9th AIAA/ISSMO Symposium on Multidisciplinary Analysis and Optimization
4-6 September 2002, Atlanta, Georgia

AIAA 2002-5451

A NEW APPROACH IN STACKING SEQUENCE OPTIMIZATION OF COMPOSITE LAMINATES USING GENESIS STRUCTURAL ANALYSIS AND OPTIMIZATION SOFTWARE

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ABSTRACT

A simple approach to stacking sequence optimization of composite laminates is presented. The original stacking sequence problem is solved using an equivalent sizing optimization problem with continuous design variables.

INTRODUCTION

The composite laminate design process typically involves optimization of the following four parameters:

1. Ply (or lamina) material,
2. Ply thickness,
3. Ply orientation, and
4. Stacking (or lay-up) sequence.

The true optimization of a composite laminate, simultaneously considering the coupling effects of the four design parameters mentioned above, is a mathematical challenge in structural optimization.

The optimization of ply material is, perhaps, the most complex of all because of the inherent possibility of designing a hybrid laminate consisting of two or more material types. Recent work by Grosset, Venkataraman and Haftka[1] is an attempt to address the issue of multi-material optimization for hybrid composite laminates.

Numerous analytical techniques are available to optimize the ply thickness and orientation of a composite laminate. While majority of the commercially available structural optimization codes treat ply thickness and orientation as continuous design variables, GENESIS[2] has been recently enhanced to handle them as discrete and/or continuous design variables during the laminate design process.

Once ply material, thicknesses and orientations in the laminate are known, a certain sequence of layers, known as laminate lay-up or stacking sequence, is assumed. However, this *assumed* laminate stacking sequence might not produce the optimal laminate design for a composite structure. This is especially true when the response of the laminated structure is NOT dominated by its membrane properties.

One straightforward approach to stacking sequence optimization may be to evaluate all the candidate designs after material, thickness and orientation optimization has been performed, and then pick the best one. This approach, however, is computationally intensive for most practical applications, because the total number of possibilities in a laminate stacking sequence design is normally huge. For example, if a

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composite laminate has n_l number of layers, the number of possible candidate designs is (n_l!). Thus, if there is a composite laminate with 6 layers, the number of possible candidate designs is 720 (6!).

This work attempts to address the problem of stacking sequence optimization independently, i.e., by separating it from the optimization of ply material, thickness and orientation. The proposed approach is computationally efficient to perform laminate stacking sequence optimization for large composite structures using the current optimization capabilities of the GENESIS program[3].

Previous Research

Researchers in the past have tried to address the stacking sequence optimization issues. Haftka and Walsh[4] used an integer-programming approach to perform stacking sequence optimization for buckling of laminated plates. It was formulated as a linear problem by using ply-identity design variables, which may have values of either 0 or 1. Once it was designed as linear integer design optimization problem, it would be solved using widely available linear integer optimization software. Here the problem chosen was a very specific one which they could formulate as a linear problem. If both thickness and orientation of composite layers are used as design variables, the problem is no longer a linear one but involves nonlinear formulations. Also the software that was used to solve the linear programming formulation used branch-and-bound method, which cannot handle a large number of design variables.

Nagendra, Haftka and Gürdal[5] used an approach similar to the one described in reference[4] to solve the stacking sequence optimization of simply supported laminates with stability and strain constraints. Here the buckling constraints are linear in terms of the ply-identity variables, but strains are nonlinear functions of ply-identity variables. A linear approximation was developed for the strains so that problem could be solved using a linear programming software.

Riche and Haftka[6] used a genetic algorithm to optimize the stacking sequence of a composite laminate for buckling load maximization. While using genetic algorithms for optimization, it is very important to have the right values of parameters used in the optimization. They studied various genetic parameters such as population size, probability of mutation and probability of crossover using numerical experiments. The use of genetic algorithm produced several near optimal designs.

Riche and Haftka[7] further improved the algorithm discussed in reference[6] by incorporating knowledge of the physics of the problem into the genetic algorithm. Improved selection, mutation, and permutation operators were proposed. These improvements reduced the cost of genetic search by more than 50%.

Todoroki and Haftka [8, 9] used a genetic algorithm to obtain stacking sequence of the laminates that had the set of lamination parameters closest to a set of target parameters. The study was based on a constrained combinatorial optimization formulation. Constraints were enforced in genetic optimization by introducing a new repair strategy. The relationship between the reliability of the genetic algorithm and the probability of repair was investigated. It was concluded that the repair strategy should always be used with composite optimization that usually includes 45^0 plies.

In general, genetic algorithms are effective in producing global optimum solutions when only a few design variables are involved, and analyses are not so expensive. The kind of problems that we hope to deal with will involve a larger number of discrete design variables and computationally intensive finite element analyses. Although Genetic Algorithms are effective in producing near global optimum solutions, it may not be very effective to optimize the problems that involve a large number of design variables and expensive analyses.

A NEW APPROACH

The proposed approach to perform the laminate stacking sequence optimization presented here is an indirect approach. This approach consists in changing the stacking sequence problem into an equivalent sizing optimization problem.

This approach uses current sizing optimization capabilities in GENESIS. This approach is meant to be used along with the existing GENESIS input data. The software itself is not changed. This approach allows all existing analysis capabilities in GENESIS version 7.0 to be used simultaneously with the stacking sequence optimization. Among these capabilities are the optimization of natural frequencies, buckling load factors and stress fields.

The proposed approach consists of four steps. The first three steps are for setting-up the equivalent sizing optimization problem. The last step is used to interpret the results to obtain the optimized stacking sequence. These four steps are described next.

STEP1: Replacing Analysis Data

The first step of the proposed approach is to replace the initial N layers with N^2 layers. The first N layers have the same orientation as the initial design but their thicknesses are divided by the total number of layers. Then, this set is repeated (N-1) times as shown in Figure 1.

STEP 2: Setting the Optimization Problem

Step 2.1: Defining the design variables

For each composite with N layers, N^2 design variables are created. Each design variable will have an initial value of $(1.0/N)^{(1/3)}$ with a lower bound of 0.0 and upper bound of 1.0.

Step 2.2: Linking design variables to the thickness

Each thickness will be related to a unique design variable using the following relationship:

$$T_{ij} = T_i X_{ij}^{3} \quad \text{where i, j=1, N} \qquad (1)$$

Other relationships are also possible. The key here is that when X_{ij} is 0.0, T_{ij} is 0.0, and when X_{ij} is 1.0, $T_{ij}=T_i$. This relationship is similar to the power rule used in topology optimization. The cubic power is arbitrary, but experience has shown that it works well in many problems studied by the authors.

Step 2.3 Linking the design variables to enforce design variable values to zeros or ones.

To the original optimization problem, 2N constraints are added per laminated composite. These constraints link layers with the same orientations together. This linking is done in such a way that at the end of the optimization, one of the design variables corresponding to each orientation will be driven to 1.0 and the rest (N-1) to zero. These constraints are shown next:

$$\sum_{j}^{N} X_{ij}^{3} \le 1.0 + Tol \quad \text{and} \qquad (2)$$

$$\sum_{j}^{N} X_{ij}^{3} \ge 1.0 - Tol \quad \text{i=1,N} \qquad (3)$$

In the above constraints, TOL is a small tolerance to help the optimizer converge at a faster speed. A value of 0.02 has produced good results. A value of 0.0, or one very close to 0.0 (0.001) my lead to sub-optimal solutions. The power 3 in Equation 3 is arbitrary, but the experience has shown that it works well in most cases.

T_1	Orientation 1
T_2	Orientation 2
T_3	Orientation 3
T_4	Orientation 4
Original Initial Design	

$T_{11} = T_1/N$	Orientation 1
$T_{21} = T_2/N$	Orientation 2
$T_{31} = T_3/N$	Orientation 3
$T_{41}= T_4/N$	Orientation 4
$T_{12} = T_1/N$	Orientation 1
$T_{22} = T_2/N$	Orientation 2
$T_{32} = T_3/N$	Orientation 3
$T_{42}= T_4/N$	Orientation 4
$T_{13} = T_1/N$	Orientation 1
$T_{22} = T_2/N$	Orientation 2
$T_{33} = T_3/N$	Orientation 3
$T_{43} = T_4/N$	Orientation 4
$T_{14} = T_1/N$	Orientation 1
$T_{24} = T_2/N$	Orientation 2
$T_{34} = T_3/N$	Orientation 3
$T_{44} = T_4/N$	Orientation 4
Proposed Initial Design	

Figure 1: Original and Proposed Layer Setup

STEP 3: Optimization

The additional design variables and constraints are added to the original problem, and the problem is optimized using GENESIS.

STEP 4: Interpreting the Results

At the end of this process, if the added constraints are not violated, N design variables should have a numerical values approximately equal to 1.0, and the rests will have numerical values of zeros or nearly zeros.

A value of 1.0 in X_{ij} means that the layer that originally was in location *i* should be stacked in location *j*. In other words, the values of 1.0 indicate the optimal stacking sequences. This can be exemplified in a simple case: Assume that X_{12}=1.0, X_{21}=1.0, X_{34}=1.0 and X_{43} =1.0, the results would then be as shown in Figure 2.

$T_{11} = 0.0$	Orientation 1
$T_{21} = T_2$	Orientation 2
$T_{31} = 0.0$	Orientation 3
$T_{41} = 0.0$	Orientation 4
$T_{12} = T_1$	Orientation 1
$T_{22} = 0.0$	Orientation 2
$T_{32} = 0.0$	Orientation 3
$T_{42} = 0.0$	Orientation 4
$T_{13} = 0.0$	Orientation 1
$T_{23} = 0.0$	Orientation 2
$T_{33} = 0.0$	Orientation 3
$T_{43} = T_4$	Orientation 4
$T_{14} = 0.0$	Orientation 1
$T_{24} = 0.0$	Orientation 2
$T_{34} = T_3$	Orientation 3
$T_{44} = 0.0$	Orientation 4
Optimized Stacking Sequence	

T_2	Orientation 2
T_1	Orientation 1
T_4	Orientation 4
T_3	Orientation 3
Final Design	

Figure 2: Optimized Stacking Sequence and Final Design

SYMMETRIC LAYOUTS

If the layout is symmetric, we will need to create only (N/2)**2 design variables, and (2N)/2 additional constraints.

Examples of Composite Stacking Sequence Optimization

Three optimization problems of variable complexities are presented to illustrate the use of the proposed approach.

1. Frequencies Optimization of a laminated plate

The first problem aims at maximizing the fundamental torsion frequency of a 16-ply balanced symmetric laminated plate while requiring the fundamental bending frequency to be at least 100 Hz.

The plate is 60 mm by 60 mm. Each layer is 0.15 mm. The material properties are as follows:

E_1=168.2 GPa, E_2=1.15 GPa, V_{12}=0.3,
G_{12}=0.601 GPa, G_{1Z}=0.601 GPa, G_{2Z}=0.43 GPa,
Density= 0.15E-3 Kg/mm^3.

The plate was modeled using a 3,750 degrees of freedom finite element mesh (625 grids, 576 QUAD4 elements and 1 PCOMP property).

The initial laminate design of $[0_2/90_2/-75/75/30/-30)]s$ produced ω_T = 62.41 Hz, and ω_B = 176.62 Hz.

The equivalent sizing problem has 64*2 $[(16/2)^2*2]$ layers and hence 64 design variables. There are 16 [(16/2)*2] added constraints. This initial proposed design $8[0_2/90_2/-75/75/30/-30)]s$ produced the following results: ω_T = 82.17 Hz, and ω_B = 176.26 Hz.

The equivalent sizing problem was solved using two optimizers available in GENESIS: BIGDOT and DOT.

Using BIGDOT, the optimization converged in 12 design cycles overcoming an initial maximum constraint violation of 292.2%. The maximum constraint violation came from the added constraints described on Equations (2) and (3). The optimization run produced the following design: ω_T = 153.08 Hz, and ω_B = 103.37 Hz.

Using step 4, the results are interpreted as a composite with the following stacking sequence: $[\pm 30/-75/75/90_2/0_2]_s$. Using this interpreted stacking sequence, GENESIS produced the following results: ω_T = 153.46 Hz, and ω_B = 103.41 Hz. These results are very similar to the optimized ones because the design variables were very close to zeros or ones.

Table 1: Results using BIGDOT Optimizer

Design Type	Number of Layers	ω_T	ω_B
Original Initial Design	8*2	62.41	176.62
Proposed Initial Design	64*2	82.17	176.26
Optimized Proposed Design	64*2	153.08	103.37
Final Design	8*2	153.46	103.41

The results with BIGDOT optimizer are shown on Table 1. It shows that the torsional frequency of the original initial design was improved from 62.41 Hz to 153.46 Hz, an improvement of 146%.

Using DOT, the optimization converged in 5 design cycles, also overcoming the initial maximum constraint violation of 292.2%. The optimization run in this case, produced the following results: ω_T = 150.13 Hz, and ω_B = 103.63 Hz.

Again using step 4, the results are interpreted as a composite with the following stacking sequence: $[\pm 30/-75/75/0_2/90_2]_s$. Using this new interpreted stacking sequence GENESIS produced the following results: ω_T = 150.13 Hz, and ω_B = 103.63 Hz. These results are almost identical to two significant digits, to the optimized ones because all the design variables were nearly zeros or ones.

Table 2: Results using DOT Optimizer

Design Type	Number of Layers	ω_T	ω_B
Original Initial Design	8*2	62.41	176.62
Proposed Initial Design	64*2	82.17	176.26
Optimized Proposed Design	64*2	150.13	103.63
Final Design	8*2	150.13	103.63

The results with DOT optimizer are shown in Table 2. It shows that the torsional frequency of the original initial design was improved from 62.41 Hz to 150.13 Hz, an improvement of 141%.

A Comparison of the frequencies obtained by the two optimizers shows that the BIGDOT optimizer obtained a torsional frequency slightly better than DOT. Using BIGDOT, however, GENESIS took more design cycles. The two optimizers apparently converged to two different local optimums.

It is noteworthy to mention here that a similar problem was solved using ply angles as design variables. In that problem, starting with an initial laminate design of $[(\pm 45)_2/90_2/0_2]_s$, ω_T = 108.2 Hz and ω_B = 125.5 Hz, a final design with ply orientated as $[\pm 30/\pm 75/90_2/0_2]_s$ was found. The first two fundamental frequencies for this optimized design were computed as ω_T = 153.54 Hz and ω_B = 103.50 Hz. It can be seen that the two frequencies of interest obtained using discrete optimization are almost identical to the one obtained with BIGDOT. The only difference was the order in which the layers with angles -75.0 and 75.0 were located.

2. Vibration and torsion buckling of a laminated cylinder

The second problem aims at maximizing the buckling load factor λ of a 12-ply balanced symmetric laminated cylinder while requiring the fundamental torsional frequency to be at least 100 Hz. Also, the failure index at each layer at each element should be below 0.75.

The height of the cylindrical structure is 53.4 mm. Its diameter is 3.60 mm. Each layer thickness is 0.007 mm.

The material properties are as follows:

E_1=168.2 GPa, E_2=1.15 GPa, V_{12}=0.3,
G_{12}=0.601 GPa, G_{1Z}=0.601 GPa, G_{2Z}=0.43 GPa,
Density= 0.15E-3 Kg/mm^3, X_T=1.61 GPa,
X_C= -1.43 GPa, Y_T=28.5 MPa, Y_C= -263.5 MPa, and S=66.7 MPa.

The structure was modeled using a 15,561 degrees of freedom finite element mesh (2594 grids, 2568 QUAD4 elements, 1 PCOMP property).

The problem has three load cases. The first is a frequency load case, the second is a static load case and the third one is a buckling load case. On the static load case, a moment of 30,000.0 N-mm was applied on one end of the structure, the other end was constrained.

An initial laminate design of $[\pm 20/\pm 30/\pm 65]_s$ produces λ = 0.55 and ω_t = 150.63 Hz.

The equivalent sizing problem has 36*2 [(12/2)^2*2] layers and hence 36 design variables and 12 [(12/2)*2] added constraints.

The initial laminate design of $8*[\pm 20/\pm 30/\pm 65]_s$ produced λ = 0.91 and ω_t = 150.62 Hz. and. The maximum failure index in the first design cycle was 0.65.

The optimization process, overcoming an initial 223.7% maximum constraint violation converged in 6 design cycles. The final results were λ = 1.31, and ω_1 = 150.61 Hz. The maximum failure index constraint on the last design cycle was 0.66.

The initial maximum constraint violation came from the added constraints described on Equations (2) and (3). The DOT optimizer was used in this case.

Again, the results are interpreted as a composite with the following stacking sequence:
$[\pm 65/\pm 20/\pm 30]_s$.

Using this interpreted stacking sequence, a new GENESIS analysis run produced the following results: λ = 1.31 Hz and ω_t = 150.61 Hz. These results are almost identical to the optimized ones to two significant digits, because all the design variables were nearly zeros or ones, as required by the added constraints.

Table 3:Results Using DOT Optimizer

Design Type	Number of Layers	λ	ω_T
Original Initial Design	6*2	0.55	150.63
Proposed Initial Design	36*2	0.91	150.62
Optimized Proposed Design	36*2	1.31	150.61
Final Design	6*2	1.31	150.61

The original design, the initial proposed design, the optimized proposed design and the final design are shown on Table 3. It shows that the buckling load factor improved from 0.55 to 1.31. This improvement represents a 138% increase of the buckling load factor, changing the condition of the structure from unstable ($\lambda \leq 1.0$) to stable (λ>1.0) one. The stacking sequence changes did not affect the first frequency or the failure index responses considerably.

3. Bending stiffness of an all composite, floor-pan structure

This problem aims at maximizing the bending stiffness of an all composite floor-pan structure. This problem corresponds to a typical unitized-body automotive. The floor-pan structure has been divided into six zones, namely Z-1 to Z-6. Each zone has a 12-ply balanced symmetric composite layout.

The material is unidirectional carbon/epoxy prepreg material.

This problem was modeled with a 531,049 degrees of freedom mesh (6 PCOMP data).

The equivalent sizing problem has 216*2 [6*(12/2)^2*2] layers and hence 216 design variables and 72 [6*(12/2)*2] added constraints.

The stacking sequence optimization problem converged in 7 design cycles. A 4% gain was obtained by the stacking sequence optimization. Table 4 shows the results for three designs.

DESIGN-1 corresponds to a trial initial design. DESIGN-2 corresponds to the optimization results obtained in reference[2], where the results were obtained by treating ply angles as discrete design variables. DESIGN-3 shows the optimized stacking sequences obtained starting from DESIGN-2.

These results show the strong influence of both the ply orientations and stacking sequence on the mechanical response of a large automotive composite structure. An approximately 21% increase in bending stiffness is achieved when using optimized lay-ups as given in DESIGNS-2+ DESIGNS-3 rather than using the quasi-isotropic lay-ups of unidirectional prepregs in DESIGN-1.

Table 4: Lay-up Optimization of Six Zones of the Floor-pan to Achieve Bending Stiffness Target.

ZONE	DESIGN-1	DESIGN-2	DESIGN-3
Z-1	$[0_2/90_2/\pm 45]_s$	$[\pm 45/\pm 60/0_2]_s$	$[\pm 60/0_2/\pm 45]_s$
Z-2	$[0/90/\pm 45]_{2s}$	$[\pm 15/\pm 60/0_4]_s$	$[\pm 60/\pm 15/0_4]_s$
Z-3	$[0/90/\pm 45]_{2s}$	$[\pm 30/\pm 45/0_4]_s$	$[30/\pm 45/-30/0_4]_s$
Z-4	$[0_2/90_2/\pm 45]_s$	$[\pm 60/0_4]_s$	$[\pm 60/0_4]_s$
Z-5	$[0/90/\pm 45]_{2s}$	$[\pm 75/\pm 30/0_4]_s$	$[\pm 75/\pm 30/0_4]_s$
Z-6	$[0_2/90_2/\pm 45]_s$	$[\pm 30/90_2/\pm 15]_s$	$[30/\pm 15/-30/90_2]_s$
Bending Stiffness, N/mm	831	*968*	1007
Remarks	Starting lay-ups for DESIGN-2.	Orientations optimized; Starting lay-ups for DESIGN-3	Stacking sequence optimized.

CONCLUDING REMARKS

A new approach to solve stacking sequence optimization problems was presented. The new approach replaces the complexity of the stacking sequence problem with a simpler and easier sizing optimization problem.

The approach presented here is numerically very efficient. The major reason for that efficiency is that this approach inherits sizing optimization efficiencies. On most problems, the approach converged in 12 or less design cycles, just as typical sizing optimization problems converges in GENESIS.

The research presented in this paper focused on design variables associated to stacking sequence, so the equations presented here only reflect that type of variables. It should be noted that with small changes on the presented equations, other design parameters such as thicknesses and angles could be treated simultaneously. For example, on Equation 1 the constant term T_i could be changed to be a variable and therefore, the thickness associated with orientation i could have been designed. The orientations themselves could have also been designed simultaneously.

The problems presented here used only one type of material. However, there is nothing built–in to the method to preclude the use of use multiple materials. On the limit, each layer could have been designed with a different material.

Equation 1 for updating the layer thicknesses, and Equations 2 and 3 for forcing design variables to be close to 0 or 1, could be studied further. Both sets of equations worked well on the studied problems, but other equivalent equations may be derived and made use of. The TOL parameter in these equations turns out to be important. A value of 0.02 produced good results. Small values such as 0.001 caused the optimizer to converge prematurely to a sub-optimal solution.

The current approach was developed for use with GENESIS software. However, it is general enough to be used with other similar structural optimization programs.

ACKNOWLEDGMENTS

The work described herein was partly supported by Visteon Corporation. Additionally, the authors would like to thank Mr. Gary Quinn of Vanderplaats Research and Development, Inc. for his help during this work.

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