Canfield⁵ and consists of using the following expression:

$$w^2 = \frac{U}{T} \tag{6}$$

where U is the linear modal strain energy and T the linear kinetic energy.

Canfield proposed to approximate U and T separately and calculate the approximate load factor from these values. The RQA method was chosen because of its generality (it can be used for any type of element) and because with it handles problems with repeated frequencies can be solved.

Other intermediate responses used are the dynamic forces and moments to calculate stresses.

Constraint Screening

Constraint screening is a technique to reduce the computational time. The idea is to disregard, in a given design cycle, all constraints that are far from being violated. In GENESIS this technique is used extensively.

APPROXIMATIONS FOR DAMPED SYSTEMS

For modal frequency responses, GENESIS approximates the modal matrices first and from them calculates the needed approximated responses. This approximation explicitly captures the nonlinearities associated to resonance¹⁴.

SENSITIVITY ANALYSIS

The sensitivities of the required intermediate responses with respect to the intermediate design variables are calculated using analytical expressions in most cases. These sensitivities are calculated once per design cycle and are kept constant during the approximate problem.

APPROXIMATE PROBLEM

Response approximations

In GENESIS, most response approximations use the conservative approximation approach first developed by Starnes and Haftka¹⁵ and later refined by Fleury and Braibant¹⁶:

$$G(X) = G(X_0) + \sum h_i(x_i)$$
 (7)

where,

$$h_{i}(x_{i}) = \begin{cases} \frac{\partial G}{\partial x_{i}} \Big|_{X=X_{0}} (x_{i} - x_{0i}) & \text{if } x_{i} \frac{\partial G}{\partial x_{i}} \Big|_{X=X_{0}} > 0 \\ -\frac{\partial G}{\partial x_{i}} \Big|_{X=X_{0}} (\frac{1}{x_{i}} - \frac{1}{x_{0i}}) x_{0i}^{2} & \text{if } x_{i} \frac{\partial G}{\partial x_{i}} \Big|_{X=X_{0}} \le 0 \end{cases}$$

G(X) is the function being approximated.

- X_0 is the vector of intermediate design variables where the approximation is based.
- x_i is the *i*th intermediate design variable
- x_{0i} is the base value of the *i*th intermediate design variable

Sensitivities of response approximation

The optimizer requires the calculation of the derivatives of the actual responses with respect to the actual design variables. That calculation is divided into four parts: a) the partial derivatives of the actual responses with respect to the intermediate responses; b) the partial derivatives of the intermediate responses with respect to the intermediate design variables; c) the partial derivative of the actual response with respect to the intermediate design variable; and d) the derivative of the intermediate design variable with respect to the actual design variables.

An Example that illustrates this type of calculation is described next. The example is for natural frequencies.

Using the RQA method, the derivatives of the actual response with respect to the intermediate responses are given by:

$$\frac{\partial w^2}{\partial U} = \frac{1}{T} \tag{8}$$

$$\frac{\partial w^2}{\partial T} = -\frac{U}{T^2} \tag{9}$$

These derivatives are calculated analytically using the above equation and are updated each iteration in the approximate problem phase.

The partial derivatives of the intermediate responses with respect to the intermediate design variables are calculated once, per design cycle, in the sensitivity module and are not changed during the approximate optimization phase.

With the RQA method, the partial derivatives of the actual response with respect to the intermediate design variables are zero because the Rayleigh quotient is not an explicit function of the intermediate design variables.

The partial derivatives of the intermediate design variables with respect to the actual design variables are calculated using the explicit relationships between the intermediate design variables and the actual design variables. For example, for a rectangular beam with actual designable variables H (height) and B (width), the following derivatives are calculated for the intermediate design variable, I_{vv} :

$$\frac{\partial I_{yy}}{\partial B} = \frac{H^3}{12}$$
(10)

$$\frac{\partial I_{yy}}{\partial H} = \frac{BH^2}{4}$$
(11)

These derivatives are calculated using the finite difference method, and they are updated each iteration during the approximate problem phase.

The chain rule of partial differentiation is used to combine these four parts to calculate the approximate derivatives of the actual responses with respect the actual design variables.

Move Limits

The use of approximation techniques requires limiting how much the design variables can move in each design cycle. Therefore, temporary bounds on the design variables are applied. These temporary bounds are constructed using the following relationships:

$$X_{Li} = X_i - \max(DELX \cdot |X_i|, DXMIN)$$
(12)

$$X_{Ui} = X_i + \max(DELX \cdot |X_i|, DXMIN)$$
(13)

Where: X_{Li} and X_{Ui} are the temporary bounds for the design variable, X_i , in the current design cycle. DELX is typically 0.5 and DXMIN is typically 0.1 in shape and sizing optimization. In topology optimization DELX is typically 1.0E-6 and DXMIN is typically 0.2.

If the temporary bounds lie outside the real bounds, then the real bounds are used.

GENESIS also uses automatic move limits adjustments to improve the performance of the program¹⁷.

CONVERGENCE CRITERIA

The optimization process is terminated when one of the following three criteria is satisfied:

Soft convergence

The optimization process is stopped if the approximate optimization problem did not change the design variables. This type of termination is termed *soft convergence*.

Hard convergence

The optimization process is stopped if the objective function is not changing and there are no violated constraints. This type of termination is termed *hard convergence*.

Maximum number of iterations

In shape and sizing optimization using the approximation concepts described, takes typically 10 design cycles to get close to the final results. So even if the previous criteria are not satisfied the optimization is stopped. In topology optimization the default maximum number of iterations is 25.

ADDITIONAL CONSIDERATIONS

For shape optimization in GENESIS, the user may choose to use mesh-smoothing¹⁸. This option reduces the distortions of the mesh and allows for greater shape changes without re-meshing.

For optimization problems involving natural frequency responses the user may choose the use of mode tracking. With mode tracking the user can optimize for a particular mode (for example first torsional mode).

On shape and size optimization the GENESIS user can specify continuous and/or discrete design variables. Discrete design variables were added to GENESIS recently. This capability was released in version 7.0. GENESIS uses the BIGDOT optimizer to perform the discrete variables optimization¹⁸.

EXAMPLES

Examples using dynamic optimization on GENESIS can be found in several papers^{19, 20}. Next, three examples that are described in detail in reference 19 are discussed.

<u>Stiffness optimization of a car body using topology</u> <u>optimization</u>

The first example corresponds to the stiffness optimization of a car body. The goal of the problem is to find the optimal location of reinforcements. The objective function of the problem is to maximize the first torsional frequency This problem was solved using an added second layer of elements in top of the existing shell elements. To solve this problem 34,560 design variables were used.

Figure 1 shows the result of topology optimization on the second layer of added elements. In dark (red) the densities are 1.0 and indicate the places to add the reinforcements. In light (light blue) the densities are 0.0 and indicate the places where no reinforcements are needed.



Figure 1. Optimal Locations of Reinforcement

The optimization raised the first torsion frequency by 0.83 Hz. To get this result the optimizer was allowed to add up to 7.8kg.

The information provided by the topology optimization can be used to either to size-optimize the steal parts themselves of by adding patches of composite elements. The second alternative shows how to reinforce existing car bodies without changing the original design. The next example shows this second alternative.

<u>Increasing stiffness of a car body using sizing op-</u> <u>timization</u>

The objective of this problem was to maximize the first torsional natural frequency of a car body. In this problem the optimal reinforcement locations obtained using topology optimization on previous example were used. On the optimal locations, six patches made of composite material were glued to the car body. The problem consisted on optimizing the thicknesses and the angles of the composite layers. In each patch, 4-thickness design variables and 4 angle design variables were used for a total of 48 design variables.

Patch 1 is used to reinforce the bottom of the windshield frame, the patch 2 is used to reinforce the rear of the car, the patch 3 reinforces the floor and the transmission axle tunnel, patches 4 and 5 reinforce the bottom of the door's frame and the patch 6 reinforces the front of the transmission tunnel (under the patch 1).

In a first optimization run, the first torsional frequency was increased by 2.09 Hz. On a second optimization run, the first torsional frequency was increased by 2.64 Hz. To get these results a constraint on the patches mass was 5 and 10kg respectively.



Figure 2. Composite Patches

Spot Weld optimization of car body using sizing optimization

The purpose of the problem is to find a trade-off table with optimal location of spot welds. The objective function of each case is to maximize the sum of the first bending and first torsional frequencies.

This problem was solved using sizing optimization with 4316 design variables. Each variable designed a CVECTOR element that was added between a grid and an existing spot-weld.

| Quantity of kept welds (%) | First tor- | First | |
|-------------------------------------|------------|---------|-------------|
| | sional | bending | Sum of two |
| | frequency | fre- | frequencies |
| | (Hz) | quency | (Hz) |
| | | (Hz) | |
| 30 | 24.983 | 35.100 | 60.083 |
| 40 | 26.662 | 37.330 | 63.992 |
| 50 | 29.831 | 40.755 | 70.586 |
| 60 | 30.499 | 42.100 | 72.599 |
| 70 | 31.312 | 44.947 | 76.259 |
| 80 | 31.762 | 45.718 | 77.480 |
| 100 | 31.962 | 46.185 | 78.147 |

Table 1. Relation Between Rigidity andNumber of Welds.

This problem was optimized six times to study the effect of taking out different numbers of welds. Table 1 shows the results for all optimization cases and the case where all welds were used (100%). This table gives the designer a trade-off table to choose between the number of welds and a desirable level of rigidity. From this table it can be seen that removing 20% of welds no significant stiffness is reduced. On the other hand, deleting 40% or more welds the stiffness start reducing significantly. It should be mentioned here that the frequency results presented here comes from actually removing the welds and re-running the problems using analysis only. In other words, the numbers here are not affected by the stiffness of the **CVECTOR** elastic elements.



Figure 3. Optimal Weld Location, 70% Kept Welds



Figure 4. Optimal Result, 30% Removed Welds

Figure 3 and 4 shows the kept welds and the suggested removed welds for one of the optimization runs.

CONCLUSIONS

The current capabilities on dynamics finite element analysis and optimization in the GENESIS program were discussed. Examples that illustrate some of these capabilities were presented.

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