Methods For Generating Perturbation Vectors For Topography Optimization of Structures

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1. Abstract

Methods to automatically generate perturbation vectors for topography optimization of structures are presented. The perturbation vectors are created so that grids of designable regions, typically modeled with shell or composite finite elements, can move either normal to their original locations or in a specified direction. Manufacturing requirements such a minimum sizes of bead patterns, maximum heights and transitional distances between designable grids and non-designable grids are considered.

2. Keywords: Topography Optimization, Shape Optimization, Perturbation Vectors

3. Introduction

Topography optimization is an optimization technique that allows improving the curvature of structures that are typically, but not necessarily, assembled with shell or composite elements. Topography optimization can be treated as a special type of shape optimization (grid location optimization). A popular way to implement shape optimization is by using perturbation vectors. A perturbation vector is a vector that points where the grids associated to it would move if its corresponding design variable is 1.0 and all rest of the shape design variables are 0.0. In this type of shape optimization implementation the optimizer searches for the best solution by searching for the best linear combination of perturbation vectors scaled by their corresponding design variables. In topography optimization, grids of the designable region are allowed to move either normal to the shell or composite elements or in a specified direction. An important application of topography optimization is bead pattern optimization to increase the stiffness of shell structures. Topography optimization implemented as a special case of shape optimization requires the creation of specialized perturbation vectors and their associated design variables. These specialized perturbation vectors are named here topography perturbation vectors.

There are several ways to create topography perturbation vectors, for instance CJ Chen from Visteon Corporation developed a method [1] in which every other element in the designable region is allowed to move perpendicular to its original position. Brian Voth, form Altair Engineering, has also developed a method to generate topography perturbation vectors. His method and perhaps enhancements to it are used to generate shape optimization data for the Optistruct software; unfortunately Voth has not published the methods he developed, but results of them can be seen in Optistruct brochures. The methods presented here were developed for the structural optimization program GENESIS [2] and they were implemented so that they can be used with other existing shape or sizing optimization capability.

4. Procedure To Generate Topography Perturbation Vectors

The proposed procedure to generate topography petrubation vectors requires three basic steps. These steps are explained next.

4.1 Surface preparation

This step consists in reordering the nodes of all the elements in the topography region so that their associated norms are consistent with the neighbor norms. This step is only for internal calculations; the elements themselves are not changed.

4.2 Normal direction calculation

This step consists of calculating the norms associated to each grid on the topographically designable region. The grid norms are calculated as a weighted average of the norm of all elements that are connected to the grid. This step is optional because occasionally the perturbation vectors can be constructed using a predefined direction instead of the normal direction.

4.3 Perturbation calculation

This step consists of calculating the perturbation vectors. These vectors are calculated using the norms calculated on step 2 and parameters that identify a desirable basic shape. The created perturbation vectors can optionally reference simultaneously multiple grids for improving manufacturability and efficiency. This paper will focus in this step.

5. Grid Location Update Equations

The basic equations used in shape optimization to internally calculate the updated grid locations [3] are:

$$\left.\begin{aligned} X_i &= X0_i + \sum_j DV_j * XP_{ij} \\ Y_i &= Y0_i + \sum_j DV_j * YP_{ij} \\ Z_i &= Z0_i + \sum_j DV_j * ZP_{ij} \end{aligned}\right\} \quad (1)$$

where X_i, Y_i and Z_i are the updated coordinates of the grid *i*. $X0_i$, $Y0_i$ and $Z0_i$ are the initial coordinates of the grid *i*. XP_{ij}, YP_{ij} and ZP_{ij} are the components of the j^{th} perturbation vector corresponding to grid *i*. Finally, DV_j contains the value of the design variable *j*.

6. Specialization of Grid Location Update Equations

In this work, the grid location update equations are specialized for topography optimization. The specialization start by allowing one design variable per designable grid and make all perturbations that affect each grid to point on the normal direction at the grid, as shown next:

$$\left.\begin{aligned} X_i &= X0_i + nxi * \sum_j T_{ij} * DV_j \\ Y_i &= Y0_i + nyi * \sum_j T_{ij} * DV_j \\ Z_i &= Z0_i + nyi * \sum_j T_{ij} * DV_j \end{aligned}\right\} \tag{2}$$

where nxi, nyi and nzi are the components of the normal vector of the surface at grid i. Tij is the magnitude of the perturbation at grid i associated to design variable j.

Defining {G}, {G0}, [N], [T] and {DV} as follows:

$$\{G\}=\begin{Bmatrix} x_1 \\ y_1 \\ z_1 \\ x_2 \\ y_2 \\ z_2 \\ \cdot \\ \cdot \\ \cdot \\ x_i \\ y_i \\ z_i \\ \cdot \\ \cdot \\ \cdot \\ x_n \\ y_n \\ z_n \end{Bmatrix},\{Go\}=\begin{Bmatrix} xo_1 \\ yo_1 \\ zo_1 \\ xo_2 \\ yo_2 \\ zo_2 \\ \cdot \\ \cdot \\ \cdot \\ xo_i \\ yo_i \\ zo_i \\ \cdot \\ \cdot \\ \cdot \\ xo_n \\ yo_n \\ zo_n \end{Bmatrix},\left[N\right]=\begin{bmatrix} nx_1 & & & & & \\ ny_1 & & & & & \\ nz_1 & & & & & \\ & nx_2 & & & & \\ & ny_2 & & & & \\ & nz_2 & & & & \\ & & \cdot & & & \\ & & \cdot & & & \\ & & \cdot & & & \\ & & & nx_i & & \\ & & & ny_i & & \\ & & & nz_i & & \\ & & & & \cdot & \\ & & & & \cdot & \\ & & & & \cdot & \\ & & & & & nx_n \\ & & & & & ny_n \\ & & & & & nz_n \end{bmatrix},[T]=\begin{bmatrix} T_{11} & T_{12} & \cdot & T_{1i} & \cdot & T_{1n} \\ T_{21} & T_{22} & \cdot & T_{2i} & \cdot & T_{2n} \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ T_{i1} & T_{i2} & \cdot & T_{ii} & \cdot & T_{in} \\ & & \cdot & & \cdot & \\ T_{n1} & T_{n2} & \cdot & T_{ni} & \cdot & T_{nn} \end{bmatrix},\{DV\}=\begin{Bmatrix} DV_1 \\ DV_2 \\ \cdot \\ DV_i \\ \cdot \\ DV_n \end{Bmatrix}$$

and using Eq. (2) yields to:

$$\{G\} = \{G0\} + [N][T]\{DV\} \tag{3}$$

For simplicity, the [N] and [T] matrices will be referred as the normal matrix and the topography matrix, respectively. The multiplication of the normal matrix and the topography matrix produces the $[P_T]$ matrix. In this paper, this matrix will be named the basic topography perturbation matrix. The columns of $[P_T]$ are the basic topography perturbation vectors.

$$[P_T] = [N][T] \tag{4}$$

Combining Eqs. (3) and (4) yields the following equation:

$$\{G\} = \{G0\} + [P_T]\{DV\} \tag{5}$$

Because the normal matrix is known, from Step 2, the calculation of the basic topography perturbation matrix is reduced to calculate the topography matrix. Two methods are presented next to generate the topography matrix. The first method focuses on building the columns of the matrix and will be named here the basic method. The second method will focus on building the rows of the topography matrix and will be named here the link method.

7. Basic Method (Perturbation Based Method)

To create a perturbation vector j, say {Pj}, it is only needed to create a perturbation pattern. That perturbation pattern can be stored in column j of the topography matrix. The same perturbation pattern can be repeated to other grids to create all perturbations.

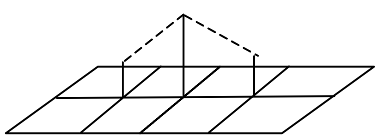

Figure 1a. Perturbation Pattern

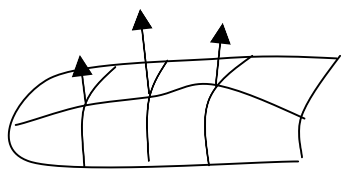

Figure 1b. Topography Perturbation Vector

The simplest pattern that can be defined is $T_{ij} = \delta_{ij}$, where δ_{ij} is Kronecker delta. This pattern will make the topography matrix to be identical to be the identity matrix. This matrix works well on some problems, but in others it could produce distorted meshes. To avoid this, it is possible to create perturbation patterns that span multiple grids. A simple pattern that achieves that is the following:

$$Tij = \begin{cases} (1 - Dij / D) * Hj & \text{if } Dij \leq D \\ 0.0 & \text{if } Dij > D \end{cases} \tag{6}$$

In the above equation Dij is the distance between grids i and j; D is a predefined influence distance and Hj is a scale factor that represents the magnitude of the perturbation for i=j. Another useful pattern is the following:

$$Tij = \begin{cases} Hj & \text{if } Dij \leq D1 \\ (1 - (Dij - D1)/(D2 - D1)) * Hj & \text{if } D1 < Dij \leq D2 \\ 0.0 & \text{if } Dij > D2 \end{cases} \tag{7}$$

In the above equation Dij is the distance between grids i and j; D1 is a predefined influence distance were the perturbation is keep constant, D2 is a influence distance and Hj is a scale factor that represents the magnitude of the perturbation for i=j.

8. Link Method (Grid Based Method)

By defining a "dependent" design variable DDVi as:

$$DDVi = Ti1*DV1 + Ti2*DV2 + \ldots + Tii*Dvi + \ldots + Tin*DVn \tag{8}$$

and by defining {DDV} as the vector that contains all dependent design variables. The following equation can be written:

$$\{DDV\} = [T]\{DV\} \tag{9}$$

In this case equations (3) can be re-written as:

$$\{G\} = \{G0\} + [N]\{DDV\} \tag{10}$$

By working with the dependant design variables DDVi, the Tij terms can be seeing as weighting factors of the independent design variables. This weighting factors should be built to act as filters that help avoiding mesh distortions. One possible set is presented next:

$$Tij = \begin{cases} (1 - Dij / D) * Hi & \text{if } Dij \leq D \\ 0.0 & \text{if } Dij > D \end{cases} \tag{11}$$

where Dij is the distance between grids i and j, D is a predefined influence distance and Hi is scale factor that affect all terms of the row associated to grid i. Another useful set of weighting factors is the following:

$$Tij = \begin{cases} Hi & \text{if } Dij \leq D1 \\ (1 - (Dij - D1)/(D2 - D1)) * Hi & \text{if } D1 < Dij \leq D2 \\ 0.0 & \text{if } Dij > D \end{cases} \tag{12}$$

In the above equation Dij is the distance between grids i and j; D1 is a predefined influence distance where all variables that are close enough are given the same weighting factors and D2 is a predefined influence distance to reduce to zero the influence of the design variable on grids that are far away. Hi is a scale factor that affects all design variables associated to grid i.

It should be mentioned here that although Eqs. (6) and (7) look similar to Eqs. (11) and (12) they are not. The scale factor in the first two equations affects the columns of the topography matrix while the scale factors in Eqs. (11) and (12) affect the rows. That difference turns out to be important for finding a procedure for properly scaling the perturbation vectors.

9. Topography Matrix Condensation

By manipulating the rows and/or columns of the basic topography perturbation matrix and/or the topography matrix several useful results can be obtained. For example, to enforce that design variables i and j always get the same value one can add columns i and j of the topography matrix and locate the resulting column in i and eliminating column j (grouping). By making two rows of the basic topography perturbation matrix be the same, one can enforce that the two grid move in the same magnitude. Grouping can be used to reduce the number of design variable and/or to produce minimum size control. Grouping along with row manipulations can be used to enforce different types of symmetries. Symmetries however, can be better enforced as a second separate procedure, a procedure that repeats the rows of the master grid in the slave grids (symmetric grids).

10. Manufacturing Considerations

10.1 Maximum Grid Movements

For manufacturing reasons very often grids cannot be allowed to move as much as the optimizer would attempt to move them. So is important to find ways to limit how much the grids can move. For the Link method, a simple procedure is used; it involves scaling the rows of [T] using the following expression:

$$Hi = H\max * \sum_{j} \frac{1}{Tij} \tag{13}$$

where Hmax is the maximum distance that the grids are allowed to move in either the normal directions or in an alternative user predefined direction.

For the basic method the procedure is not as easy and not always is possible to find a set of scalars that produce the desired effect. For the cases where that is possible, the procedure involves solving a system of equations that could be costly. Results using Eq. (13) for two different maximum heights are shown next:

Figure 2a. Small Maximum Height

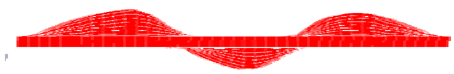

Figure 2b. Large Maximum Height

10.2 Minimum Bead Pattern Dimensions

Topography optimization can be utilized to design bead pattern on metal sheets. Often, to manufacture this type of structures it is required that bead patterns maintain a certain minimum size, so it is important to find ways to control the minimum size of the optimized results. Minimum size can be achieved by using different D1 values in equations (7) or (12) and by grouping variables that are close to each other (for example Dij<D1). The following figures show some examples using different D1 values in Eq. (12):

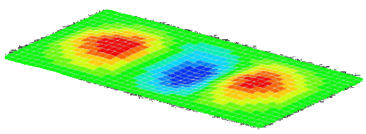

Figure 3a. Large Minimum bead size

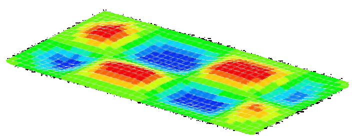

Figure 3b. Small Minimum bead size

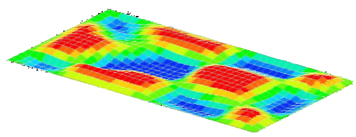

Figure 3a. Smaller Minimum bead size

The results shown in Figs. 3a, 3b and 3c were obtained using 12, 55 and 629 design variables respectively.

10.3 Transition distances

To get smooth results between the designable grids and the non-designable ones is important for manufacturing and for mesh quality. Smoothing can be optionally achieved by eliminating the perturbation vectors that directly design these grids and keeping the row that designs these grids so they can move. The transition zones can be further smoothed by using the terms D2 in (7) or (12) with D2>D1 being (D2-D1) the transition zone. As a result of this the transition grids will be able to move but not fully.

11. Final Step on Building the Topography Matrix

The eliminations of row and/or the repetition of columns discussed above can be symbolically written using two matrices: [R] and [C]. If we call [R] a 3m*n matrix with m<=n and [C] a n*q matrix with q<=n, the final topography perturbation matrix can be written as:

$$[P_T]' = [R][N][T][C] \tag{14}$$

Eq. (14) represents the condensed basic topography perturbation matrix. This matrix contains the final perturbation vectors. This matrix is the topography perturbation matrix. This matrix affect m sets of grids using q design variables. This matrix in practice is not constructed using the expression above because that would involve many unnecessary operations, like multiplying by zero. Instead, the final perturbations are constructed and kept inside the program in a sparse matrix format.

12. Examples

Two optimization examples are presented to illustrate the use of topography optimization with the proposed approach. Both problems use an 18x40 mm^2 plate. The material properties of the plate are E= 207,000 N/mm^2 and ν= 0.3. The plate is modeled using a 4662 degrees of freedom finite element mesh that contains 779 grids and 720 quadrilateral elements. The two examples show extreme design options. The first example shows two cases where there are as many design variables as designable grids. The second example shows a case were all grid in each of six topography regions are designed by one independent design variable.

12.1 Example 1

The first example aims at maximizing the torsional rigidity of the plate. The thickness of the plate is 1.0 mm. The corner grids of the tip are loaded with vertical loads of opposite directions of 1.00 N each that produce an overall torsion load of 18.00 Nmm. For manufacturing reasons, none of the edges of the plates are allowed to change and the maximum grid change in the vertical direction is 1.00 mm. Two cases are studied. In case A, the grids are only allowed to move in the positive direction of the norm. In case B, the grids are allowed to move in both directions. The optimization problem is to minimize the strain energy with a volume constraint of 735mm^3 (2% above the initial volume of 720 mm^3). For both cases, one topography region that contains all grids is used.

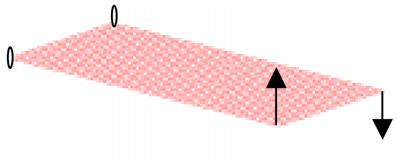

Figure 4. Initial Design

12.2 Results for Example 1

In both cases, 663 grid perturbation vectors and 663 independent design variables were automatically generated. In case A, the initial strain energy was reduced from 1.331E-2 Nmm to 9.963E-3 Nmm (25.1% improvement). On case B, the strain energy was reduced to 8.491E-3 Nmm (36.2% improvement). In both cases, the volume constraint was active (735mm^3). Figs. 5a and 5b show the optimized configurations for the two cases. In both cases, beads patterns following +/-45 degrees directions were obtained. These patterns seem reasonable for increasing torsional stiffness.

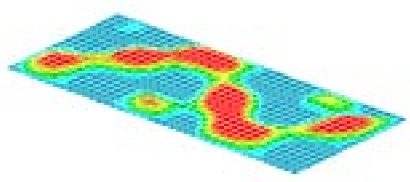

Figure 5a. Topography Optimization Results for Case A

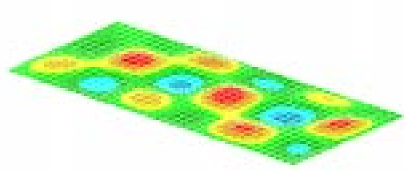

Figure 5b. Topography Optimization Results for Case B

12.3 Example 2

The second example aims at maximizing the bending rigidity of the plate. The thickness of the plate is 0.6 mm. The tip edge is loaded with evenly distributed vertical load of 2.0 N/mm (36.0 N). The locations of the grids on the larger edges of the plates are not allowed to change, whereas the locations of the grid of the short edges are. The Optimization problem is to minimize the strain energy with a volume constraint of 511mm^3 (18% above initial volume of 432 mm^3). In this case, six topography regions were used.

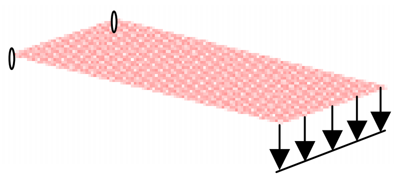

Figure 6. Initial Design

12.4 Results for Example 2

Six topography regions (strip of elements along the longer direction) produced 420 grid perturbation vectors with corresponding 6 independent design variables. In the final design, 4 design variables took a positive value and the rest took values close to zero. Fig. 7 shows the final optimized configuration. The initial strain energy was reduced from 495.2 N-mm to 299.4 N-mm (39.5% improvement). In the final design, the volume constraint was active (volume increased from 432 mm^3 to 511 mm^3, an 18% increase).

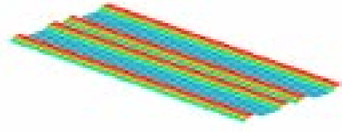

Figure 7. Topography Optimization Results for Example 2

13. Practical Considerations

Once the topography perturbation vectors are built and used to solve an optimization problem two problems could arise. The first one is that the design variable may not move because initial sensitivities could be zero. The first problem occurs naturally in flat plates where moving the variables to a positive or negative direction is the same. To solve that problem is easy: instead of setting the initial value of all design variables to zero they are set to small random value. This has the minor weakness that the initial design is not the same as the original design, which is usually preferred in standard shape optimization problems. The second problem is mesh distortion. Although using perturbation patterns such the one in Eq. (8) or filters such as the ones in Eq. (12) help reducing mesh distortion they are not capable of completely eliminate the problem and when this problem occurs it is not easy to fix it without a smoothing algorithm. The only easy fix is to put a limit on how much the grids move, that could be achieved using a smaller maximum height constraint (Hmax) or reducing the bounds of the design variables.

14. Numerical Considerations

Of the two methods presented, the basic and the link, the link method turned out to be better because it can deal in a general way with the common requirement of maximum height constraints. For problems where maximum height is not a requirement the basic methods work as well. It is interesting to mention here that topography optimization typically takes about 7 to 15 design cycles to converge. This efficiency however comes mostly from the approximate problem already built in the GENESIS program.

15. Conclusions

A general procedure to automatically generate topography perturbation vectors for shape optimization of structures has been presented. Manufacturing requirements such a minimum sizes of bead patterns, maximum heights and transitional distances between designable grids and non-designable grids were considered. Using automatically created perturbation vectors simplifies the shape optimization process and better and more innovative designs can be found.

16. Acknowledgements

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17. References

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