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MODERN STRUCTURAL OPTIMIZATION CONCEPTS APPLIED TO TOPOLOGY OPTIMIZATION

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ABSTRACT

*†The latest optimization techniques used in size and shape optimization can be applied to topology optimization. This paper describes the use of intermediate design variables, intermediate responses, constraint screening, move limit adjustments and analytical sensitivities to create an approximate topology optimization problem to efficiently solve the real topology optimization problem. Other concepts unique to topology optimization such as checkerboard controls are also discussed.

INTRODUCTION

Topology optimization is used to find a preliminary structural configuration that meets a predefined criterion. This type of optimization sometimes gives a design that can be completely new and innovative. Topology optimization methods have been discussed in a large number of publications and they can be categorized into two general approaches. The first approach, the assumed microstructure approach, tries to find the microstructure parameters (e.g., size and orientation of holes) of each designed element in a finite element model [1,2]. The second approach assumes no microstructure, but rather heuristically designs the material properties (e.g., Young's modulus and density) of each finite element directly to find optimal material distributions [3,4,5]. Commercially available codes are scarce and not as general as industry would like, but the trend is improving. The code, Optistruct, originally developed by Kikuchi et al., and now being enhanced by Altair Computing, uses the first method. Gae [6] has used the second approach to develop an interface that can be applied to existing finite element programs such as GENESIS or NASTRAN. Also, using the second approach, an interface to MSC/NASTRAN was developed at Ford Motor Company [5,7].

This paper discusses the work done to completely integrate topology optimization with finite element analysis in the general-purpose structural analysis and optimization program, GENESIS. This program is a finite element program fully integrated with sizing and shape optimization [8].

APPROXIMATION CONCEPTS

Approximation concepts for traditional structural optimization (sizing and shape) were introduced by Schmit et al. in the mid-seventies [9,10] and have been successfully implemented in research and commercial programs. In the eighties, these concepts were refined to improve the quality of approximations. GENESIS has incorporated these capabilities for shape and sizing optimization. This paper discusses the application of these refined approximations to topology optimization.

FINITE ELEMENTS

In this work, all the existing elements in GENESIS that can reference isotropic materials (MAT1) were used to design a material distribution. In other words, the rod, bar, beam, shear panel, triangular and quadrilateral shell, axisymmetric, and 3-D solid elements are included. Other elements, such as rigid or interpolations elements, are available for the finite element analysis, but are not designable. Static and normal mode analysis can be used to calculate the desired responses.

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INTERMEDIATE DESIGN VARIABLES

In non-microstructure based topology optimization, Young's modulus (E) and density (ρ) can be used as intermediate design variables. The actual design variable used, x, is a parameter that links the E and ρ using the following relationships

$$E = E_0 f(x) \tag{1}$$

$$\rho = \rho_0 x \tag{2}$$

where: E_0 and ρ_0 are respectively the fully solid Young's modulus and density.
f(x) is the stiffness function.

Typical stiffness functions are:

$$f(x) = x^n \tag{3}$$

$$f(x) = x + \beta(1 - x) \tag{4}$$

$$f(x) = \frac{\beta}{\beta x + 1 - x} \tag{5}$$

Equation (3) is the power rule, equation (4) represents the Voigt rule, and equation (5) represents the Reuss rule. These relationships are discussed in detail in references [3,4]. All of these relationships, as well as linear combinations of equations (4) and (5), are available in GENESIS.

INTERMEDIATE RESPONSES

Canfield [11] proposed the use of the Raleigh quotient to approximate frequencies using the following expression:

$$\omega = \sqrt{\frac{U}{V}} \tag{6}$$

In other words, he proposed to approximate the modal strain energy U and the modal kinetic energy V separately and calculate the frequency from these values. In this paper, the Raleigh quotient is also applied to the topology optimization problem. This approximation is considered a second-generation approximation, because it is an improvement over approximating the frequency directly.

CONSTRAINT SCREENING

Constrain screening is a technique to reduce the computational time. The idea is to disregard in a given design cycle all constraints that are far from being violated. In this work, this technique has been applied to reduce the number of displacement constraints. The procedure implemented is very simple: If a displacement constraint is violated, or is satisfied, but is within 50% of its bound, then it is retained. Otherwise it will be discarded.

MOVE LIMITS

The use of approximation techniques requires limiting how much the design variables can move in each design cycle. Therefore, temporary bounds on the design variables are applied. These temporary bounds are constructed using the following relationships:

$$X_{Li} = X_i - \max(DELX \cdot |X_i|, DXMIN) \tag{7}$$

$$X_{Ui} = X_i + \max(DELX \cdot |X_i|, DXMIN) \tag{8}$$

where: X_{Li} and X_{Ui} are the temporary bounds for the design variable X_i in the current design cycle. DELX is typically 0.5 and DXMIN 0.1 in shape and sizing optimization. In topology optimization GENESIS uses DELX=1.0E-6 and DXMIN = 0.2.

If the temporary bounds exceed the real bounds, 0.0 and 1.0, then the real bounds are used.

SENSITIVITY ANALYSIS

The sensitivity of the required responses with respect to the intermediate design variables are calculated using the following relationships:

For displacements (u):

$$Ku = P \tag{9}$$

$$K\frac{\partial u}{\partial E_i} + \frac{\partial K}{\partial E_i}u = 0 \tag{10}$$

$$K\frac{\partial u}{\partial \rho_i} = \frac{\partial P}{\partial \rho_i} \tag{11}$$

For strain energy (U):

$$U = \frac{1}{2}\sum_{j=1}^{N} u^T K_j u \tag{12}$$

$$\frac{\partial U}{\partial E_i} = -\frac{1}{2}u^T\frac{\partial K}{\partial E_i}u \tag{13}$$

$$\frac{\partial U}{\partial \rho_i} = u^T\frac{\partial P}{\partial \rho_i} \tag{14}$$

For modal energies (U and V):

$$U = \frac{1}{2}\sum_{j=1}^{N} \phi^T K_j \phi \tag{15}$$

$$V = \frac{1}{2}\sum_{j=1}^{N} \phi^T M_j \phi \tag{16}$$

$$\frac{\partial U}{\partial E_i} = \phi^T \frac{\partial K}{\partial E_i}\phi \tag{17}$$

$$\frac{\partial U}{\partial \rho_i} = 0 \tag{18}$$

$$\frac{\partial V}{\partial E_i} = 0 \tag{19}$$

$$\frac{\partial V}{\partial \rho_i} = \phi^T \frac{\partial M}{\partial \rho_i}\phi \tag{20}$$

The adjoint method is used to efficiently calculate the displacement and strain energy sensitivities.

APPROXIMATE PROBLEM

This work for most response approximations uses the conservative approximation approach first developed by Starnes and Haftka [12] and later refined by Fleury and Braibant [13]:

$$G(X) = G(X_0) + \sum h_i(x_i) \tag{21}$$

where

$$h_i(x_i) = \begin{cases} \left.\dfrac{\partial G}{\partial x_i}\right|_{X=X_0} (x_i - x_{0i}) & \text{if } \; x_i \left.\dfrac{\partial G}{\partial x_i}\right|_{X=X_0} > 0 \\ -\left.\dfrac{\partial G}{\partial x_i}\right|_{X=X_0} \left(\dfrac{1}{x_i} - \dfrac{1}{x_{0i}}\right) x_{0i}^2 & \text{if } \; x_i \left.\dfrac{\partial G}{\partial x_i}\right|_{X=X_0} \le 0 \end{cases}$$

G(X) is the function being approximated.
X_0 is the vector of design variables where the approximation is based.
x_i is the i^{th} design variable
x_{0i} is the base value of the i^{th} design variable

This work considers six basic types of responses: mass, displacement, strain energy, modal strain energy, modal kinetic energy and natural frequency. The modal strain energy and modal kinetic energy are used only as intermediate responses to calculate the natural frequency.

OPTIMIZATION

Objective Function:

Any of the considered responses can be used as the objective function for minimization or maximization. Typically, the choice is minimize the strain energy in static load cases or maximize the frequencies in the natural frequency load cases. In addition, it is possible to optimize for a linear combination of responses and/or the reciprocals of the responses. Since multiple load cases are considered in GENESIS, the terms of the linear combination can be responses from different load cases and load types. For example, a linear combination of strain energy and the reciprocal of a frequency is allowed.

Constraints:

Any of the considered responses can be constrained to a user-specified limit. However, in most of the problems the choice is a fraction of the structural mass.

Optimizer:

A new optimizer, BIGDOT, which is being developed by Gary N. Vanderplaats to solve problems with a large number of design variables, is used to solve the approximate topology optimization problem. The user can optionally select the well-established DOT optimizer [14]. Because DOT originally was developed to be efficient for several hundreds of design variables, it is not well suited for topology optimization, where the number of design variables can easily be on the order of tens of thousands. That is why the new program BIGDOT is being developed and is the default.

CONVERGENCE CRITERIA

The optimization process is terminated when one of the following three criteria is satisfied:

Soft convergence:

The optimization process is stopped if the approximate optimization problem did not change the design variables. This type of termination is termed *soft convergence.*

Hard convergence:

The optimization process is stopped if the objective function is not changing and there are no violated constraints. This type of termination is termed *hard convergence*.

<u>Maximum number of iterations</u>:

Typically, 25 design cycles are enough to get close to the final results, so even if the previous criteria are not satisfied the optimization is stopped. To get "black and white" solutions, the user may need to increase the default value of the maximum number of iteration from 25 to a higher number (e.g. 35 or 50). It is interesting to note that in shape and sizing optimization the maximum number of iteration default value is 10.

CHECKERBOARDING INSTABILITY

The literature [15,16] shows that undesirable checkerboarding instabilities occur if the displacement field shape functions are not of sufficiently high order in relation to those of design variable field. This can be easily overcome using a spatial filtering algorithm, which effectively lowers the order of the shape function of the design field. In this development, the volume averaged spatial filter described in [4] is used.

Figure 1 shows the example of the checkerboard solution, which is difficult to interpret and is generally not manufacturable. The same problem was solved using filtering algorithm and the obtained solution is shown in Figure 2. The details of this example problem and full model solution are presented in the example results section.

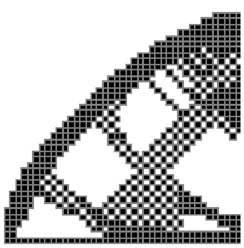

Figure 1. Example of checkerboard solution.

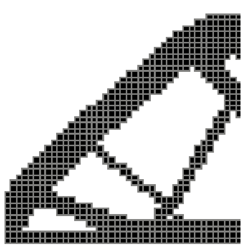

Figure 2. Example of checkerboard-free solution .

ADVANTAGES OF DENSITY BASED METHODS

Density (non-microstructure) based methods have an advantage over assumed microstructure methods in that the later requires significantly more design variables for the same number of designed elements. In addition to that, any elements in a finite element library that are based on Young's modulus, including linear elements such as trusses, bars, and beams, can be easily incorporated in topology optimization.

PROGRAM CHART

Figure 3 shows the flowchart of the topology optimization method in the GENESIS program.

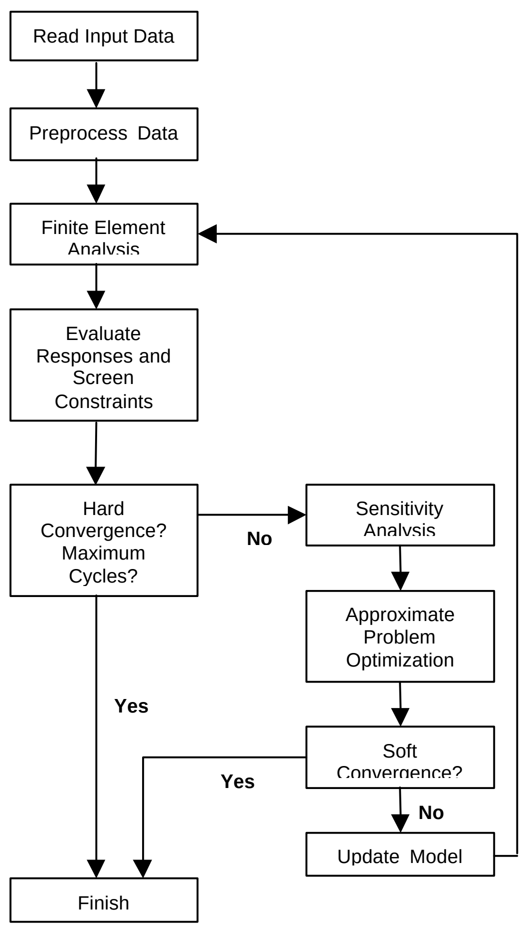

Figure 3. Flowchart of topology optimization.

EXAMPLE RESULTS

Michell Truss

The Michell truss problem is commonly used to verify the topology design algorithm. Figure 4 shows the entire design domain including the circular non-designable region, whose perimeter is completely fixed. The designable domain was discretized with 8800 CQUAD4 elements and the non-designable region's mesh was simply eliminated (hatched area shown in Figure 4). A single point load was applied to produce in-plane bending.

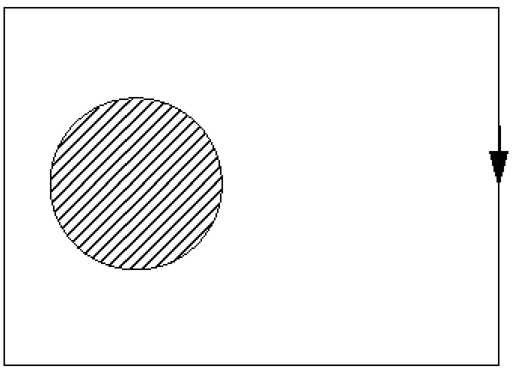

Figure 4. Designable domain and boundary/loading conditions.

This topology optimization problem consists of minimizing the strain energy using a 20% mass fraction constraint. The details of design specification for this problem can be found in [17].

Figure 5 shown below contains the final results. The dark elements represent the elements with final large density and they are the one that should be kept in the design. The rest of the elements should be removed from the design.

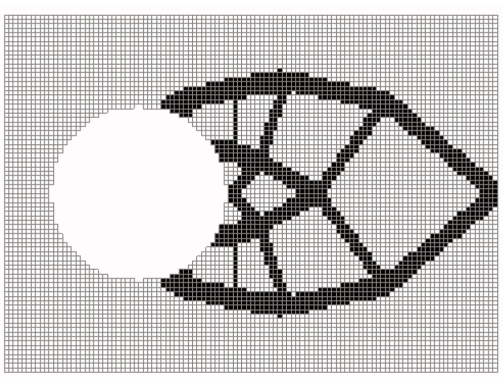

Figure 5. Michell truss topology optimization result.

3-D Space Truss

The design domain (20x20x10 rectangular solid) whose four bottom corners are fixed and middle bottom point is subjected to concentrated force is illustrated in Figure 6. In this example, a 90 degree cyclic symmetry plane which also shown in Figure 6 is used. The designable domain was discretized with 20x20x10 hexahedral solid elements. The number of independent design variables was reduced to 1000 because of the symmetry reduction. The objective of this problem is to minimize strain energy with 25% of mass fraction. The obtained solution with views from two different angles is shown in the Figures 7 and 8.

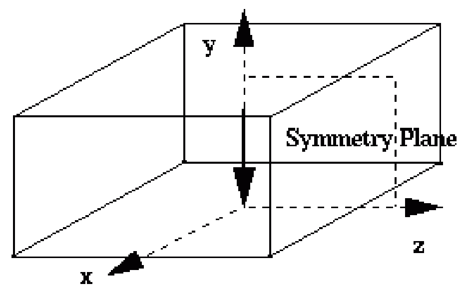

Figure 6. Designable domain and loading conditions for the 3D space truss problem.

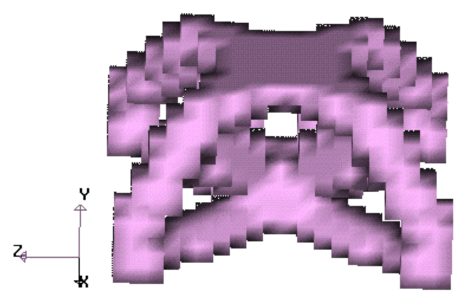

Figure 7. Front view of the 3 D space truss solution.

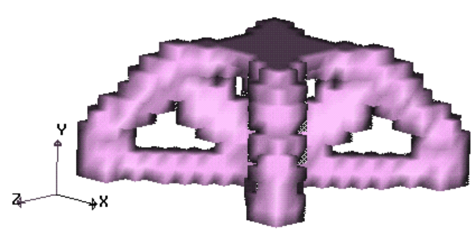

Figure 8. Corner view of the 3D space truss solution.

Truck Frame

The truck frame model is shown in Figure 9. The truck frame topology design problem consists of 29966 SOLID, 7478 SHELL, and 20 BAR elements and has 112312 DOF in its entire model. The topologically designable region is the front end, as shown in Figure 10, which has 10910 designable (mostly hexahedral solid) elements. This frame structure is subjected to 12 different static loadcases and topology optimization is used to minimize strain energy with 34% of material usage in the designable region. Figure 11 shows the results of the topology optimization for this model.

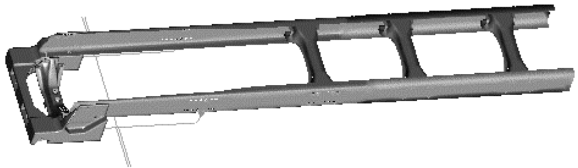

Figure 9. Full truck frame model.

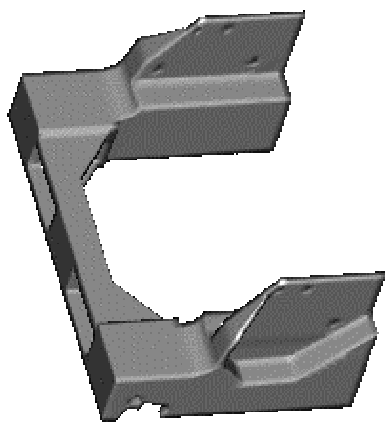

Figure 10. Package space of the topologically designed region of the truck frame.

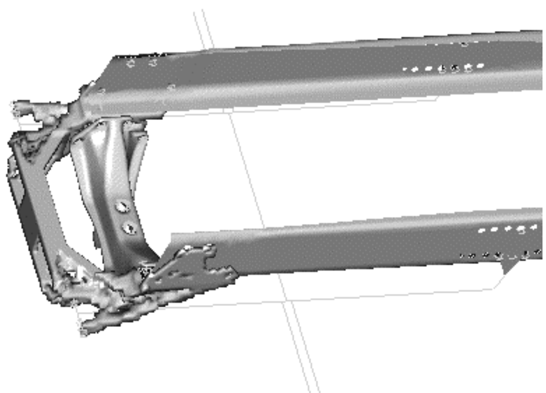

Figure 11. Front end of truck frame model after topology optimization.

Spot-Welded Bracket

With the density-based method, any element that can reference an isotropic material property can be designed. In this example, bar elements are topologically designed to determine the optimal locations to apply welds. The model simulates a support bracket, and consists of two plates bent into “L” shapes, that are welded together along the long faces. A weld is modeled by a short stiff bar element. In addition to the weld locations, the welded faces are designed. Figure 12 shows the package space of this model. The lower flange is supported, and three different load sets are applied to the top flange to simulate loading conditions of bending, shear, and compression

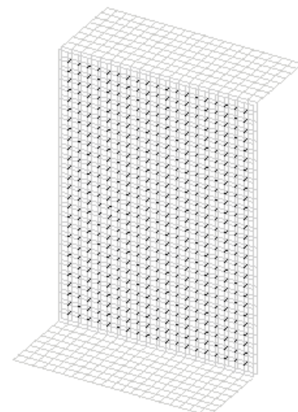

Figure 12. Weld optimization design model.

The bar property is limited to 10% of the total potential bar mass, and the plate property is limited to 45% of the total potential plate mass. Figure 13 shows the final result, with the locations of the retained welds highlighted. Figure 14 shows the final topology design results of the plates (separated for visualization).

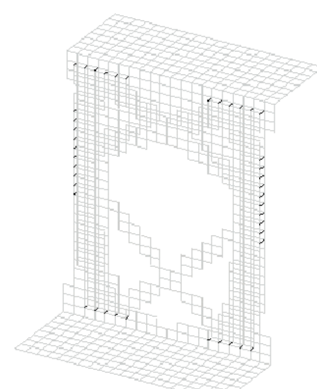

Figure 13. Final topology design of weld optimization model.

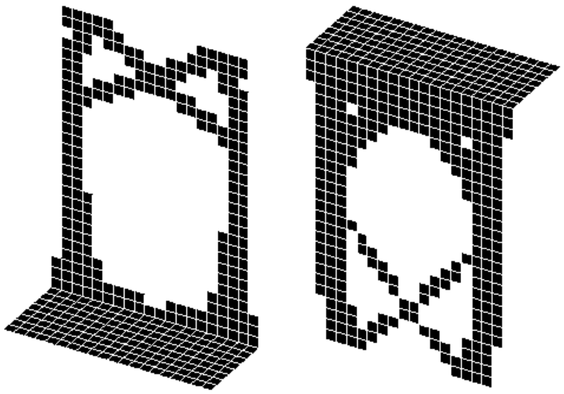

Figure 14. Topologies of lower (left) and upper (right) brackets in weld optimization model.

MBB Beam

The MBB beam problem has been previously presented in [18]. This beam is simply supported at the bottom corners and a point load is applied on the middle of top floor as shown in Figure 15. The design domain (120x40 rectangular) which is half of the entire model with symmetric boundary conditions was discretized with 4800 quadrilateral elements. Figure 16 shows the solution, which is very interpretable.

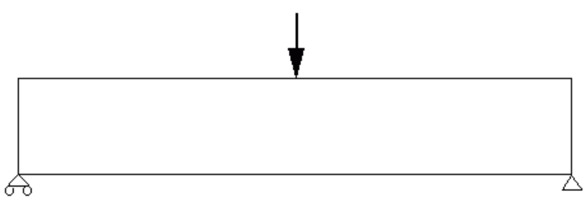

Figure 15. Full designable domain and boundary/loading conditions.

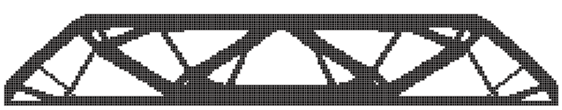

Figure 16. MBB beam problem solution.

CONCLUSIONS

Modern structural optimization concepts used in shape and size optimization can be successfully used in a topology optimization. In this work, those concepts were discussed as they were implemented in the commercial program GENESIS.

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